WAKEFIELDS IN THE TRACE 3-D CODE

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Abstract

TRACE 3-D is an interactive code that calculates and displays the envelopes of a bunched beam through a user-defined transport system. Accelerating elements and linear space-charge forces are included. The beam is described by a 6-D sigma matrix of second moments. We have extended the capabilities of this code to include effects, such as wakefields, related to the variation of the beam bunch in the longitudinal direction. This nonlinear capability was implemented by adding centroid tracking and describing the beam by a collection of slices, each described by a 6-D centroid and sigma matrix. External forces, space-charge forces, and wakefields act on the collection of beam slices. Results are presented in terms of an overall sigma matrix, computed by combining the slice distributions. The new TRACE 3-D has been integrated with an improved graphic user interface (GUI) based on the Shell for Particle Accelerator Related Codes. This new approach to modeling wakefields demonstrates the flexibility of extending the capabilities of moment codes to handle important physical effects, and the rapid incorporation of the new capabilities into the graphic interface illustrates the ease of customizing the new GUI. The wakefield model and features of the new interface are presented.

Introduction

The TRACE 3-D program [1] is one of the standard codes used in the design of linear accelerators and transport lines. A new version of TRACE 3-D has been developed that computes the short-time (single bunch) wakefield effects that can alter the bunch distribution. In order to model the effects of wakefields, we imagine the beam bunch to be divided into a number of slices longitudinally. Each slice of the bunch is then described by its own 6-D centroid and 6×6 sigma matrix. The effects of wakefields are modeled using a multipole expansion of the forces which act on the beam as a function of the longitudinal position within the bunch. Monopole, dipole and quadrupole terms (wake functions) are included. These terms can be expressed as transfer (R) matrices which act on the centroids and sigma matrices of each slice. The approach has been outlined by Chan [2], and follows the development of Chao and Cooper [3] for the code LTRACK, but we do not assume that the beam is traveling at the speed of light.

To implement this model in TRACE 3-D, new wakefield "optical" elements have been developed that are used to describe the monopole, dipole and quadrupole wakefield functions [2]. These are currently modeled as a second-degree polynomial in the longitudinal position, although other parameterizations could be readily implemented. One of the new wakefield elements is then placed in the beamline after each physical element (e.g. misalignment or RF gap) that is responsible for a wakefield.

The wakefield version of TRACE 3-D has been integrated with a GUI designed specifically to support particle beam simulation and analysis programs. Known as the Shell for Particle Accelerator Related Codes (S.P.A.R.C.), this GUI provides a unique software environment customized to the needs of the accelerator community [4]. Earlier versions of TRACE 3-D have been integrated with the S.P.A.R.C. GUI for several years [5]. New capabilities have been added to S.P.A.R.C. that are aimed at improving the customization of the GUI to meet the differing needs of users. These new features were utilized to create a prototype GUI for use with the new TRACE 3-D.

Overview of the Beam Model

The initial beam bunch is assumed to be a uniformly filled, upright ellipsoid in (x,y,z) with

$$- z_{max} \le z \le z_{max} \quad . \tag{1}$$

The bunch is divided into $2N$+1 equal-length slices, labeled from -N (head) to +N (tail). Slice number 0 is centered at z=0. Let z_i be the z value at the upstream face of slice i and define z_{N+1} to be $-z_{max}$. Since the number of particles, dn, in a slice of length dz is proportional to the square of the distance from the bunch center. Introducing the variable $\zeta=(z / z_{max})$ one has

$$dn \sim [1 - (\zeta)^2]\, dz \quad . \tag{2}$$

With this distribution, the z-centroid of slice i is given by

$$\langle z\rangle_i=[6\zeta_i^2-3\zeta_i^4-6\zeta_{i+1}^2+3\zeta_{i+1}^4]/[12\zeta_i^2-4\zeta_i^4-12\zeta_{i+1}^2+4\zeta_{i+1}^4]z_{max}. \tag{3}$$

The fraction of particles in slice i is given by

$$n_i = [3\zeta_i - \zeta_i^3 - 3\zeta_{i+1} + \zeta_{i+1}^3] / 4 \quad . \tag{4}$$

To estimate the z'-centroid of each slice, it is assumed that the ratio of the centroid values $\langle z'\rangle_i / \langle z\rangle_i$ is the same as z'_e/z_m, where z_m, is the maximum value of z for the z-z' ellipse and z'_e is the value of z' at $z = z_m$. From the definition of the Twiss (Courant-Snyder) parameters for the z-z' ellipse, then

$$\langle z'\rangle_i = - [\alpha_z / \beta_z] \langle z\rangle_i \quad . \tag{5}$$

