



# Space Charge Effects on the Third Order Coupled Resonance

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IPAC 2017, Kopenhagen

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B. Franczak, P. Forck, T. Giacomini, I. Hofmann, F. Kesting,  
M. Kirk, H. Kollmus, T. Mohite, A. Parfenova, P. Schuett,  
P. Spiller
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S. Gilardoni, A. Huschauer, M. Martini, E. Metral,  
R. Steerenberg, G. Rumolo, F. Schmidt, R. Wasef,  
F. Zimmermann
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- Univ. A.I. Neishtadt  
Loughborough
- Univ. Bologna G. Turchetti, C. Benedetti, A. Bazzani

The case for accelerators and projects

Space charge effect in nonlinear rings

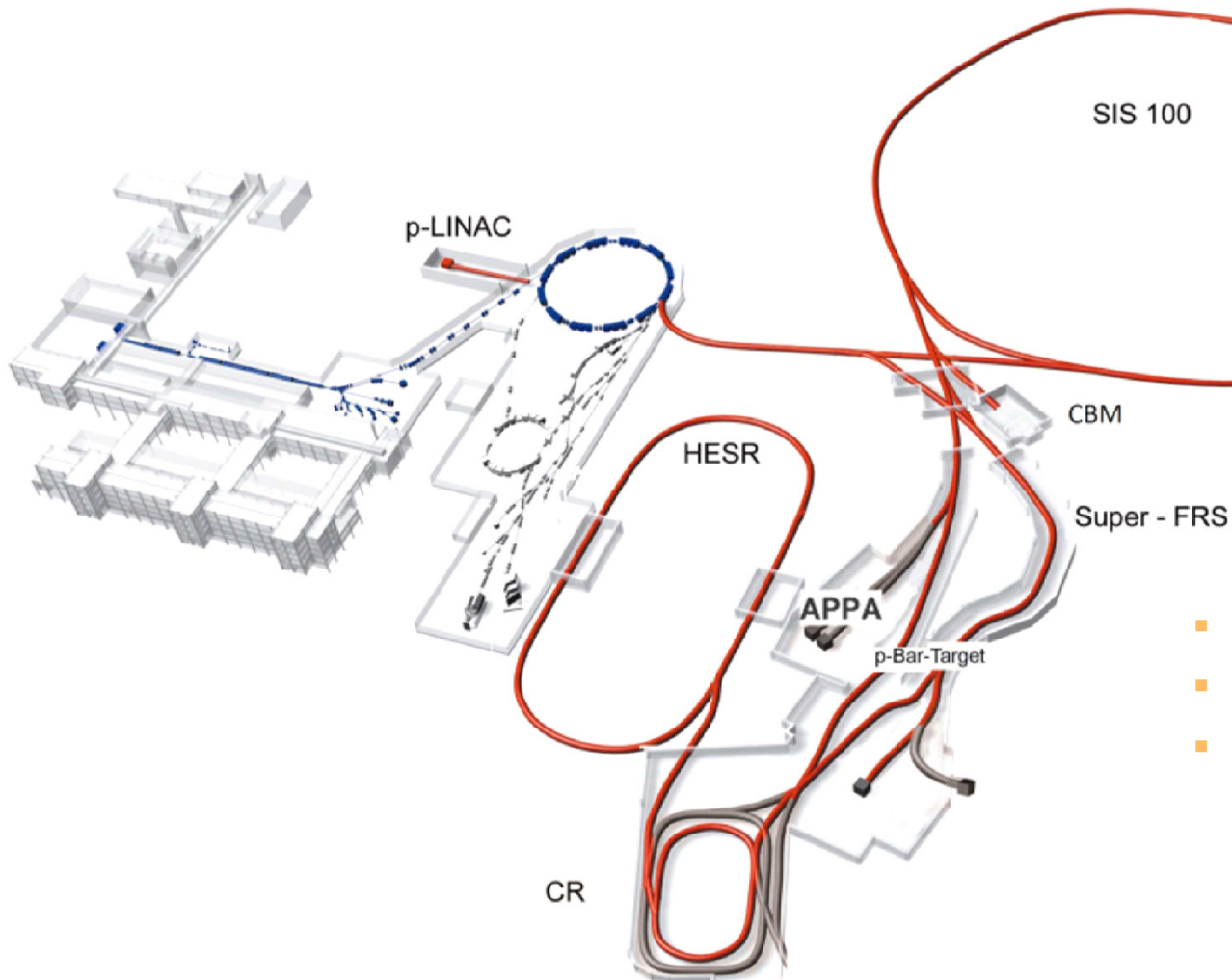
One dimensional resonances

The coupled resonances

Experiment and simulations

Conclusion / Outlook

# Motivation: the FAIR project



- SIS100 beam parameters:
- Every ion from p to U
- $U^{28+}$  -ions for RIB production:
  - $5 \times 10^{11}$  / cycle
  - Rep. rate: 0.5 Hz
  - Energy: 400–2715 MeV/u

# Motivation: the FAIR project

## FAIR @ IPAC 2017

### GSI

- Winfried A. Barth - TUPVA055 Further Investigations for a Superconducting cw-LINAC at GSI
- Lars Bozyk - TUPVA056 Ionization Loss and Dynamic Vacuum in Heavy Ion Synchrotrons
- Stephanie Deveaux - MOPIK127 FAIR Risk Management as a Proactive Steering Tool for the Large Scale Multi Project
- Manuel Heilmann - MOPVA054 High Power RF Coupler for the CW-Linac Demonstrator at GSI
- Manuel Heilmann - TUPVA057 Design Study for a Prototype Alvarez-Cavity for the Upgraded Unilac
- Egbert Fischer** - **WEOCB2 Superconducting Magnets at FAIR**
- Michael Frey - THPIK015 Prototype Results of the ESR Barrier-Bucket System
- Carl M. Kleffner - TUPVA058 The Status of the FAIR pLinac
- Harald Klingbeil - THPIK016 Status of the SIS100 RF Systems
- Sergio Mauro - THPIK017 Field Uniformity Preservation Strategies for the ESS DTL: Approach and Simulations
- Carsten Omet - WEPVA029 SIS100 Tunnel Design and Status
- David Ondreka - TUPVA059 Overcoming the Space Charge Limit: Development of an Electron Lens for SIS18
- Thomas Reichert - MOPAB034 SIS-100 BPM System: Design and Realization
- Stephan Reimann - THPAB096 Automatized Optimization of Beam Lines Using Evolutionary Algorithms
- Anna Rubin - THPVA003 Status of the Beam Dynamics Design of the New Post-Stripper DTL for GSI-FAIR
- Mariusz Sapinski - TUPVA060 Upgrade of GSI Hades Beamline in Preparation for High Intensity Run
- Marcus Schwickert - MOPAB035 Status of Beam Diagnostics for SIS100Bernd
- Robert Schlei - TUPIK045 Closed Orbit Feedback for FAIR - Prototype Tests at SIS18
- Peter J. Spiller - WEPVA030 FAIR SIS100 - Features and Status of Realisation
- Ralph Jeffrey Steinhagen - TUPIK046 Beam-Based Feedbacks for FAIR - Prototyping at the SIS18
- Markus Vossberg - TUPIK047 FAIR Control Centre (FCC) - Concepts and Interim Options for the Existing GSI Main Control Room
- Natalya Winters - MOPIK128 Integrated Project Planning as a Central Steering Tool for the Large Scale Multi Project FAIR
- Stepan Yaramyshev - TUPVA061 Beam Dynamics Study for the HIM&GSI Heavy Ion sc cw-Linac

