HIGH CURRENT BEAM TRANSPORT

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1. Introduction

In recent years there has been a resurgence of interest in the problem of transporting high current beams, and in the nature of the limitations imposed by space-charge forces. The motivation has largely come from the heavy ion fusion programme. Recent advances in computer size, and the development of 'particle in cell' codes have facilitated computational studies, and experiments are providing complementary information in this important but difficult field. Space charge forces have two effects: first, they modify (and weaken) the focusing forces, and second, they allow dynamic instabilities of the class known as 'microinstabilities' in plasma physics. These are characterised by the growth of coherent longitudinal or transverse oscillations in the beam. Many classes of phenomenon associated with space-charge forces can be distinguished. In the present paper an attempt is made present some of the essential physical ideas, especially those relevant to the beam transport experiments and computations to be presented at this meeting. These are all concerned with the high current regime, where the behaviour is 'space-charge dominated'. These terms are later defined more precisely, here we merely note that conventional accelerators mainly operate under emittance dominated conditions.

2. The Paraxial Approximation

2.1 The Paraxial Equation and Emittance Concept

The paraxial equation defines particle trajectories in 'perfect' lens and deflection systems without aberration. It represents the approximation known in light optics as 'gaussian optics'. In axially symmetrical magnetic lenses motion in two orthogonal planes through the axis is coupled. This coupling can be removed by specifying co-ordinates in the 'Larmor Frame', which rotates about the axis with a local angular velocity $-qB_z/2m$, where B_z is the component of magnetic field along the axis. The canonical angular momentum in the laboratory frame corresponds to mechanical angular momentum in the Larmor frame, so that, (neglecting thermal velocities) particles drawn from a cathode outside the magnetic field move in a plane in the Larmor frame. In applications of the paraxial equation to accelerator focusing systems there is not in general axial symmetry, but in the absence both of magnetic fields along the orbit and skew quadrupoles, the transverse components motion in the z-x and y-z planes are uncoupled. The fact that the paraxial equation in the Larmor frame is linear and of second order implies the existence certain invariants, and the possibility of using transfer matrices in beam transport calculations.

In practice one is interested in beams of particles, consisting of ensembles of trajectories. These are conveniently characterised in terms of the variation with time of the particle distribution in six dimensional phase-space with co-ordinates x, p_x etc. For a nearly monoenergetic beam in a paraxial system the distribution in the three phase planes x-p_x etc are uncoupled, and can be considered independently. An analytically elegant, though in practice unrealistic, distribution function for the particles in transverse phase space is that of Kapchinsky and Vladimirsky, (the K-V distribution). This is a hollow three-dimensional ellipsoidal shell in the four dimensional phase-space that has the convenient property that all two dimensional projections are elliptical, with uniform density and sharp edges. Expressing the transverse momentum as derivatives of x and y with respect to z, $x' = p_x/p_z = p_x/\beta\gamma m_0 c$, the area of these ellipses in x-x' and y-y' planes are denoted by $\pi\varepsilon_x$ and $\pi\varepsilon_y$ where ε is the emittance. The longitudinal emittance, discussed in section 3, can be similarly defined in terms of the z-p_z phase-space projection.

For non K-V distributions, the rms emittance $\bar{\varepsilon}$, defined as $4(\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2)^{\frac{1}{2}}$ is often convenient. This, like ε, is invariant in paraxial beam transport systems, but not when aberrations or non-linearities are present. For a K-V distribution, $\bar{\varepsilon} = \varepsilon$ as defined above, not the rms value of ε. For K-V beams with finite emittance, the envelope of all trajectories is given by the K-V envelope equation, formed by adding a term to the paraxial equation:

$$a'' + \kappa_x(z)a - \varepsilon_x^2/a^3 = 0$$
$$b'' + \kappa_y(z)b - \varepsilon_y^2/b^3 = 0 \qquad (1)$$

where a and b are the beam radii in the x-z and y-z planes, and κ_x and κ_y are functions of the focusing field. For a uniform focusing channel the orbits are sinusoidal, with wavelength $(2\pi\kappa)^{-\frac{1}{2}}$. For non K-V distributions, the rms emittance can be used, and a and b represent twice the rms radius of the beam.

