

EXPERIMENTAL DEMONSTRATION OF THE HENON-HEILES QUASI-INTEGRABLE SYSTEM AT IOTA*

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Abstract

The Integrable Optics Test Accelerator is a research electron and proton storage ring recently commissioned at the Fermilab Accelerator Science and Technology facility. Its research program is focused on testing novel techniques for improving beam stability and quality, notably the concept of non-linear integrable optics. In this paper, we report the first results of experimental investigation of a quasi-integrable transverse focusing system with one invariant of motion, a Henon-Heiles type system implemented with octupole magnets. Good agreement with simulations is demonstrated on key parameters of achievable tune spread and dynamic aperture preservation. Resilience to perturbations and imperfections in the lattice is explored. We conclude by outlining future research plans and discussing applicability to future high intensity accelerators.

INTRODUCTION

One of key factors limiting beam intensity in modern circular accelerators are collective instabilities. They can be suppressed either by a spread in betatron tunes through Landau damping, or in case of slow instabilities by an external damper. Tune spreads are typically produced with standalone octupoles distributed around the ring, as is the case for LHC [1]. The disadvantage of using octupoles, and most other nonlinear elements, is the appearance of resonant behavior, leading to chaotic and unbounded motion, and eventual particle loss [2]. Recently, a new nonlinear focusing system was proposed by Danilov and Nagaitsev (DN) [3] that is predicted to achieve significant tune spreads without such negative effects though careful shaping of the magnetic potential and special requirement on lattice optics. To test this concept, the Integrable Optics Test Accelerator (IOTA) storage ring was constructed at Fermilab, and has just finished its year 1 commissioning and scientific run, the first results of which we present in this paper.

INTEGRABLE OPTICS

Modern accelerator designs are based on a strong-focusing linear lattice design, which has no tune spread and is fully integrable - that is, it has the same number of conserved dynamic quantities (Courant-Snyder invariants) as degrees of freedom, and so particle motion is regular for any initial conditions. Due to misalignments, field errors, and the

need to correct chromaticity and induce tune spread, real accelerators are slightly nonlinear, and so no longer have any invariants. Their regular motion is limited to a finite region, called the dynamic aperture (DA) - preserving its size is critical for achieving good accelerator performance. Mathematically, transverse particle dynamics are described by the Hamiltonian

$$H = \frac{1}{2} \left(K_x(s)x^2 + K_y(s)y^2 + p_x^2 + p_y^2 \right) + V(x, y, s)$$

with $K_{z=x,y}$ being the linear focusing strength, and $V(x, y, s)$ containing any nonlinear terms (in general dependent on time ($\equiv s$) and transverse (x, y) position). DN approach is to seek solutions for V that yield two invariants of motion and also are implementable with conventional magnets. First invariant comes from appropriate time scaling of $V(x, y, s)$, such that it becomes a time-independent potential $U(x_N, y_N)$ in normalized coordinates, namely

$$z_N = \frac{z}{\sqrt{\beta(s)}} \quad p_N = p\sqrt{\beta(s)} - \frac{\beta'(s)}{2\sqrt{\beta(s)}}$$

It is furthermore possible to derive a specific form of $U(x_N, y_N)$ (DN solution) that yields another invariant of motion. Such system is both nonlinear and fully integrable, and its experimental demonstration is the ultimate goal of IOTA. However, this is a difficult task due complicated field shape, and small tolerances on optics and field errors [4]. Conveniently, the first nonlinear multipole in the DN solution is an octupole, which produces tune shift ΔQ_z quadratic with particle oscillation amplitude, and has potential of the form

$$V(x, y, s) = \frac{\alpha}{\beta(s)^3} \left(\frac{x^4}{4} + \frac{y^4}{4} - \frac{3x^2y^2}{2} \right)$$

where $\alpha(\text{m}^{-1})$ is the strength parameter. Using only this multipole component instead of full DN potential gives a system of so-called Henon-Heiles type [5], first studied in the context of galaxy dynamics, and known to have rich dynamical behavior. It has a single invariant of motion, and is hence only quasi-integrable, with finite DA. However, even a single invariant is highly beneficial for particle stability, and unlike the DN potential, this system is easily implementable with conventional magnets and predicted to be highly robust to misalignments and other lattice errors [6], making it the perfect first test of nonlinear optics at IOTA and a stepping stone towards fully integrable systems.

EXPERIMENTAL SETUP AT IOTA

IOTA is a research electron and proton storage ring recently commissioned at Fermilab's Accelerator Science and

* This work was supported by the U.S. National Science Foundation under award PHY-1549132, the Center for Bright Beams. Fermi Research Alliance, LLC operates Fermilab under Contract DE-AC02-07CH11359 with the US Department of Energy.