### TUD

- Alexander Andreev - THPAB097 Phase Calibration of Synchrotron RF Signals
- Jens Harzheim - WEPVA047 Input Signal Generation for Barrier Bucket RF Systems at GSI
- Erika Kazantseva - WEPAB026 SUSPSIK049 BRho-Dependent Taylor Transfer Maps for Super-FRS Dipole Magnets
- Benjamin Frederic Reichardt - TUPIK048 Longitudinal Beam Stabilization at FAIR by Means of a Derivative Estimation
- Thibault Ferrand - THPVA041 Progress in the Bunch-to-Bucket Transfer Implementation for FAIR
- Herbert De Gerssem - THPIK018 Simulating Cross-Magnetization Effects in Combined-Function Accelerator Magnets
- Kerstin Gross - THPAB098 Test Setup for Automated Barrier Bucket Signal Generation
- Nicolai Schweizer - THPVA042 Modular Robot for Visual Inspection of the Vacuum Beamline of a Particle Accelerator
- William Stem - THPVA004 Pushing the Space Charge Limit: Electron Lenses in High-Intensity Synchrotrons?
- Dinu Mihailescu Stoica - THPAB100 On the Impact of Empty Buckets on the Ferrite Cavity Control Loop Dynamics in High Intensity Hadron Synchrotrons

### IAP-Frankfurt

- Ali Mohammad Almomani - TUPVA064 Updated Cavities Design for the FAIR p-Linac
- Markus Baschke - TUPAB147 The Final RF-Design of the 36 MHz-HSI-RFQ-Upgrade at GSI
- Daniel Koser - THPIK021 SUSPSIK091 Structural Mechanical Analysis of 4-Rod RFQ Structures in View of a Newly Revised CW RFQ for the HLI at GSI

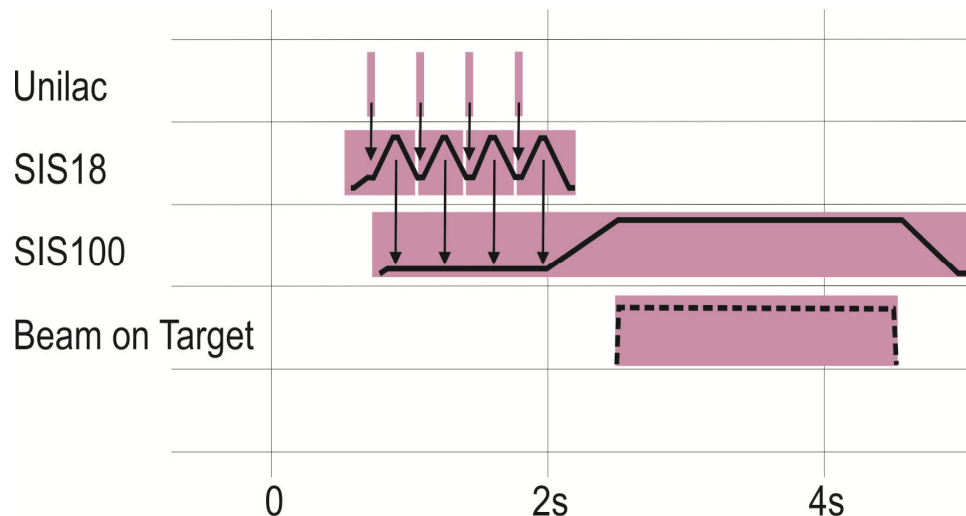
### ITEP

- Sergey Markovich Polozov - TUPAB013 Beam Dynamics Study and Electrodynamics Simulations for the CW RFQ



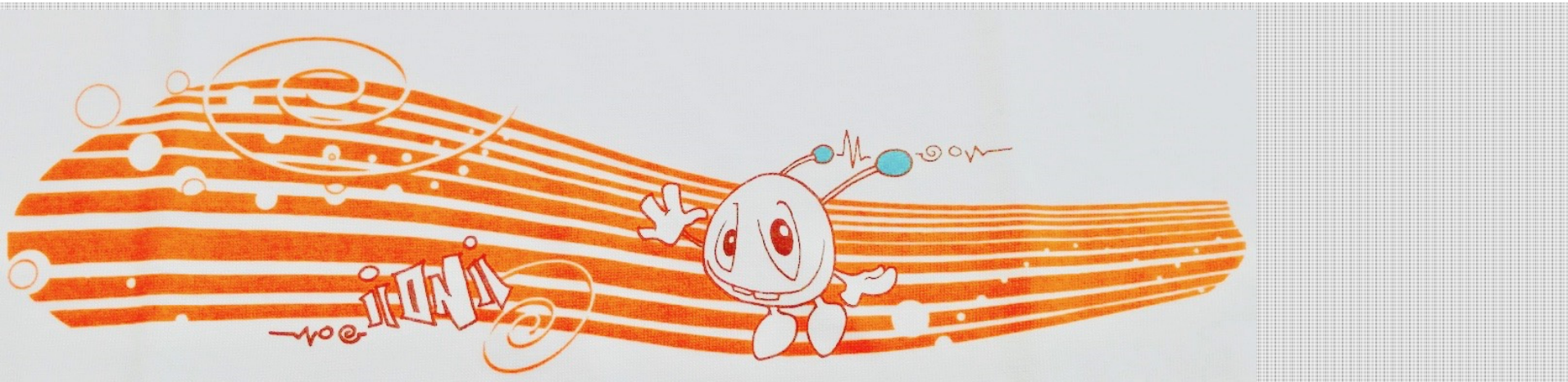
# Intensity Requirements in SIS18 for FAIR

FAIR stage	Today	Stage 0 (Existing Facility after upgrade)	Stage 1 (Existing Facility supplies Super FRS, CR, [HESR])	Stage 2 (SIS100 Booster)
Reference Ion	$U^{73+}$	$U^{73+}$	$U^{73+}$	$U^{28+}$
Maximum Energy	1 GeV/u	1 GeV/u	1 GeV/u	0.2 GeV/u
Maximum Intensity	$4 \times 10^9$	$2 \times 10^{10}$	$2 \times 10^{10}$	$1.5 \times 10^{11}$
Repetition Rate	0.3 - 1 Hz	1 Hz	1 Hz	2.7 Hz



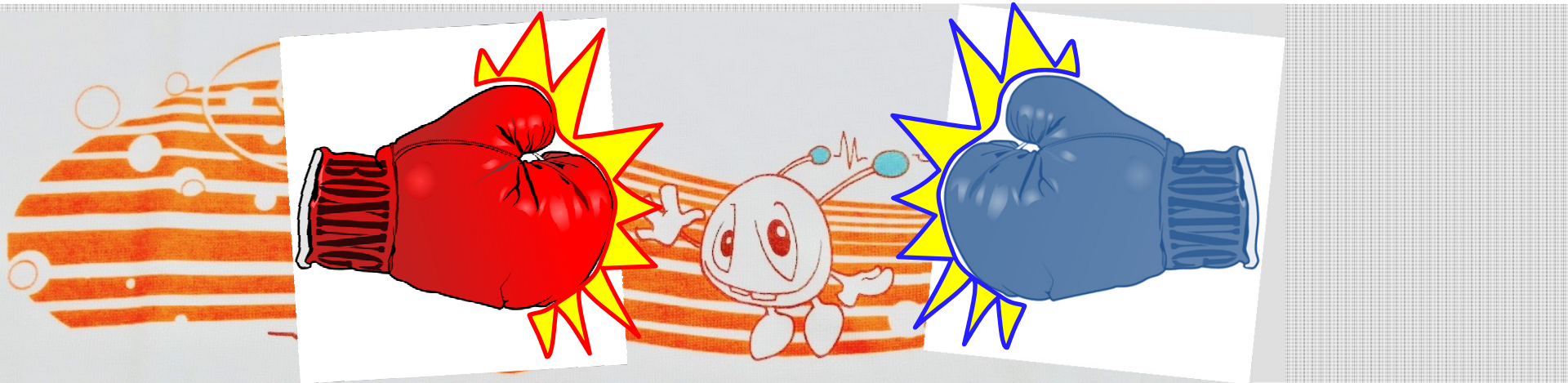
SIS100:  
 $5 \times 10^{11}$   $U^{28+}$  ions per cycle  
 $3 \times 10^{11}$   $U^{28+}$  ions per second

