A SIMPLE MODEL OF ION EXTRACTION FROM A PLASMA SOURCE

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Abstract
A simple model of ion extraction from a plasma source has been developed. Based on a non-relativistic paraxial approximation, the model takes into account the finite ion temperature of the plasma, the focusing at the plasma meniscus, the space charge defocusing, the lens effects due to the apertures and the influence of an axial magnetic field. A personal computer has been used to implement the model for a triode extraction system followed by a drift space. (Space charge neutralization in the drift space has been taken into consideration.) The model has been verified by a comparison with experimental data. It should prove especially useful in the development of ion sources with a large axial magnetic field and, in particular, ECR ion sources.

Introduction
High-current, high-brightness ion sources are now used for everything from materials modification by ion implantation to neutral-beam heating of fusion plasmas, in addition to the traditional accelerator applications. The increased use of these devices has revived interest in developing an understanding of the factors that influence the extraction of ions from a plasma. Computer codes that address the problem by simultaneously solving the Poisson equation for the electrostatic potential and the Vlasov equation for the ion density have reached maturity during the last decade. However, they still have two principal drawbacks. Firstly, a lot of storage and plenty of CPU time on a mainframe computer are required to generate meaningful results. Secondly, and probably more significantly, the user is deprived of the physical insight that can often be gained from a simple model. Specifically, the representation of the plasma sheath by a space-charge limited spherical diode, coupled to a solution of the paraxial equation, has proven relatively successful. The present paper reports a simplified, though equally effective, version of this approach with an extension to include the influence of an axial magnetic field.

The Model
The non-relativistic paraxial equation for a cylindrically symmetric ion beam of radius, \( r \), with a charge-to-mass ratio, \( \eta \), and a current, \( I \), propagating in an electric potential, \( \Phi \), and a magnetic induction, \( B \), is given by

\[
2r'\Phi' + r'\Phi'' + \left( \frac{\Phi'' - \eta B_0^2 r_0^4}{4r} \right) - \frac{\eta B_0^2 r_0^4}{4r^3} - \frac{I}{2\eta \epsilon_0 (2\eta \Phi)^{3/2}r} = 0
\]  

(1)

where \( B_0 \) is the magnetic induction at the initial radius, \( r_0 \), and \( \epsilon_0 \) is the permittivity of free space.

Integrating using Ruler's method, the second derivatives at the axial displacement, \( z_i \), are approximated by

\[
r_i'' = \frac{r_{i+1}' - r_i'}{\Delta z_i}
\]  

(2)

and

\[
\frac{\Phi_{i+1}' - \Phi_i'}{\Delta z_i} = \frac{\phi_{i+1}''}{\delta z_i}
\]  

(3)

where

\[
\Delta z_i = z_{i+1} - z_i.
\]

Substituting and solving for the beam divergence, \( r_{i+1}' \), at the axial displacement, \( z_{i+1} \), yields

\[
r_{i+1}' = r_i'\left( 1 - \frac{\Phi_i';\Delta z_i}{2\Phi_i} \right) - \frac{\phi_i'' + \eta B_0^2 \Delta z_i}{4\Phi_i} + \frac{I\Delta z_i}{8\Phi_i^3} + \frac{\eta B_0^2 r_0^4 \Delta z_i}{4\eta \epsilon_0 (2\eta \Phi_i)^{3/2} r_i} = 0
\]  

(4)

The beam radius, \( r_{i+1} \), at \( z_{i+1} \) can then be deduced from

\[
r_{i+1} = r_i + r_i'\Delta z_i.
\]  

(5)

Equations (4) and (5) can be solved iteratively for the beam divergence and radius at any axial displacement, provided that the initial values are defined. Obviously, the beginning radius, \( r_2 \), is the radius of the aperture in the electrode at the beam-plasma interface. The initial divergence, \( r_0' \), is attributed entirely to the shape of the plasma meniscus. Assuming that the interface can be treated as a spherical diode with space-charge-limited flow and that the radius of curvature of the meniscus is large compared to the length of the extraction gap, \( d_i \), then the initial divergence follows from

\[
r_0' = 0.625 \left( \frac{r_0}{d_1 P_0} - 1 \right)
\]  

(6)

where the perveance, \( P \), for an extraction voltage, \( V_1 \), is defined as

\[
P = I/V_1^{3/2}
\]  

(7)

and \( P_0 \), the Child-Langmuir perveance, is given by

\[
P_0 = \frac{4}{9} \frac{r_0^2}{d_1} \epsilon_0 (2\eta)^{1/2}.
\]  

(8)

The electric potential in the extraction gap is assumed to follow the Langmuir distribution for space-charge-limited flow, \( \Phi \propto 1/r^{3/2} \). Elsewhere, the potential is taken to vary linearly with axial displacement. Since the self magnetic field of the beam is insignificant, the induction on the axis can be determined experimentally in the absence of the beam.