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Technology (FAST) facility. It has a circumference of 40m, and is designed to use either 2.5 MeV protons provided by an RFQ injector, or 150 MeV electrons from FAST linac. The lattice, shown in Fig. 1, has mirror symmetry with two 1.8m nonlinear element-compatible drifts and can be configured to satisfy all the necessary integrability constraints. An extensive suite of beam diagnostic systems is installed, including wall current monitors/DCCTs, electrostatic beam position monitors (BPMs), and synchrotron light cameras/photomultipliers. This provides capabilities both for the beam-based alignment at necessary accuracy, and the subsequent turn-by-turn dynamics analysis. Two independent single-turn kickers, vertical and horizontal, allow for beam 'pings' to anywhere in the available aperture.

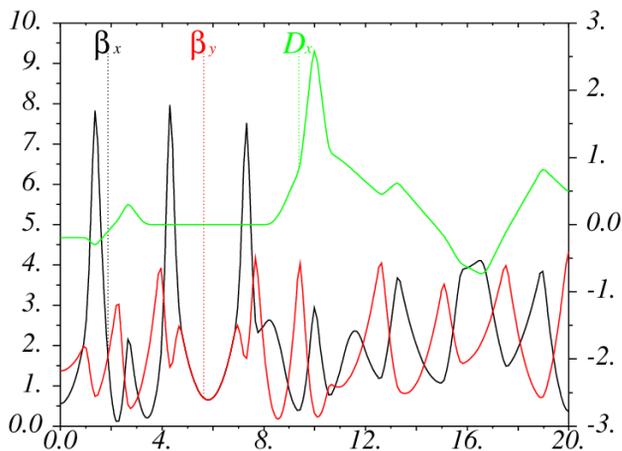


Figure 1: Half of IOTA lattice at working point $Q_{x,y}=5.3$. All units in meters, $\beta_{x,y}$ on the left, D_x on the right. Lattice is mirror symmetric across 20m marker.

The octupole insert is shown in Fig. 2 - it is comprised of 17 equidistant air-cooled iron yoke octupoles with 28mm aperture and 7cm length, that are designed to approximate piecewise the ideal continuous octupole potential. A manual laser-guided alignment method (with pinhole markers) was used for relative alignment, with insert endpoints laser tracker referenced to the rest of the ring. During commissioning, model-independent beam-based measurements were used to estimate the center-to-center shifts at 200 μm rms transversely, within the design specifications. Each magnet was individually powered by a 2A bipolar supply, reaching normalized gradient of $K_3 = 1.4\text{kG}/\text{cm}^3$ at peak current.

Data Collection

The key practical advantage of integrable systems is improved DA over standalone elements, and hence higher possible tune shifts without beam losses. To measure these figures of merit, we first measured the DA envelope by repeatedly pinging the beam to a certain amplitude with octupole insert on or off, and recording the current. Iteratively, kick strength was maximized until degradation in lifetime relative to control run was observed at very low currents ($<0.1\text{mA}$, to minimize intra-beam scattering). Corresponding oscillation



Figure 2: IOTA octupole insert in segment B2L. RF cavity and synchrotron radiation diagnostics in the background.

amplitude, where not equal to physical/control one, was then taken as a DA limit.

Then, for above identified points, pings were repeated at high beam currents ($>1\text{mA}$) to obtain accurate and linear turn-by-turn data, even in presence of beam losses. Arrays of 2000 turns from 21 available BPMs were recorded for each kick, of which 17 were used for analysis and rest excluded due to either incompatible configuration or excessive noise levels. Due to large chromaticity and momentum spread, signal decoherence was very fast (150 turns), and extensive data processing techniques were required for sufficient tune resolution ($<10^{-4}$). Core algorithm was implemented in Python 3, based on iterative coupled mode unmixing technique [7] and subsequent NAFF analysis [8] of combined (M BPMs \times N turns) hybrid datasets [9]. For all runs, beam was kicked in both planes simultaneously, ensuring sufficient signal to determine both tunes regardless of coupling strength. Reference simulation results were obtained by long-term particle tracking and frequency map analysis (FMA) [10] with LIFE-TRAC code, using complete experimental lattice but with no field errors or misalignments except intentional ones.

RESULTS

Dynamic Aperture

A selection of extinction scans are plotted in Fig. 3, demonstrating different beam loss rates observed for varying combinations of octupole insert strength and kick amplitude. Note how small, 10% changes in either parameter immediately spoiled beam lifetime (green and purple lines), indicating that 1.0A/4.5kV and 1.1A/4.2kV are very close to DA limit and hence good candidates for further kick analysis.

Tune Shift

Points of interest identified above were tested to find one with the largest tune shift. Previous simulations have found optimum insert strength to occur with $\sim 1\text{A}$ in the central octupole - experimental data is in agreement, with best tune shift obtained at 1.1A/4.2kV, corresponding to physical amplitude of 3.85mm in the middle of the nonlinear drift. We observed tune shifts of $\Delta Q_x = -0.030 \pm 0.003$ and $\Delta Q_y = +0.013 \pm 0.001$, as shown in Fig. 4, with near -2:1

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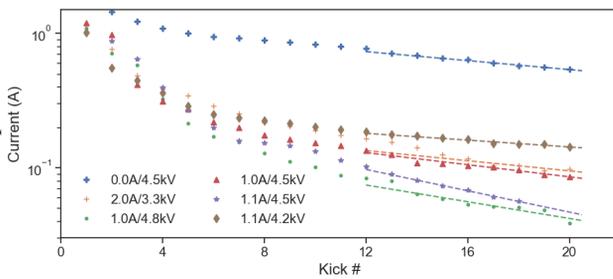


Figure 3: Semilog plot of extinction scan currents. Kicks were performed every 20 seconds. Dotted lines are linear fits to low current data.