High intensity bunch stored for many turns





High intensity bunch stored for many turns



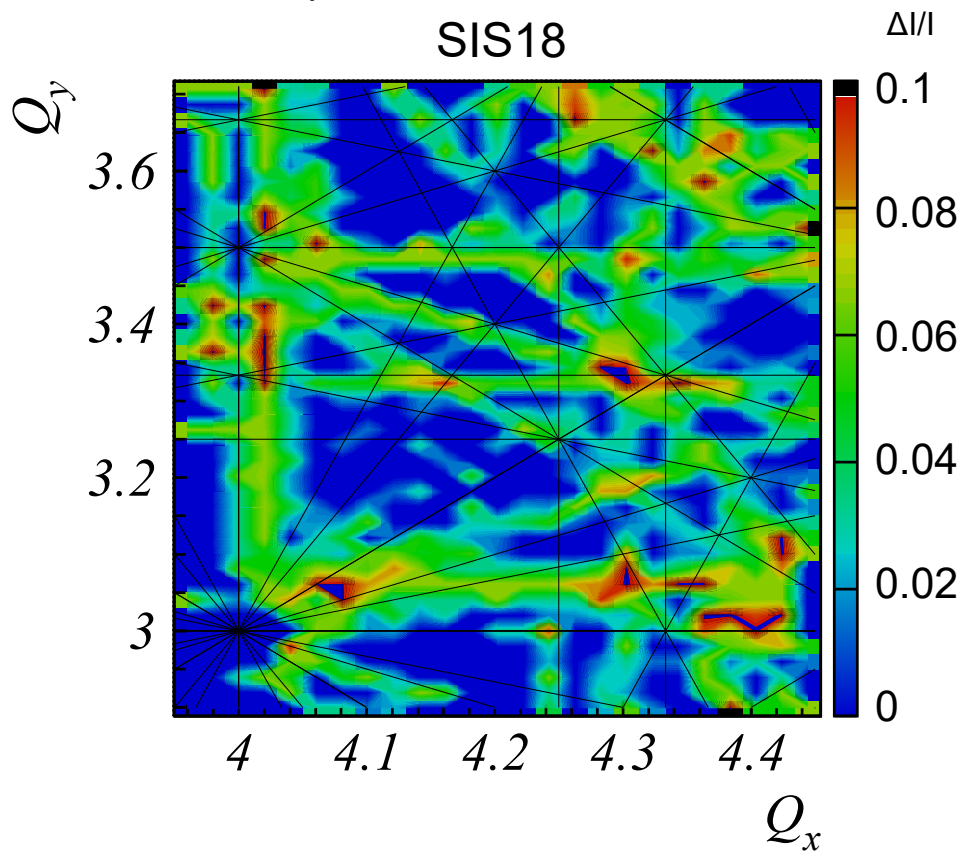
Particles subject to  
Space charge

Space charge tune-shift  
Amplitude dependent detuning  
Structure resonances  
Collective effects  
impedances