The transverse emittances, ε_x and ε_y, are adjusted at each slice according to

$$\varepsilon_{x,y,i} = [1 - (\langle z\rangle_i / z_{max})^2]\,(\varepsilon_{x,y} / f) \quad , \tag{6}$$

where the factor f is the ratio of the average to maximum emittance value

$$f = \Sigma_i\,[1 - (\langle z\rangle_i / z_{max})^2]\,n_i \quad . \tag{7}$$

The values of σ_{55} and σ_{66} for the i-th slice are estimated from

$$\sigma_{55i} = (\Delta z / 2)^2 \quad , \tag{8}$$
$$\sigma_{66i} = [(\varepsilon_z / \beta_z) - (\langle z\rangle_i / \beta_z)^2]/e \quad , \tag{9}$$

where $\Delta z = 2z_{max}/(2N+1)$, and e is ratio of the average to maximum σ_{66} value

$$e = (\beta_z / \varepsilon_z)\,\Sigma_i\,\left[(\varepsilon_z / \beta_z) - (\langle z\rangle_i / \beta_z)^2\right] \quad . \tag{10}$$

The centroids and sigma matrices for each slice are transformed through the beamline using the usual transfer matrix formalism. Each TRACE 3-D optical element [1] is described by a 6×6 R-matrix, $R(\Delta s)$, that transforms the beam over a distance Δs in the element according to

$$\langle \boldsymbol{X}(s+\Delta s)\rangle_i = R(\Delta s)\,\langle \boldsymbol{X}(s)\rangle_i \quad , \tag{11}$$
$$\boldsymbol{\sigma}(s+\Delta s)_i = R(\Delta s)\,\boldsymbol{\sigma}(s)_i\,R(\Delta s)^{\mathrm{T}} \quad , \tag{12}$$

where $\langle \boldsymbol{X}(s)\rangle_i$ and $\boldsymbol{\sigma}(s)_i$ are the 6-D centroid and 6×6 sigma matrix for the beam slice at position s. Existing TRACE 3-D subroutines are used to compute the R-matrix elements for the standard optical elements, but new subroutines have been written to carry out the transformations described by (11) and (12). As described in the next section, new subroutines for modeling the wakefield optics have also been written.

The longitudinal slices for the beam bunch are recombined to compute effective bunch centroids and sigma matrix elements. The overall bunch centroid is given by

$$\langle \boldsymbol{X}\rangle = \Sigma_i\,n_i\,\langle \boldsymbol{X}\rangle_i \quad . \tag{13}$$

The individual elements of the overall bunch sigma matrix are given by

$$\sigma_{ij} = \Sigma_k n_k[\sigma_{ijk} + 5\langle u_i\rangle_k\langle u_j\rangle_k] - 5[\Sigma_k n_k\langle u_i\rangle_k][\Sigma_k n_k\langle u_j\rangle_k] \quad , \tag{14}$$

where u_i represents (x,x',y,y',z,z') for i=1,6 and

$$\sigma_{ijk} / 5 = \langle u_i\,u_j\rangle_k - \langle u_i\rangle_k\,\langle u_j\rangle_k \quad , \tag{15}$$

is the sigma matrix for the k-th slice. The individual slice centroids and overall sigma matrix are used to compute space charge effects with a modified space charge model that takes into account the effective force on each slice centroid. The overall bunch centroids and overall sigma matrix are utilized for generating graphic output displays of the beam envelopes and centroid locations.

Wakefield Optical Elements

Three new "optical elements" have been added to TRACE 3-D to model the monopole, dipole and quadrupole wakefield functions. These elements are inserted into a beamline model immediately after each element that is responsible for generating a wakefield. The monopole wakefield changes the energies of the bunch slices, the dipole wakefield causes deflections of the transverse centroids of the slices, while the quadrupole wakefield effects the sigma matrices of the bunch slices in addition to the energy and transverse centroids. The three wakefield multipoles are expressed in terms of wake function strengths per unit length, $W_0(s)$, $W_1(s)$, and $W_2(s)$. Each wakefield acts on a bunch over the length, L, of the element responsible for generating the wakefield. The product of this length and the multipole strengths are used for computing the wakefield effects [6] and are modeled as second degree polynomials:

$$LW_0(s) = p_0(1) + p_0(2)s + p_0(3)s^2 \quad , \tag{16}$$
$$LW_1(s) = p_1(1) + p_1(2)s + p_1(3)s^2 \quad , \tag{17}$$
$$LW_2(s) = p_2(1) + p_2(2)s + p_2(3)s^2 \quad . \tag{18}$$

The three coefficients for a given multipole, $p_m(1)$, $p_m(2)$ and $p_m(3)$, are user inputs for the corresponding wakefield optical element.