2.2 Space-Charge, Perveance, and Limiting Currents

One of first applications of high current beams was to microwave tubes. In 1941 Brillouin described the flow pattern now known as 'Brillouin flow', and this strongly influenced beam design[1]. The focusing field is uniform, magnetic, and in the z-direction. The flow is laminar, the particle orbits are helical, and the density profile is uniform. The inward Lorentz $v_\theta \times B_z$ force is balanced by the sum of the outward centrifugal and space charge forces. There is the additional constraint that the canonical angular momentum is zero. This implies that the electrons circulate about the axis at the Larmor frequency. In the Larmor frame the B_z force is transformed to an inward electric force proportional to radius: the particles move in straight lines parallel to the axis, the inward electric field being just balanced by the external space-charge force. The emittance in the Larmor frame is zero.

An important quantity characterizing these beams is the 'perveance', defined in terms of the beam current and (non-relativistic) gun voltage as $k = I/V^{3/2}$. In planar diodes, where the particles move parallel to the electric field, Child's law states that k is constant for a space charge limited cathode. The perveance is, however, a convenient quantity for all beams. If the external fields are suitably scaled, then the flow pattern is preserved and k remains constant. Commonly a 'Pierce gun' is used to form a beam which is then propagated down a 'drift tube', in which focusing is provided.

A fundamental limitation to such flow, discussed by Pierce[1], occurs when the difference of potential, ϕ, between the beam axis and the drift tube is equal to the beam voltage V. When this happens, some electrons are reflected towards the gun, and a 'virtual cathode' is formed. The critical perveance for this effect depends on the geometry: it is of order 50×10^{-6} amps/(volt)$^{3/2}$, (or 50 microperys) for electrons. Typical klystron guns have k of order 1 microperv. The perveance is an important parameter also for beam transport systems, but different authors denote it by different symbols and the term perveance is not yet in general use. A convenient dimensionless form, generalized to relativistic energies and un-neutralised beams may be written in various ways as

$$K = 2Nr_c/\beta^2\gamma^3 = 2I/\beta\,\gamma^2 I_A = \omega_p^2 a^2/2\,\beta^2\gamma^2 c^2 \qquad (2)$$

where r_c is the classical particle radius $q^2/4\pi\varepsilon_0 m_0 c^2$, I_A is the Alfvén current $I_A = (4\pi\varepsilon_0 m_0 c^3)\beta\gamma/q$ and ω_p is the plasma frequency measured in the beam frame, $\omega_p^2 = nq^2/m_0\gamma\varepsilon_0$. For electrons, at non-relativistic energies, (γ =1), K=15000k.

The perveance enables the force tending to disperse the beam to be quantified. For a beam with a K-V distribution it can be included in the envelope equation, and, since the forces in an elliptical charge distribution depend on both axes, the two equations 1 are coupled to give

$$a'' + \kappa_x(z)a - \varepsilon_x^2/a^3 - 2K/(a+b) = 0 \qquad (3)$$

together with a second equation with y instead of x and b and a interchanged. The space charge force varies linearly with radius r to a maximum value at the beam edge. The field gradient itself is proportional to the current and inversely proportional to the square of the beam radius, resulting in a force at the beam edge inversely proportional to the beam radius.

For a given type of focusing system, limiting currents can be found by balancing the forces tending to focus the beam against those tending to disperse it, namely the emittance and perveance terms, both of which have negative sign. The space-charge can readily be seen as representing an outward force; the finite emittance can be interpreted as a force arising from the pressure gradient in the beam. (This can be found quite simply from the appropriate pressure tensor). This balance of forces evidently gives a limiting current much smaller than the basic perveance limitation given earlier. Indeed, in most accelerator situations the difference of potential between the axis and the beam edge is quite negligible compared with the beam energy. Instabilities can further reduce the limiting currents; these topics are discussed later.