Particles are subject to the  
nonlinear motion

Error and structure resonances  
Dynamic aperture  
Chromatic effects

Error / Structure Resonances  
SIS18

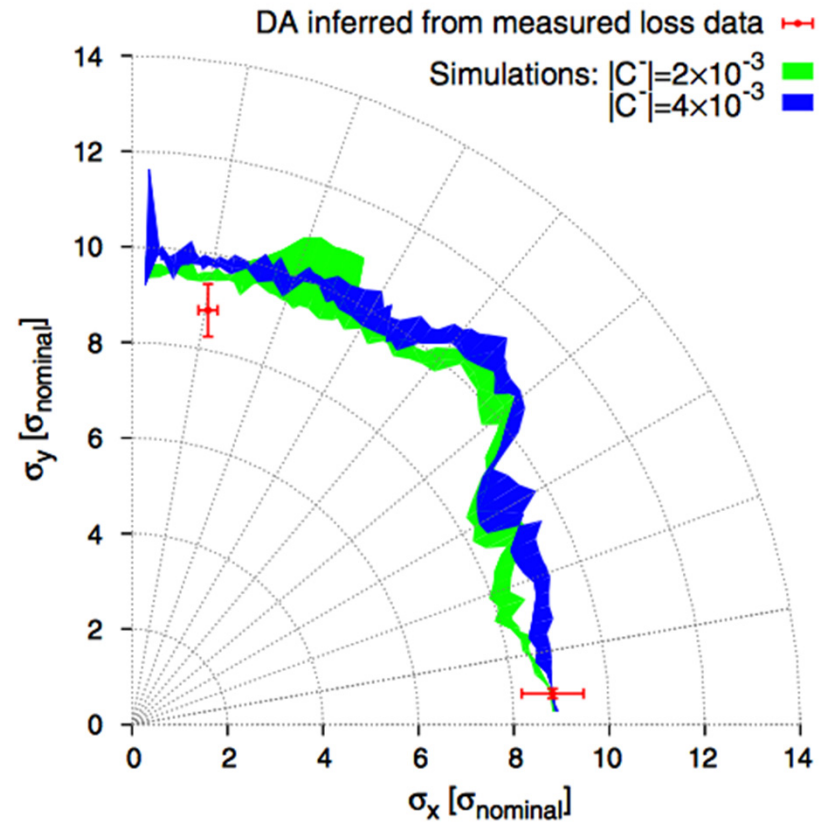


Lattice induced nonlinear resonances

$$n_x Q_{x0} + n_y Q_{y0} = m$$

G. Guignard, CERN 78-11, (1978); A. Bazzani et al., CERN94-02 (1994).

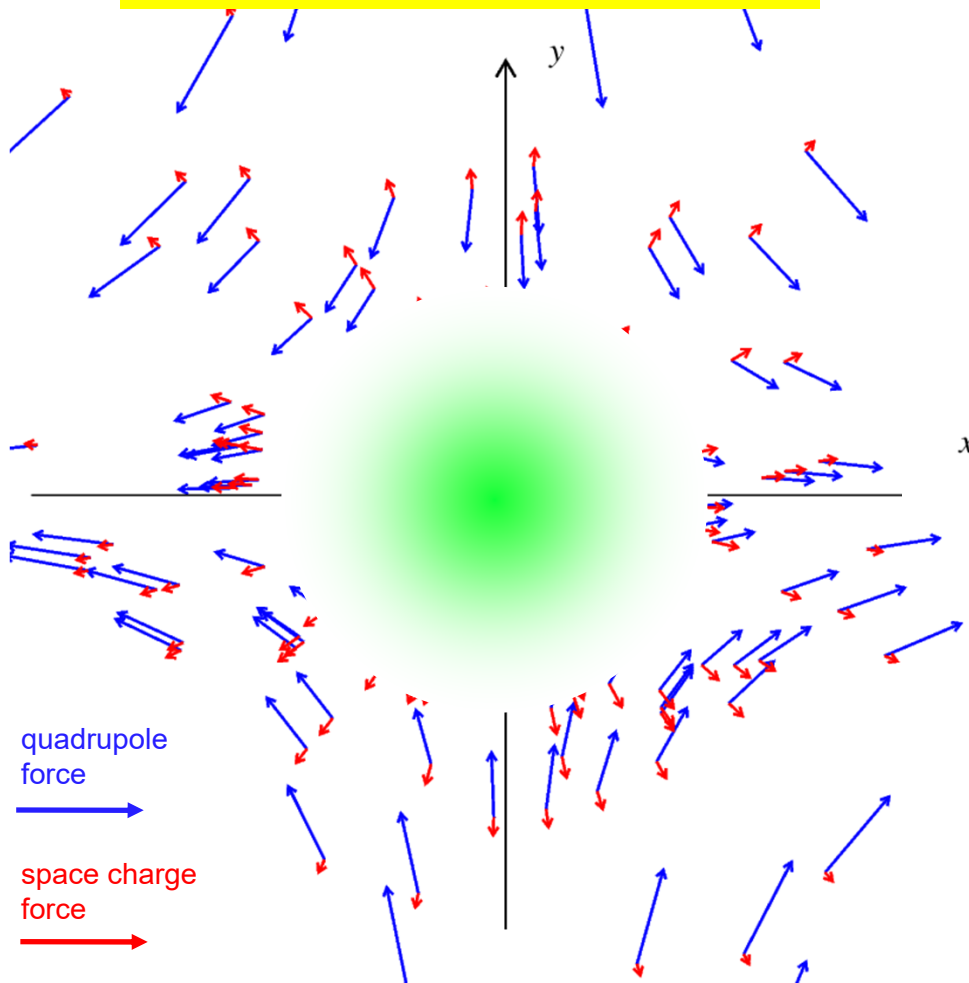
Dynamic Aperture: LHC



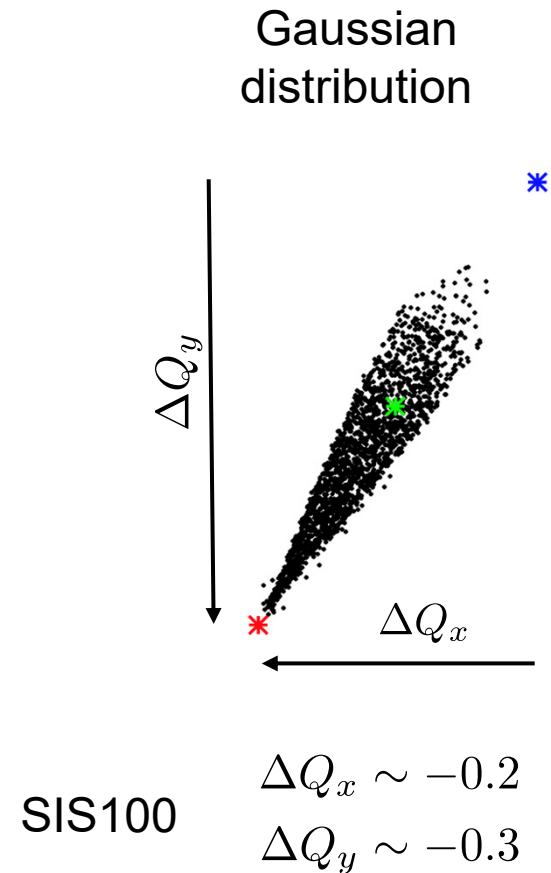
E. H. Maclean, R. Tomás, F. Schmidt, and T. H. B. Persson  
Phys. Rev. ST Accel. Beams 17, 081002