The effects on the energy and centroid of the k-th slice are given by [6]:

$$\Delta E_k = -\Sigma_{i<k}\,n_i\,LW_0(\langle z\rangle_i - \langle z\rangle_k) \quad , \tag{19}$$
$$\Delta\langle x'\rangle_k = \mathrm{C}_k\,\Sigma_{i<k}\,n_i\,LW_1(\langle z\rangle_i - \langle z\rangle_k)\,\langle x\rangle_i \quad , \tag{20}$$

with an expression similar to (20) for $\Delta\langle y'\rangle_k$. The sigma matrix for the k-th slice is transformed with a R-matrix, with elements that differ from the identity matrix given by [2]:

$$R_{21} = -R_{43} = q_1 \quad , \tag{21}$$
$$R_{23} = R_{41} = q_2 \quad , \tag{22}$$

with

$$q_1 = \mathrm{C}_k\,\Sigma_{i<k}\,n_i\,LW_2(\langle z\rangle_i - \langle z\rangle_k)[\sigma_{11i} - \sigma_{33i} + \langle x^2\rangle_i - \langle y^2\rangle_i] \quad , \tag{23}$$
$$q_2 = \mathrm{C}_k\,\Sigma_{i<k}\,n_i\,LW_2(\langle z\rangle_i - \langle z\rangle_k)[2\sigma_{13i} + 2\langle x\rangle_i\,\langle y\rangle_i] \quad . \tag{24}$$

The q_1 and q_2 terms correspond to normal and skew quadrupole moments, respectively. The coefficient C_k appearing in (20), (23) and (24) is a function of the relativistic energy factor of the k-th slice, γ_k, and is given by

$$\mathrm{C}_k = r_e\,(m_e/M)/\gamma_k \quad , \tag{25}$$

where r_e is the classical radius of the electron, m_e is the electron mass and M is the particle mass.

Integration with the GUI

User defined optics elements, such as the wakefield elements described above, may be easily integrated into the S.P.A.R.C. GUI for TRACE 3-D using a new TableBuilder application. The TableBuilder is used to create customized data input windows called Piece Windows [5]. Custom Piece Windows for user defined elements provide the same functionally as other Piece Windows, including options for the choices of parameter units, including several "smart units" options, and lower and upper user guidance limits. The guidance limits are soft, that is, any parameter value may always be entered. The limits are utilized to provide the user with a visual alert when his or her input value may have impractical consequences.

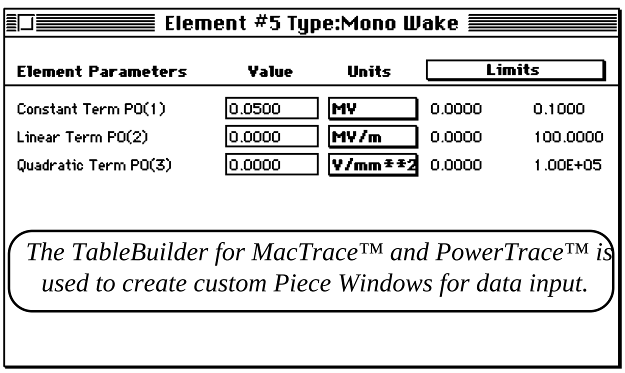

Figure 1. Example of custom Piece Window for a wakefield element, created using the TableBuilder application.

Once a custom Piece Window such as that shown in Figure 1 has been generated, the graphic construction of beamlines that include the user defined elements is the same as for beamlines with any other optical elements [4,5]. The setting up of arrays and other input for TRACE 3-D is accomplished by the GUI and is transparent to the user.

Several other improvements to the GUI have also been implemented and a few more are under development in order to fully support the new TRACE 3-D capabilities. Several additional smart units options have been added to the Global Parameters [5]. For example the Beam Energy may be input in terms of the relativistic velocity (β), relativistic energy (γ), or particle momentum (in GeV/c), in addition to eV, keV, MeV or GeV. The radiofrequency may be entered as either a frequency or a wavelength, with several options for each. The S.P.A.R.C. expert rule system [4,5] provides all conversions and gives users feedback in any of the available units options for his input.

Other Enhancements

A few other optical elements have been added to TRACE 3-D as part of this work, and some additional parameters have been added to existing elements to support misalignment modeling. In particular, the rotate element has been modified so that it can model either rotations (yaw and pitch, as well as roll) or displacements of the beam axis. Roll and displacement parameters, including an option to generate random values, have been added to the quadrupole. An electrostatic quadrupole has been added to the program. We also note that together with a suite of other electrostatic elements (prisms, einzel lenses and accelerator tubes) developed as part of other work, versions of TRACE 3-D are available for studying a broad spectrum of bunched and continuous beam accelerator systems.

Summary

A new version of the TRACE 3-D code has been developed for modeling wakefield effects and similar phenomena related to variations of a beam bunch in the longitudinal direction. A number of new optical elements have been added to support the modeling of wakefields and misalignments. The new version of TRACE 3-D has been integrated with an enhanced version the S.P.A.R.C. GUI that allows users to customize the integrated TRACE 3-D / GUI program to meet individual needs.

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