2.3 Flow in a Uniform Focusing Channels

It is instructive to examine the role of the parameters by considering some special simple solutions to Eq.3. An axially symmetric beam with a=b is considered first, with uniform focusing such that $\kappa(z)$ is independent of z and equal to $(1/\lambda)^2$. Two very simple and well known solutions represent waists in regions without focusing. The appropriate equations are found by including only the relevant terms in Eq.3. In the absence of space-charge, the emittance dominated waist is given by $a''a^3 = \varepsilon^2$. This is a hyperbola, with asymptotic angle $\Theta_e = \varepsilon/a$. With space charge but no emittance, the space-charge spreading curve is given by $a''a = K$. There is no simple analytical expression for a(z), but the tangential hyperbola at the waist has asymptotic angle $\Theta_s = K^{\frac{1}{2}}$. With both space-charge and emittance, the corresponding angle is given by

$$\Theta^2 = \Theta_s^2 + \Theta_e^2 = K + a^2/\varepsilon^2 \qquad (4)$$

The beam is designated 'emittance' or 'space-charge' dominated according to whether Θ_e is much greater or much less than Θ_s. The ratio of these angles, $K^{\frac{1}{2}}a/\varepsilon$, is a function of the beam radius; in a converging or diverging beam there can be a transition between these two regimes.

Another interesting simple solution is that appropriate to a uniform focusing system, where κ is independent of z. For a uniform beam a" = 0; all the trajectories are sinusoidal. In the absence of space-charge, the wavelength is given by $\lambda_0 = a^2/\varepsilon$. If space charge is present, then $\lambda = (a^2/\varepsilon)(1+Ka^2/\varepsilon^2)^{\frac{1}{2}}$. From these expressions the important result follows that, for a beam of radius a

$$\frac{\lambda^2}{\lambda_0^2} = \frac{\sigma_0^2}{\sigma^2} = 1 + \frac{Ka^2}{\varepsilon^2} = 1 + \frac{\Theta_s^2}{\Theta_e^2} = 1 + \frac{\omega_p^2}{2\gamma^2\omega_b^2} \qquad (5)$$

where σ/σ_0 is the ratio of the phase change of the oscillation over some fixed length of the channel with and without space charge. (For a periodic system, section 2.4, this length is taken as that of a single period). It is interesting to see the variety of forms in which this expression can be written. In the last expression ω_b is the betatron frequency in the absence of space charge. The parameter σ/σ_0 is an important one in beam transport, as will be apparent later. When K=0 (or ω_p=0), σ/σ_0=1. When ε=0, then for finite K, σ =0. This means that the space-charge and focusing forces on a single particle balance, so that the particle moves in a straight line parallel to the axis. This is what happens in the Larmor plane in Brillouin flow, as discussed in section 2.2. Equation 5 is fundamental. It can be written in many ways, perhaps a more familiar one is that of Reiser[2]

$$I = \frac{2\pi\varepsilon_0 m_0 c^3}{q}\,\beta^3\gamma^3\,\frac{\alpha^2}{a^2}\left(1 - \frac{\varepsilon^2}{\alpha^2}\right) \qquad (6)$$

where α is the acceptance of the channel of radius a.

It is convenient here to introduce the concept of matching. In a uniform channel the beam is said to be matched when the distribution function of particles in xx'yy' space is independent of z. Although in general the distribution function varies with z, an infinite number of matched distribution functions in addition to the K-V distribution can be found. This topic is resumed later.

2.4 Flow in Periodic Focusing Channels

Periodic focusing channels are used both in microwave tubes and accelerators. The usual focusing elements are solenoids, magnetic quadrupoles or electric quadrupoles. The phase change of the transverse oscillation experienced by a particle in passing through a single period in the absence and presence of space-charge are denoted by σ_0 and σ respectively. The parameters of the distribution function, in particular the beam radius, are perforce a function of z, and a beam is said to be matched if these parameters are periodic with the lattice period. For such a beam, σ_0 depends on the form of the focusing lattice, K, and the mean beam radius, $\bar{a}$.

If σ_0 is less than about 90°, the radius of a matched beam does not vary much along its length; furthermore, if quadrupole focusing is used the eccentricity of the elliptic beam cross-section is small. Under these circumstances it is found that Eqs. 5 and 6 represent a remarkably good approximation, (the smooth approximation). For larger values of σ_0, perhaps of less interest in practice, a more detailed investigation is needed. This has been carried out by Reiser[2], in his paper a complete analysis, supported by graphs which include form factors for the lattice, is presented.