# Space charge vs. magnets force

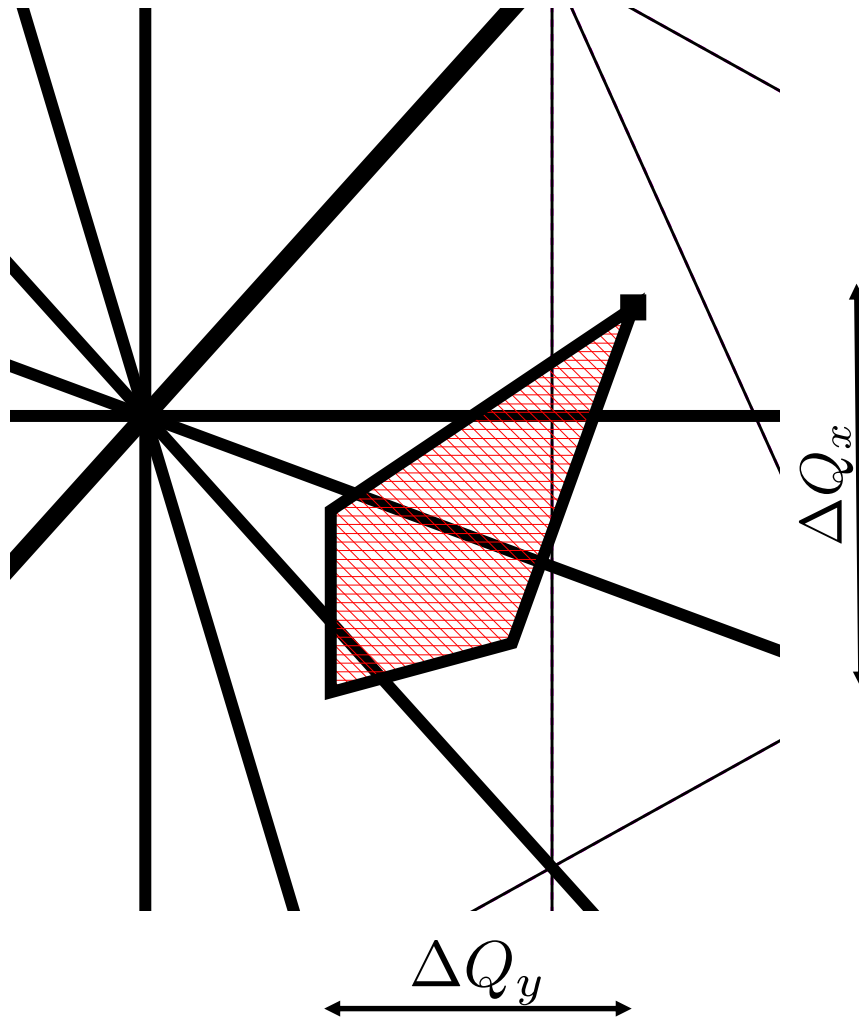
Example in a focusing quadrupole



Space charge tune-spread



# The space charge limit



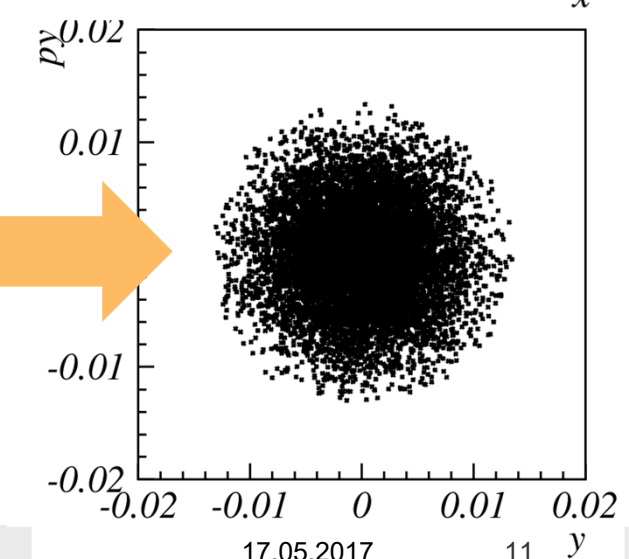
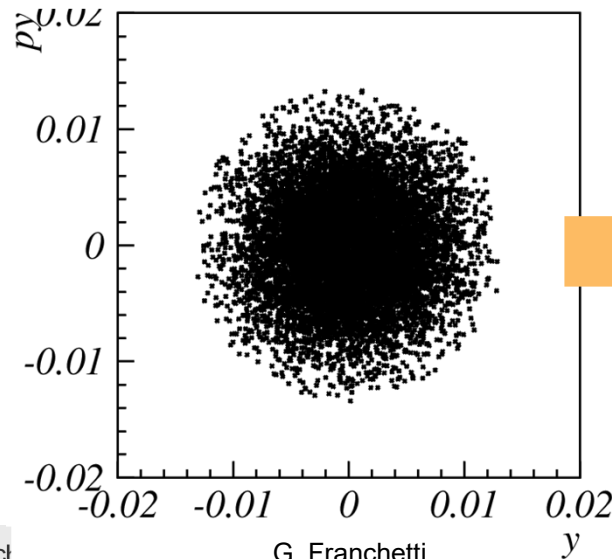
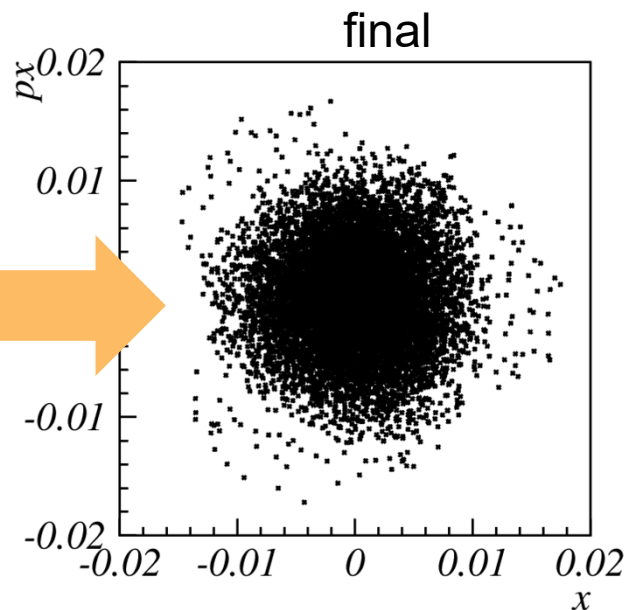
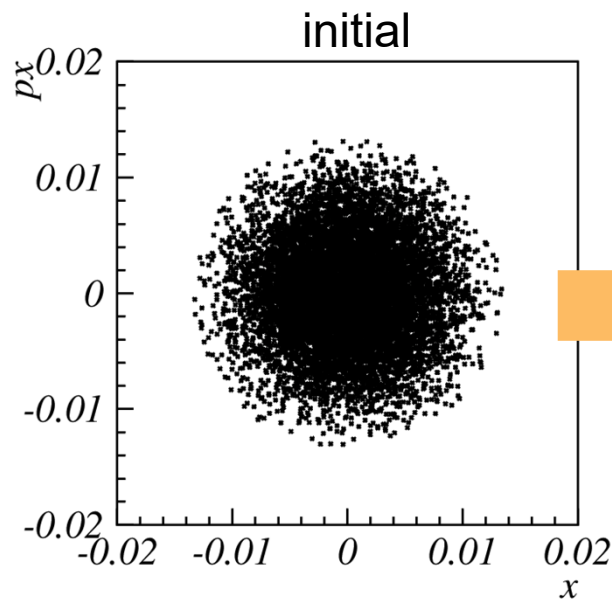
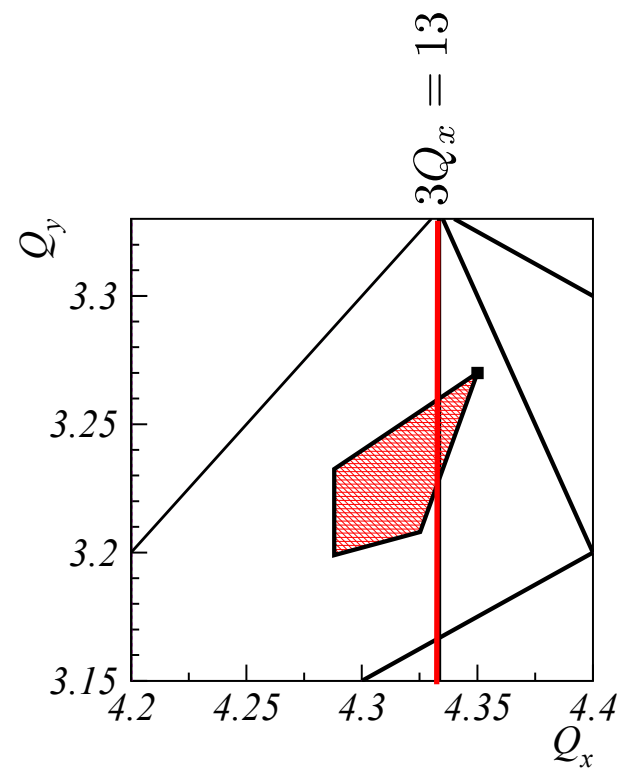
Tolerable space charge tune-shift in order not to overlap with resonances

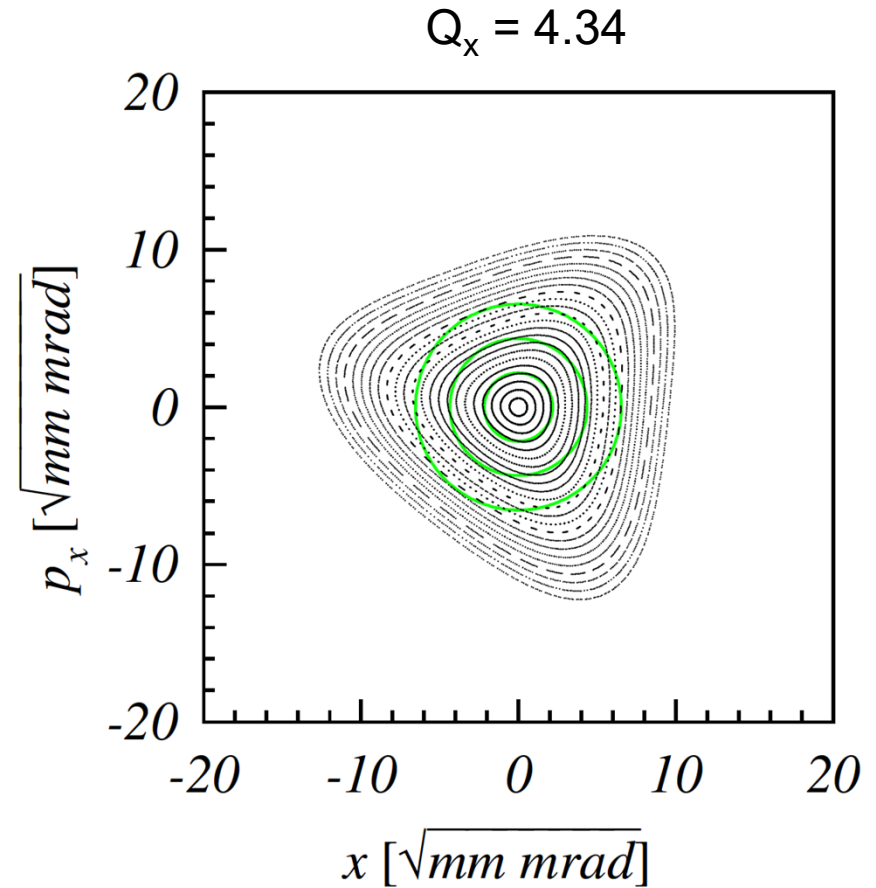
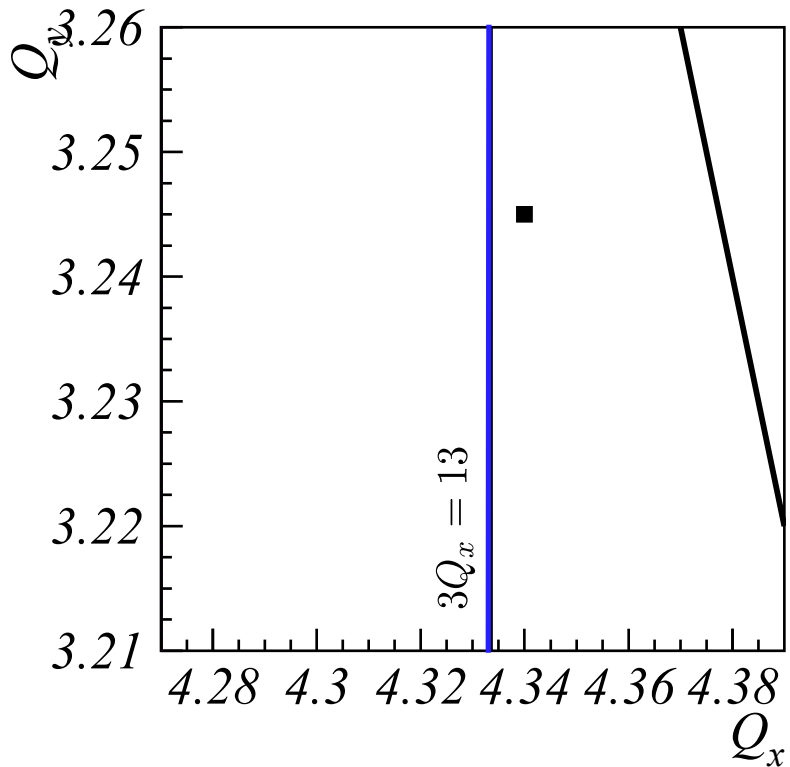
If resonances are too many, or the incoherent tune-shift is too large there is always a resonance overlapping