$\Delta Q_x:\Delta Q_y$ ratio being consistent with theoretical predictions. As compared to simulations, only 60-70% of the expected performance was achieved, which nonetheless significantly exceeded equivalent single octupole results (not shown). We attribute this shortcoming to the lack of chromaticity correction and general first-year alignment and power supply drift issues that likely introduced significant systematic lattice errors.

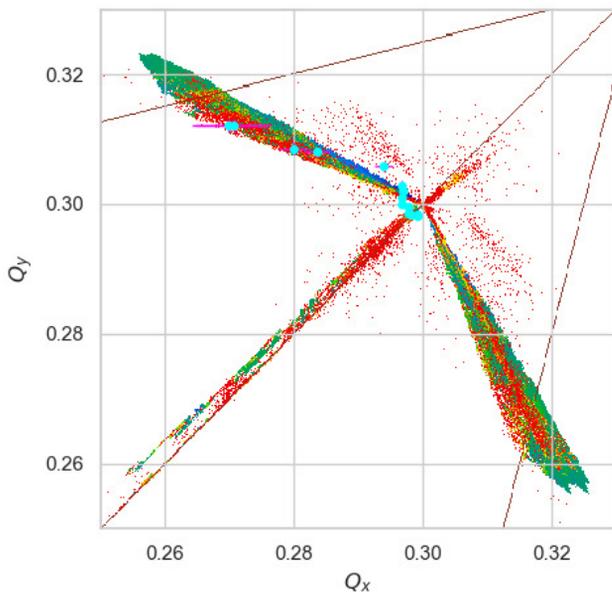


Figure 4: Experimental tune shifts (in cyan, error bars in magenta) for various kicks overlaid with FMA. Density plot denotes amount of tune diffusion, with red indicating more chaotic regions and white particle loss. Small kick/tune shift points are offset due to residual coupling.

Error Tolerances

Given the enormous parameter space, only the most critical categories of optics and magnet errors were tested experimentally. Namely, we varied the phase advance within the insert $\pm 0.01 Q_x/Q_y$ (equivalent to β^* or current profile mismatch), the longitudinal location of β -function minimum (± 10 cm), and the current in individual octupoles ($\pm 10\%$).

For each category, we found that performance degradation was below 15%, thus showing high system resiliency.

Resonance Stability

Significant tune spreads can allow for novel beam and optics manipulation methods that would otherwise result in beam losses. As a preliminary experiment, we brought the tunes within nonlinear drift to $Q_x/Q_y = 0.26$ while keeping the rest of the lattice the same, and adjusted octupole current profile accordingly. Beam pings then rapidly brought Q_x down to ~ 0.24 , across the 1/4 resonance. During subsequent damping, over half the beam survived the reverse resonance crossing. With octupoles off, we saw complete beam loss and generally poor injection efficiency. Introduction of slight coupling unexpectedly captured beam into stable, long lifetime high amplitude orbits, shown in Fig. 5. We observed that island distance to beam core (i.e. amplitude) scales with octupole current, and work is ongoing to reproduce this curious phenomenon in simulations.

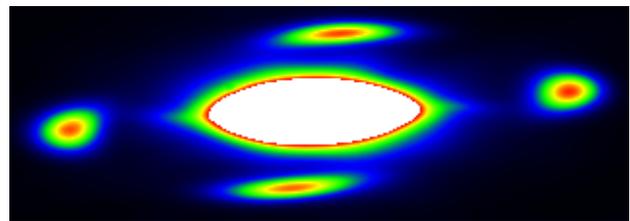


Figure 5: Synchrotron radiation image of high amplitude stable orbit beam near 1/4 resonance, with octupoles on.

SUMMARY AND FUTURE PLANS

We have presented a successful implementation of the octupole Henon-Heiles quasi-integrable system. Our results show strong agreement with theoretical predictions, achieving up to 70% of ideal case performance and significantly exceeding those of an equivalent standalone element. For next run, expected to start in fall of 2019, a number of upgrades are planned, including full ring realignment, addition of 8 more sextupoles, improvements in BPM system sensitivity, and more. These will significantly improve optics accuracy and stability - as such, we plan to first refine and improve year 1 measurements, and then explore some newly proposed alternative magnet/optics arrangements. Several ancillary experiments are also planned, such as anti-damper studies (simulating fast collective instabilities) and reconstruction of resonance crossing dynamics, which will provide supporting experimental evidence for the usefulness and practicality of nonlinear integrable inserts. Finally, we hope to apply all the above procedures to the fully integrable DN magnet, which has already been placed into the ring - several proposed accelerators, including the rapid cycling synchrotron for Fermilab proton upgrade program, incorporate nonlinear elements and will rely on this experimental data to guide final design.

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