It is evident from Eq. 6 that within the paraxial theory, the maximum current in a channel of given acceptance is greatest when ε=0 and σ/σ_0=0. Further, there is no limit to the current as the acceptance is increased. In practice, limits would occur either when the channel radius becomes so large that paraxial theory is not valid or, alternatively, when the potential difference between the beam axis and the beam edge becomes comparable to the kinetic energy of the beam particles. From these one can find the more accurate expressions for σ_0 and σ/σ_0 in terms of the lattice parameters and beam radius, and hence find the maximum current for all values of σ_0 and σ/σ_0 for which a matched beam can exist.

2.5 Limitations Arising from Beam Instabilty

The effects considered so far are associated with the steady electrostatic fields from the beam space charge. A high current beam represents a charged fluid, or 'non-neutral plasma' with many degrees of freedom. Plasma oscillation can occur, and waves, both longitudinal and transverse, can be propagated. Regarded as a plasma, the beam has a highly anisotropic velocity distribution. It can behave as an active medium, where energy associated with directed motion is available for feeding instabilities.

We now discuss plasma type instabilities in the space-charge dominated regime for an initially paraxial K-V beam, surrounded by a uniform, smooth conducting tube, so that beam-wall interactions can be neglected. The simplest situation is uniform focusing, achievable with a uniform solenoid. A rigorous analysis of the mode structure in such a beam indicates that, perhaps surprisingly, instabilities do exist when $\sigma/\sigma_0 <$ 0.4.[3] This finding has been verified computationally, details are given in ref.3. These instabilities are mainly of academic interest, since they arise from the singular nature of the K-V distribution, and saturate very rapidly, resulting in a new distribution with little change of rms emittance. Physically, it has been suggested that the instability is in a similar class to the two-stream instability[4]. At any point in the beam the transverse velocity distribution is single valued in amplitude, but isotropic in angle. This amplitude distribution broadens as the instability develops and saturates. An alternative viewpoint is that the hollow shell in 4-dimensional phase-space suffers Rayleigh-Taylor instability, and rearranges itself to form a distribution with density that decreases monotonically from the centre. It is known from thermodynamic arguments that such distributions are stable[4].

In periodic systems other types of instability become possible also. The formal theory, for solenoid and quadrupole channels, using the K-V distribution but not the smooth approximation, has been presented by Hofmann et al[3]. The matched periodic solutions are first found, and the effects on stability of various forms of perturbation are then determined making use of a linearized Vlasov analysis. Matrix elements for propagation of the perturbation are computed, and the usual rules for stability are applied. Physically, the oscillation amplitude of the individual particles can be considered as being driven parametrically by periodicity associated with the focusing structure.

The linearized analysis of ref.3 indicates the presence of instabilities, and enables the initial growth rate to be calculated. It gives no indication, however, of saturation phenomena. Many resonances have been identified, and the stable and unstable regions as a function both of σ_0 and σ/σ_0 are presented in ref.3. From these results a conservative 'working rule' was suggested: $\sigma/\sigma_0 > 0.4$, and $\sigma_0 < 60^o$.

To determine the severity of these resonances, especially with more realistic distributions, computational programmes and beam transport experiments have been undertaken. Present indications are that resonances are less severe than was originally anticipated. In the first place, saturation occurs, resulting in relatively small emittance growth. Secondly, in non-K-V beams the 'effective' σ/σ_0 varies for particles with different oscillation amplitudes. This tends to weaken the coherence of the oscillations and suppresses the instability. (C.f. Landau damping).

The analytical work so far discussed has been restricted to paraxial conditions and K-V distributions. For realistic beams, with non-uniform density profile and (possibly) momentum spread, analytical techniques are too difficult, and recourse must be had to computation or experiment. Computational work forms a bridge between the theory, which only deals with unrealistic distributions, and experiment, which shows what really happens.

Before continuing the discussion, it is profitable to examine some of the general features of non-paraxial beams.