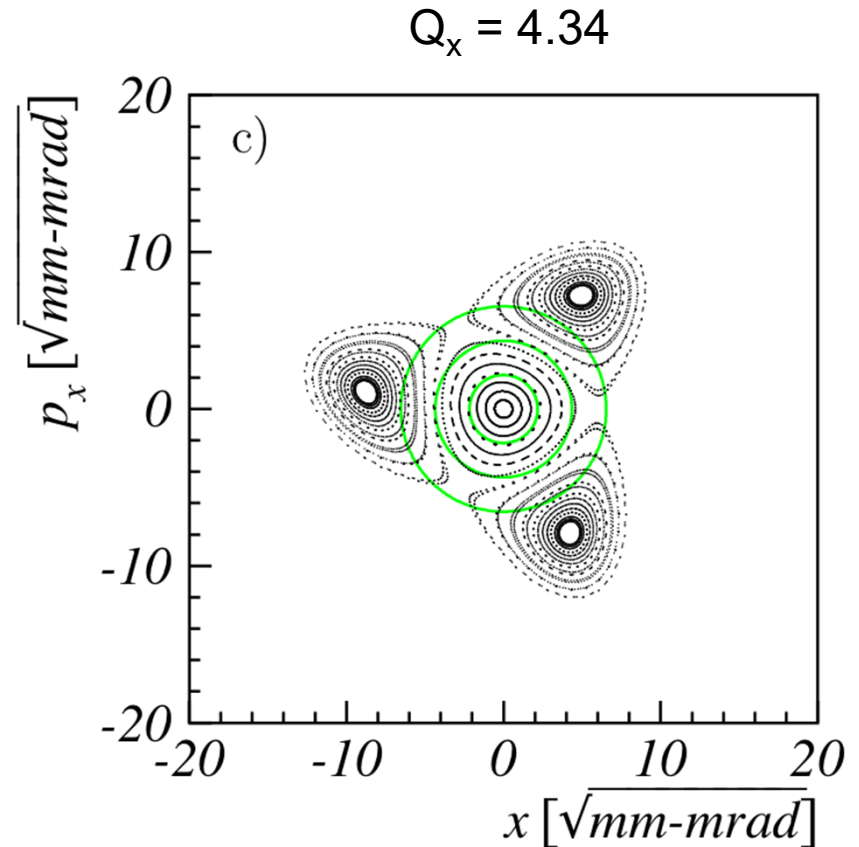
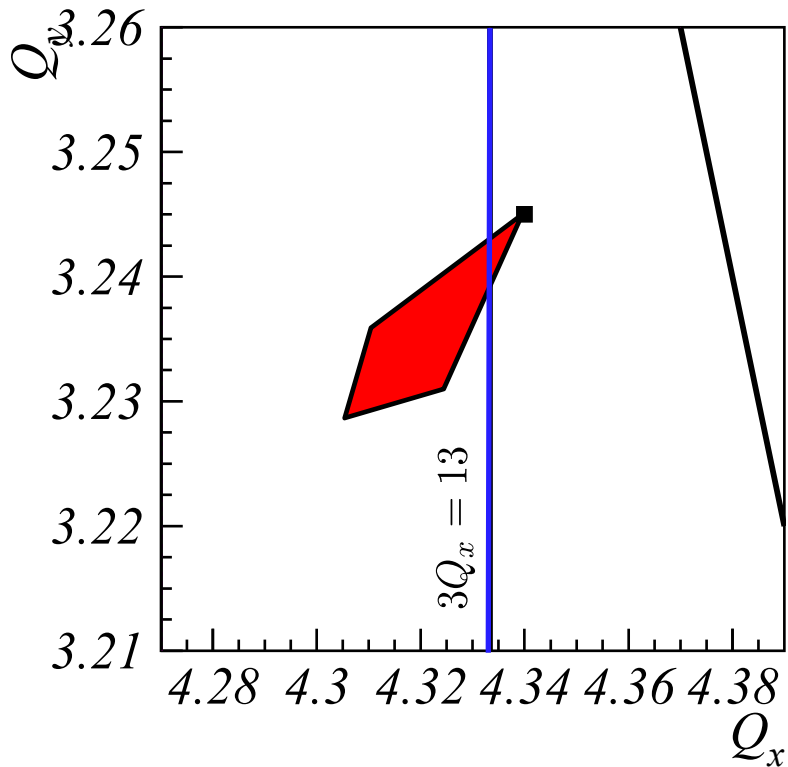
What happens if space charge tune-spread overlaps a resonance?