Thermal effects are introduced by adding in quadrature the divergence calculated using the paraxial equation and the divergence obtained by conserving the normalized emittance. The divergence attributable to the finite ion temperature, \( T_0 \), is given by

\[
\Delta \theta = \frac{1}{2} \theta_0^2 (T_0/T_1)^{1/2}
\]  

(9)
\[ r' = \frac{r_0 kT^{1/2}}{qV} \]  \hspace{1cm} (9)

where \( k \) is the Boltzmann constant, \( q \) is the charge of the ions that comprise the beam, \( V \) is the net electric potential through which the ions have been accelerated and \( r \) is the radius that the beam has achieved in being accelerated to the energy \( qV \).

The difference equations converge to the exact solution of the paraxial equation in a reasonable number of iterations, if the integration increments, \( \Delta z \), are chosen such that

\[ \Phi_{i+1} - \Phi_i = g \]  \hspace{1cm} (10)

where \( g \) is a constant.

The paraxial equation is undefined for an ion energy of zero. Hence, the beam is assumed to drift from \( z_0 \) to \( z_1 \) where an energy \( q\Phi_1 - \Phi_2 \) is assigned. The results are relatively insensitive to the choice of \( z_1 \), provided that the condition

\[ z_1 - z_0 \approx 0.2d_1 \]  \hspace{1cm} (11)

is satisfied.

A code of the model has been run on a personal computer for a triode extraction column followed by a drift space. Space-charge neutralization in the drift space has been accounted for by multiplying the last (space-charge) term in equation (4) by a neutralization factor. \( \eta \).

Results

Calculations have been made for the extraction configurations studied experimentally by Coupland et al.\textsuperscript{7} The empirical and the theoretical variation of the divergence with relative perveance, \( P/P_0 \), are compared in Fig. 1. An ion temperature corresponding to 4 eV was chosen to best reproduce the minimum divergence. The optimum perveance is well predicted by the model.

Figures 2 and 3 compare calculated and experimental values of the total beam current and the minimum divergence as a function of the extraction voltage. The arc current was adjusted to minimize the divergence for each value of the extraction voltage. Considering that the calculations involve no free parameters (the ion temperature having been chosen to fit the data of Fig. 1), the agreement is remarkable. The slight, though probably significant, deviations between the experimental and the theoretical values of the divergence are probably attributable to aberrations that are not accounted for by the present simple model.

The most novel feature of the present model is the inclusion of magnetic fields. This is particularly significant for electron cyclotron resonance (ECR) ion sources where extraction in high magnetic fields is unavoidable. The minimum divergence was calculated as a function of the magnetic field at the plasma aperture for an ECR ion source being developed at Chalk River. The measured magnetic field extended into the drift space well beyond the extraction system. The space charge was assumed to be totally neutralized in the drift space. The present model is compared in Fig. 4 to an analytical expression developed by Krauss-Vogt et al.\textsuperscript{8} on the assumption that the beam divergence is entirely attributable to the magnetic field and given by

\[ r' = B_0 x_0 [\eta/(8V)]^{1/2}. \]  \hspace{1cm} (12)

The two results are consistent; the discrepancy at low fields is due to the inclusion of the ion temperature contribution in the present model while at high magnetic fields, the two results deviate because the present model includes electrostatic lens and space charge effects. Keeping in mind that the resonance magnetic induction, \( B_{\text{ECR}} \), is given by

\[ B_{\text{ECR}} = 0.036 f_{\text{RF}} \text{ (T/GHz)} \]  \hspace{1cm} (13)
where $f_\text{RF}$ is the RF frequency of the ion source, it is immediately apparent that, at least for the Chalk River ECR ion source configuration, at S-band frequencies (e.g. 2.45 GHz) the minimum divergence is determined by the ion temperature while at X-band frequencies (e.g. 9 GHz) the beam quality is significantly affected by the magnetic field.

Conclusions

A linear representation of ion extraction from a plasma source developed by Kim et al.\textsuperscript{4} and refined by Dietrich\textsuperscript{5} has been simplified and extended to include the influence of an axial magnetic field. The resultant model has reproduced, remarkably well, the experimental measurements of Coupland et al.\textsuperscript{7} for a triode extraction system and has generated results that will prove useful in the design of ECR ion sources.

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References