3. Non Paraxial Phenomena

3.1 Uniform Focusing Channels

In the presence of aberrations the elegant simplicity of gaussian optics can no longer be invoked. Transverse motion in orthogonal planes in coupled, and in axial magnetic lenses simple transformation to a Larmor frame is in general no longer possible. The presence of space-charge complicates trajectory calculations, which have do be done in a self-consistent manner.

Despite these difficulties, there are useful approximations that can be made if the non-linearity is not large. We start by reviewing some properties associated with non-paraxial beam transport systems. Probably the simplest of these conceptually is an axially symmetric focusing channel, uniform in the z-direction, but with a focusing force represented by a potential well that is not parabolic, so that the betatron oscillation frequency is a function of the amplitude. Imagine now a parallel beam, with radius a_0, zero emittance, and zero perveance, injected into such a channel. After travelling a distance large compared with $\lambda^2/\Delta\lambda$, where $\Delta\lambda$ is the difference in betatron wavelength for large and small amplitudes of oscillation, thorough 'phase mixing' will have occurred, and the beam will have an emittance of order a^2/λ. From Liouville's theorem the fine-grained phase-space density is still conserved, (equal to zero!), but the coarse grained emittance has 'grown'. It is found also that the rms emittance has increased. This is invariant in linear systems, but can grow when non-linearity is present. We note that in this example the beam radius starts by being oscillatory, but ultimately becomes uniform with radius a_0. Strictly, at a given (large) value of z, there is a correlation between betatron phase and amplitude. In practice, however, the beam is almost indistinguishable from one with small as well as large scale emittance a^2/λ in which all amplitudes and phases are present.

In the presence of space charge the behaviour is similar. An initially oscillatory envelope 'settles down' to one which is independent of z. In general the radial distribution in density is not uniform. The same is true even for a channel with linear focusing if the density distribution is not uniform, since the overall focusing force is no longer linear.

It is of interest to consider matched distributions in a uniform channel that do not have uniform density profile. For a non-uniform beam, σ/σ_0 is not a well defined parameter, since the betatron oscillation wavelength is a function of amplitude furthermore, ω_p is a function of radius. Referring to Eq.5, K is defined, but the radius a needs to be generalized. The rms radius $\langle r^2\rangle^{\frac{1}{2}}$ is a convenient parameter. For a uniform beam, this is just half the actual radius, and so it is convenient to take $2\langle r^2\rangle^{\frac{1}{2}}$ as a, the effective beam radius. Two interesting non-uniform distributions that can be matched to a uniform focusing channel are the water-bag distribution and the thermal distribution. In the water-bag distribution, the phase-space density is uniform within a four dimensional oval volume. In the absence of space-charge this oval is a hyperellipsoid; the radial density distribution in the beam is parabolic. As the space-charge is increased this distribution becomes squarer, becoming uniform as $(1+Ka^2/\varepsilon^2)^{-1}$ varies from unity to zero.

In the thermal distribution, discussed in ref.1 for example, the gaussian transverse velocity distribution can be characterised by a temperature that is independent of radius. In the absence of space-charge the profile is gaussian, (a good approximation in most accelerator applications), and as $(1+Ka^2/\varepsilon^2)^{-1}$ is varied from unity to zero, the profile again becomes progressively more square. In plasma terms the transition from small to large ratio of beam radius to Debye length marks the transition from emittance to space-charge domination.

Neither of these distributions corresponds to those readily obtainable from a source, except perhaps in the limiting cases of Brillouin flow, (K=0), and a gaussian beam with Ka^2/ε^2 small, often assumed in conventional beam handling calculations. An interesting practical distribution, that cannot, however, be matched to a uniform channel, is one which starts off uniform in real space but gaussian in velocity space. This corresponds to the beam obtainable from a uniform, circular, hot cathode. In the absence of space-charge, this gives rise in a uniform focusing channel to a series of images of the cathode at half wavelength intervals. Midway between these images are waists with a gaussian profile, for which the transverse velocity distribution is rectangular. By adjusting the focusing field strength the rms beam radius can be held invariant, but the shape, and hence higher moments of the distribution, all vary periodically along the channel.