# Example: Coasting beam and 1D resonance

PIC simulation

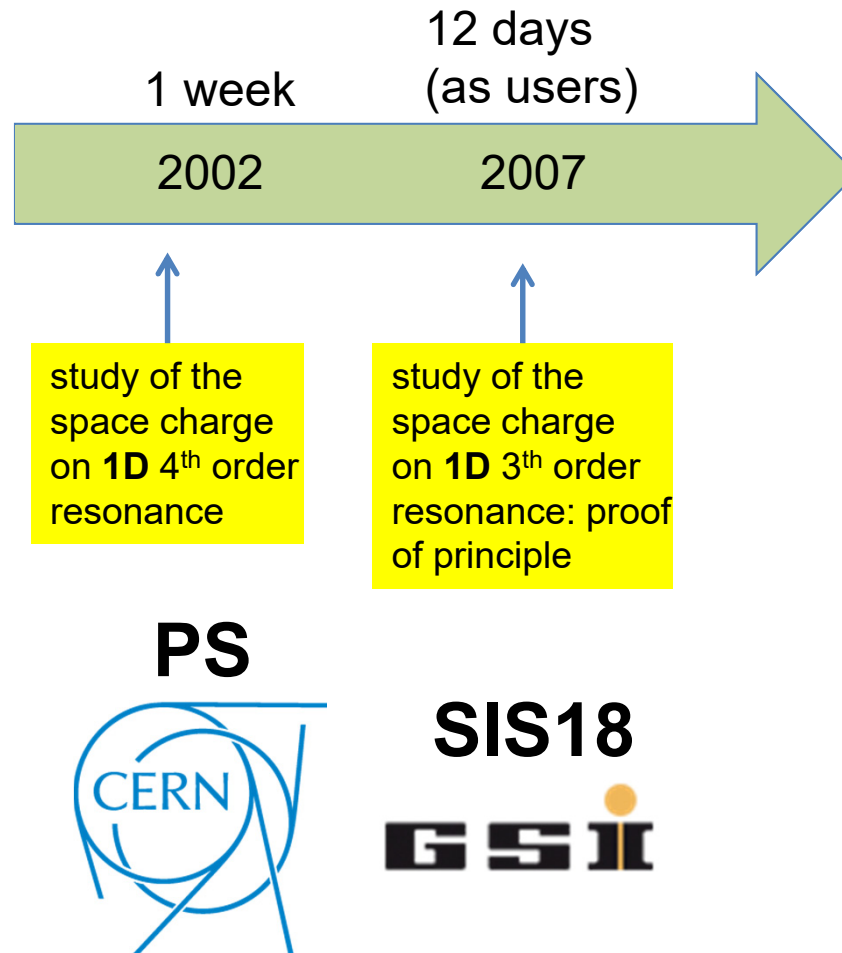






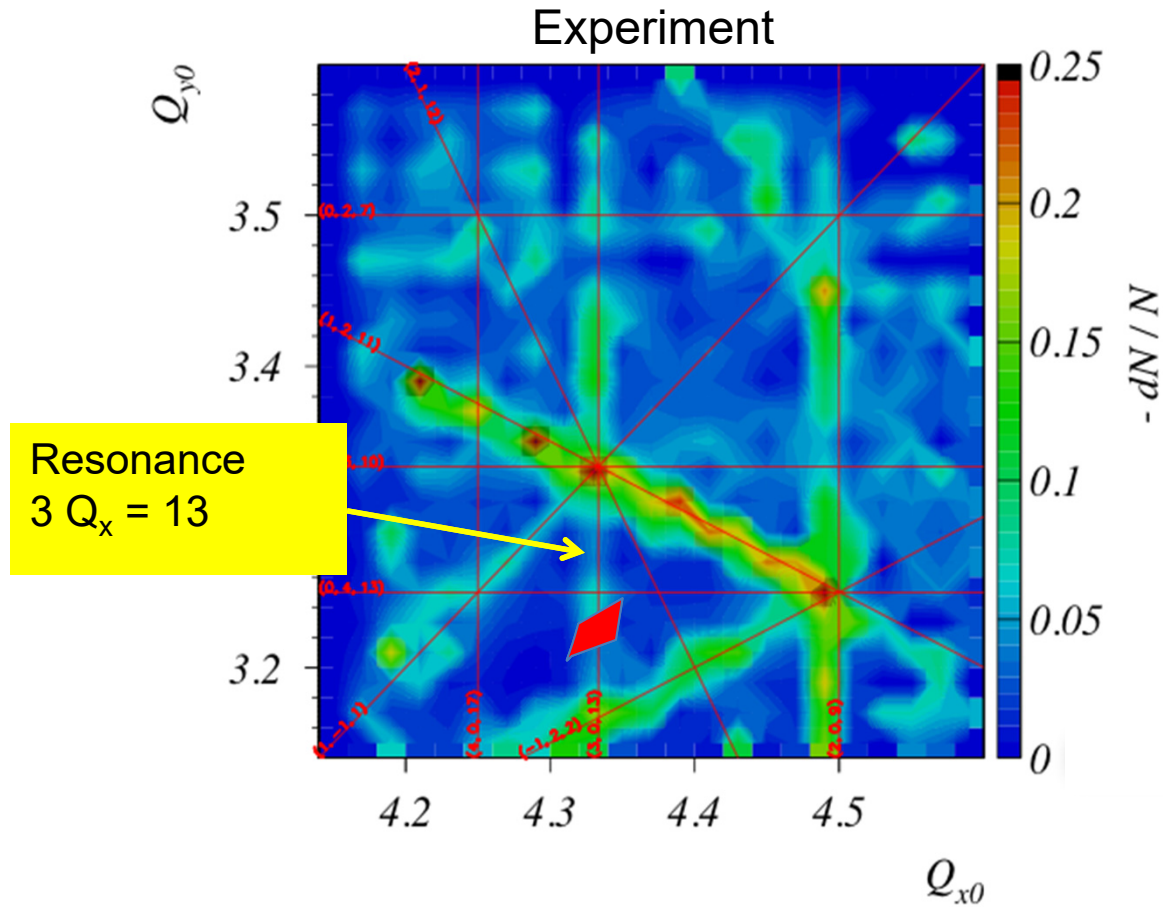
**Above the resonance:** Large stable 3<sup>rd</sup> order islands are created