In the presence of space-charge, the motion is not periodic. As soon as the profile becomes non-uniform, non-linearity sets in, and phase mixing occurs. Presumably the distribution settles down ultimately to something like the thermal distribution described above. Just what does happen, and to what extent emittance growth occurs is not known.

Under these circumstances, it is pertinent to enquire into the status of the envelope equation for non-uniform beams. Formally, it can be shown that in the presence of space charge, the envelope equation is still valid provided that a and b are interpreted as twice the rms values. The emittance, however, is not invariant in non-linear systems, so the equation is of limited value. Nevertheless, it is found that in many practical situations, where the beam is not far from being matched, the assumption of invariant ε is a good approximation. The first example in this section, however, shows that in extreme cases it can be far from accurate.

We conclude this section by emphasizing that even in a uniform focusing channel, the final form assumed by an initially non-uniform beam with space-charge is not generally known.

3.2 Periodic Focusing Channels

For $\sigma_0 > 90°$ smooth approximation appears to be a good one. The determination of matched distribution functions in the presence of space charge, however, other than the K-V distribution, is not easy. Indeed, no such distributions seem to have been explicitly described. Presumably in a long channel they are ultimately achieved, computer simulations suggest a squaring of the phase distribution with a form not inconsistent with the 'thermal' steady state beam discussed in section 3.1. Can we be sure that all moments of the density distribution for ever remain finite? Is there scope for 'chaotic' behaviour, and loss of some particles to ever larger amplitudes? These questions may be academic at present, but could become important if very high efficiency of of transmission is to be achieved in a channel of limited size.

Many simulations have been made of the behaviour of non-K-V distributions in beam transport channels, particularly water-bag and gaussian distributions[3,5]. Emittance growth occurs: not surprisingly its extent depends very much on the degree of mismatch. An initial rapid change in the first few periods, associated with a radical change of the distribution function, is often found, followed by a decreasing gradual growth. The initial growth is less for beams which have the same rms radius as the matched K-V beam with the same values of K and $\bar{\varepsilon}$. Instabilities arising from parametric interactions with the structure also occur, as in a K-V distribution, but to a somewhat lesser extent. This is hardly surprising, since the core of a real beam is almost paraxial, but the outer regions contain orbits with non-paraxial values of σ/σ_0.

No attempt is made to summarize the details of these reults here; they will be reported later at this meeting. It is of interest, however, to speculate on the form of the final 'equilibrium' distribution (if there is one), and also on the question of whether a method can be found of predicting the degree of emittance growth. It has been suggested in this connection that the transverse kinetic plus electrostatic energy might be conserved. There are, however, simple situations where this clearly is not the case. The first of these is a dilute beam with K=0. If a K-V beam is changed in radius by a matching section from one channel to another, the emittance is conserved, but the transverse energy is not. Another example is an intense beam with large K and zero emittance, confined in a magnetic focusing system of varying strength, so that it changes radius with z. Considering a path of integration radially from the axis to the wall, along the wall, back to the axis at a value of z where the beam radius is different, and along the axis, the radial components of $\int E dl$ are clearly different along the two radial parts. Furthermore, $\int E_z dl=0$ at the wall, so that $\int E_z dl$ along the axis is not zero. This evidently converts longitudinal to radial energy.

Despite these observations, there is some evidence that transverse energy conservation might be approximately true in some circumstances[6].

4. Longitudinal Motion

Longitudinal beam dynamics is usually studied in conjunction with a harmonic travelling wave, with electric field in the direction of motion. In a finite emittance beam Δz and Δp are conveniently measured with respect to the phase-stable particle. In the absence of space-charge the motion in the potential well of the travelling wave is well understood. In the presence of E_z and space-charge, the phase oscillation frequency is reduced, as with transverse motion. Only when the bunch is long, and of small radial extent in a large tube can a simple approximation corresponding to the K-V distribution for transverse motion be set up. This requries a parabolic density variation with z, and a distribution function which does not correspond to a uniformly filled phase-space ellipse[7].

Unfortunately this is not such a useful starting point for the study of beam transport system as the K-V equation is for transverse motion, since the approximations are too unrealistic. In systems of practical interest the ratio of beam to tube diameter is not small, and this implies that the fields arising from space-charge vary with radius. Furthermore, distributions of current interest in heavy ion fusion research tend to be uniform rather than parabolic, giving strong non-linearity. Finally, this non-linearity implies coupling with the transverse degrees of freedom. Approximate models, perhaps more appropriate to linacs, can be constructed with ellipsoidal charge distributions, but unfortunately there is no simple self-consistent analogy with the K-V equation.

The problem of interest in the heavy ion fusion problem concerns bunching in a drift space. In the induction linac, acceleration also occurs as the particles pass the acceleration gaps; during final focusing it is the dynamics of a drifting beam in the final transport system that needs to be understood. To the linear approximation described above, a bunch would contract and expand in a manner analogous to a transverse waist; particles would be mutually 'reflected' and no overtaking would occur. The actual behaviour in realistic system is complex. Formation of a bunch of this sort is a classic problem, well known for many years to klystron designers. No really illuminating analysis seems to exist, but with advancing computer power, progress on understanding this type of situation is beginning to be made, as we shall hear in the following presentation.

The bunching problem is analogous to that of transverse waists discussed earlier. Quasi steady state analysis, in the presence of applied E_z fields and acceleration, is more appropriate to accelerator than beam transport systems. The problem of emittance growth in linacs is of course of considerable interest, but outside the scope of the present paper.

As with transverse motion, plasma type instabilities can arise. In beams that are non-uniform and of finite length, the many non-linearities make analysis very difficult; computational and experimental approaches are needed. The problem of deciding how to proceed, especially with regard to longitudinal effects, is not easy. It is discussed in the next section.

5. Outlook and Conclusions

Understanding of the behaviour of continuous mono-energetic beams in transport systems has made good progress in recent years. Contributions have come from theory, computation, and experiment. Nevertheless a great deal that will be of practical importance remains to be learned. What, for example, is the effect of walls near the beam that are not cylindrical perfect conductors? After traversing a long channel, what is the final state of a beam, and how does this depend on the initial distribution? Since the long term future of accelerators will be as much concerned with high current as high energy, a better understanding of 'tails' and 'halos' is essential.

With regard to longitudinal motion, much remains to be done. Because of the essential non-linearities, all situations are different, and it is difficult to dis-entangle relevant characteristic parameters. Care must be taken in devising computations and experiments both that reflect conditions of real practical interest, and also are compatible with one another. Nevertheless our knowledge and experience are still so limited that any work in this field that can be done for a reasonable expenditure of money and effort is to be welcomed.

Even more intractable are problems, not discussed in the paper, arising from partial neutralization of beams by ions of the opposite sign. This topic has often been studied, and a great deal of empirical knowledge exists. Electromagnetic isotope separators, for example, rely on almost complete neutralization for their operation. In many situations it is, however, an embarrassment. Although the space-charge force is reduced, neutralization is seldom everywhere complete and this gives rise to unpredictable non-linear effects. Plasma oscillation giving a 'hashy' beam are often found. These effects are very dependent on the details of particular apparatus, especially such parameters as gas pressure and wall configuration in addition to the particle energy and species. For this reason anything other than very specific studies are difficult to justify.

In conclusion, it is evident that continuous progress is being made in understanding beam transport in the space-charge dominated regime. For longitudingal effects, the problem is so non-linear that progress will depend on ever more elaborate simulations, and well designed experiments. Careful thought must be given on how to integrate these two lines of approach; if this is well done, progress can be expected in the coming years.

6. References

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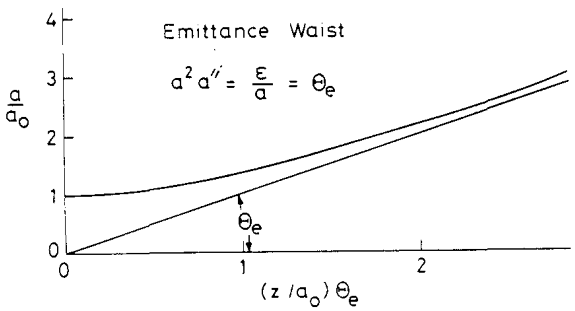
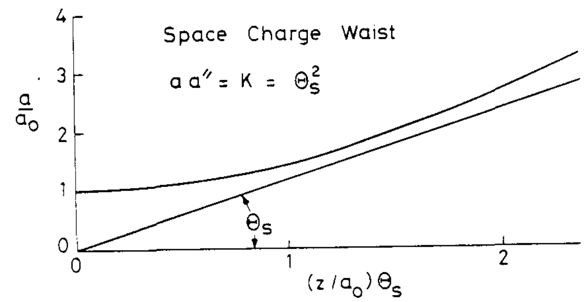




Waists arising from space-charge and emittance. The envelope equations are found from Eq.3 with a=b, including only the two relevant terms.

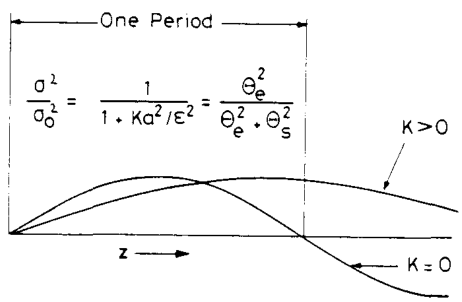
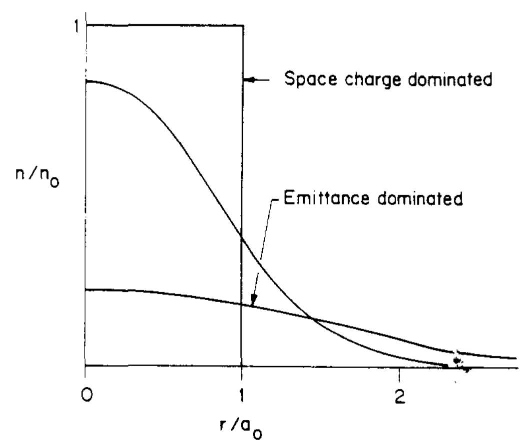


Illustration of the meaning of σ/σ_0 in a K-V beam with smooth approximation. This may be expressed either in terms on the perveance and emittance, the characteristic space-charge and emittance angles, or the plasma frequency and betatron frequencies in the absence of space charge; this may be seen from Eq.4.



Sketch showing transverse profiles of a matched distribution with thermal transverse velocities and finite emittance in a uniform channel with linear focusing. Transition from a square to a gaussian density distribution occurs as σ/σ_0 varies from zero to unity. In the intermediate regime $K \approx \varepsilon^2/a^2$, or the Debye length $\lambda_D \approx a$.

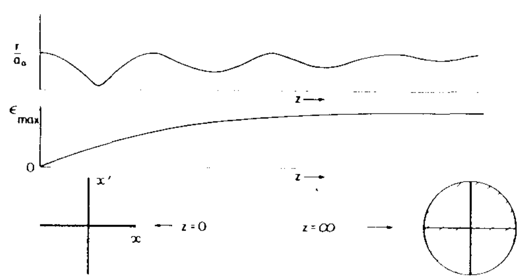


Illustration of emittance growth in an initially parallel beam with K=0 injected into a uniform focusing channel with non-linear focusing. The beam radius at infinity is the same as at injection, but it does not vary with z. Strictly, each betatron amplitude has a defined phase, but in practice the structure is almost indistinguishable from that of a beam in which all phases and amplitudes are present. Note that this is a rather special distribution: all transverse motion is radial, there are no circumferential components of velocity.

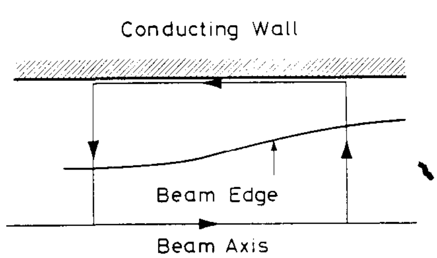


Diagram to illustrate the transition from longitudinal to transverse energy in a beam that changes its transverse configuration. $\int E\,dl$ round the rectangle is zero and since the contribution along the conducting wall is zero, the value along the axis is equal to the difference of the numerically unequal radial contributions.