# The quest of the incoherent effects of space charge

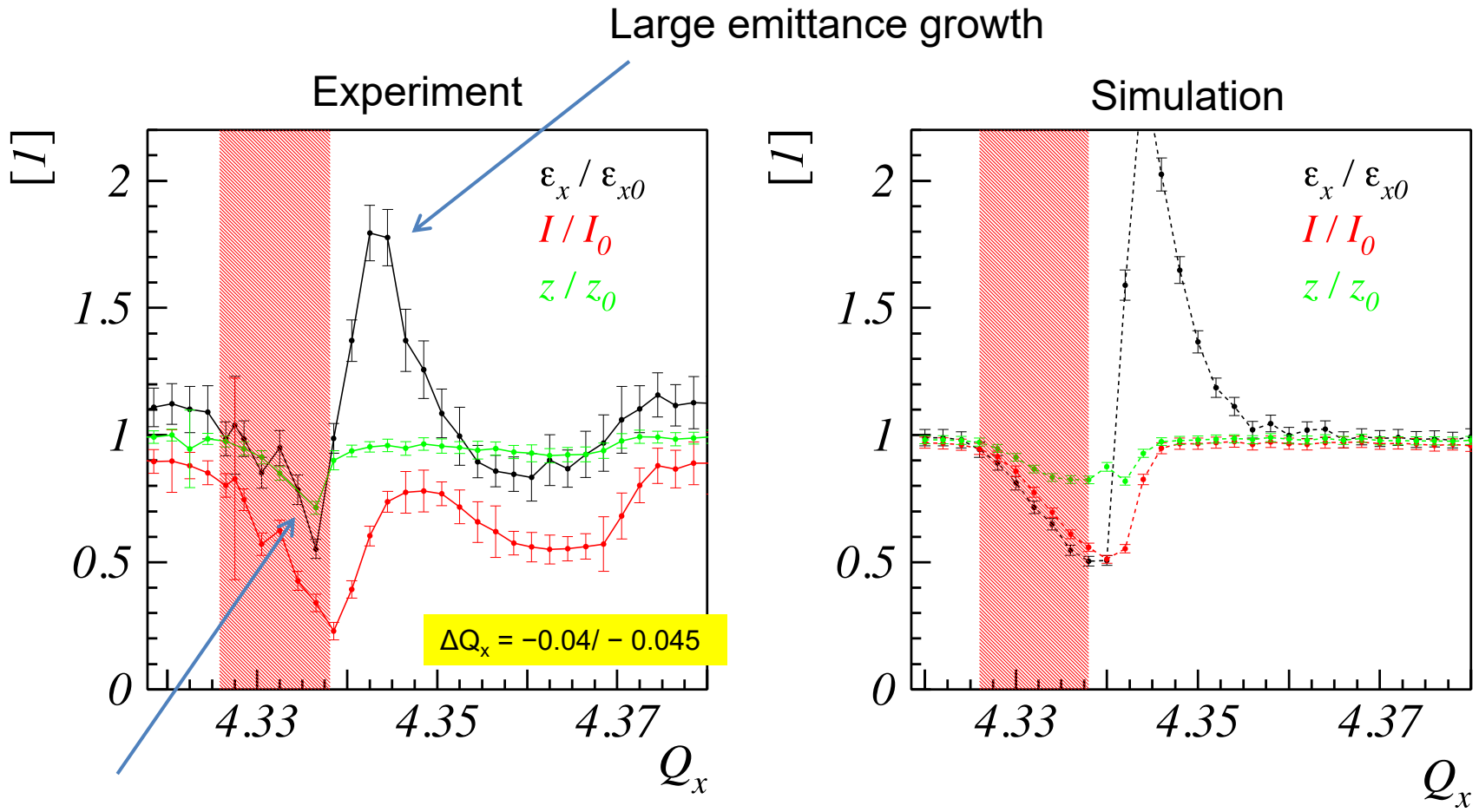




# 1D third order resonance

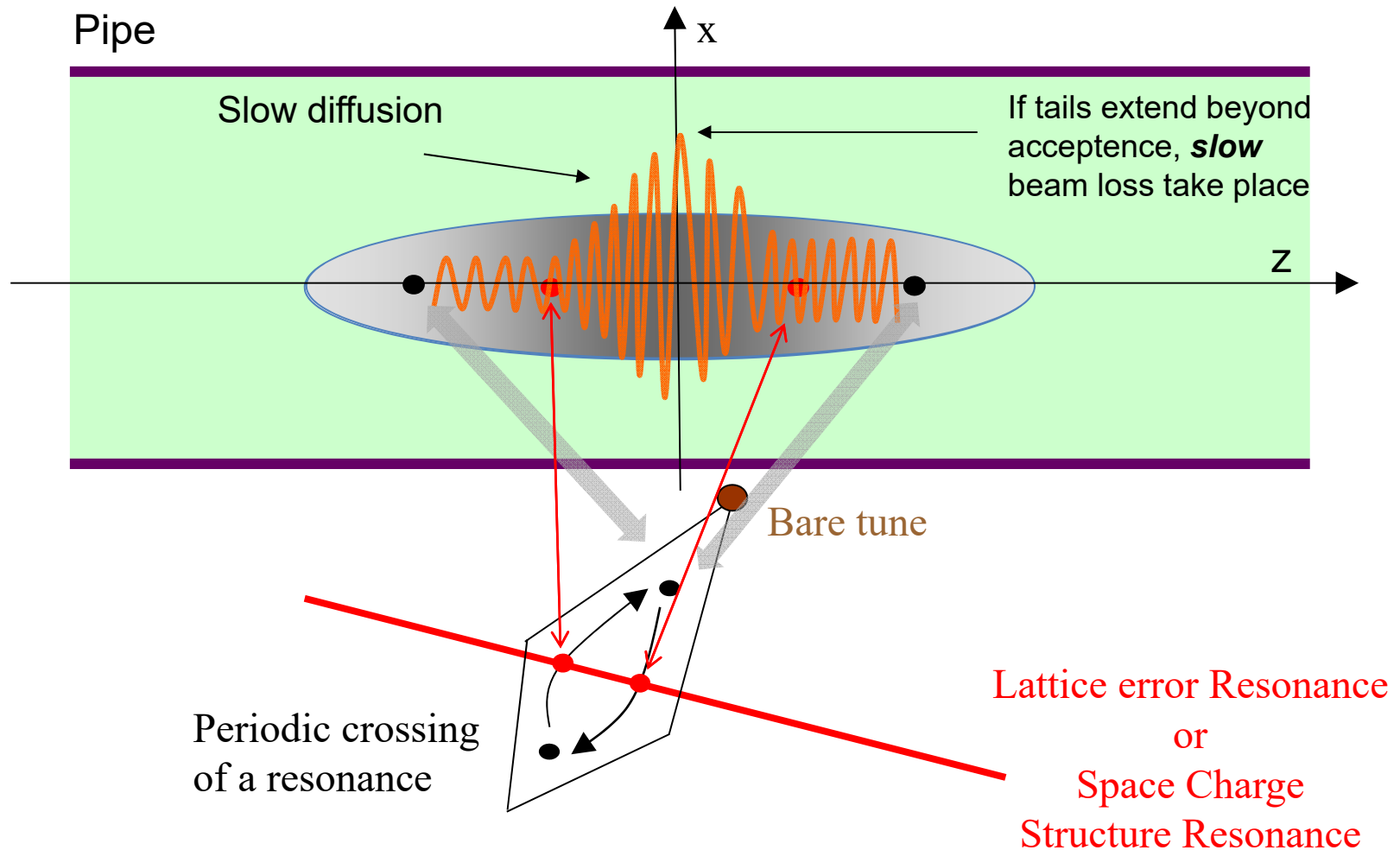


# Bunched beam at high intensity



The bunch is shorter !

G. Franchetti, O. Chorniy, I. Hofmann, W. Bayer, F. Becker, P. Forck, T. Giacomini, M. Kirk, T. Mohite, C. Omet, A. Parfenova, P. Schuett  
 Phys. Rev. ST Accel. Beams **13**, 114203 (2010).

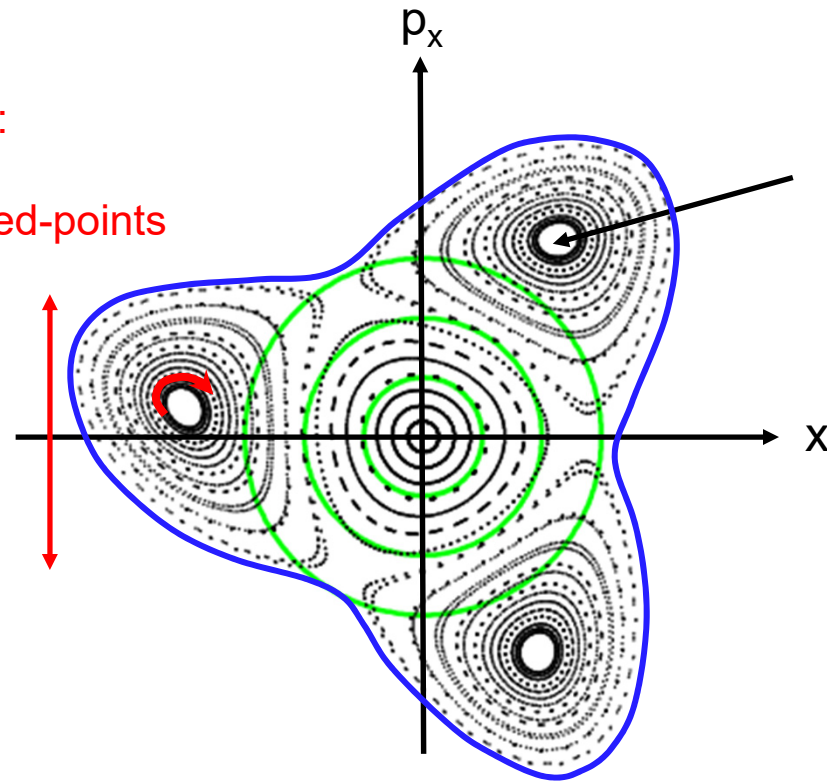


# 1D resonance and space charge

## Summary (2000 – 2010)

Resonance strength:

- Island size
- Tunes around fixed-points



Determined by  
 $n[q_{x0} + \Delta Q_x(X)] = N$

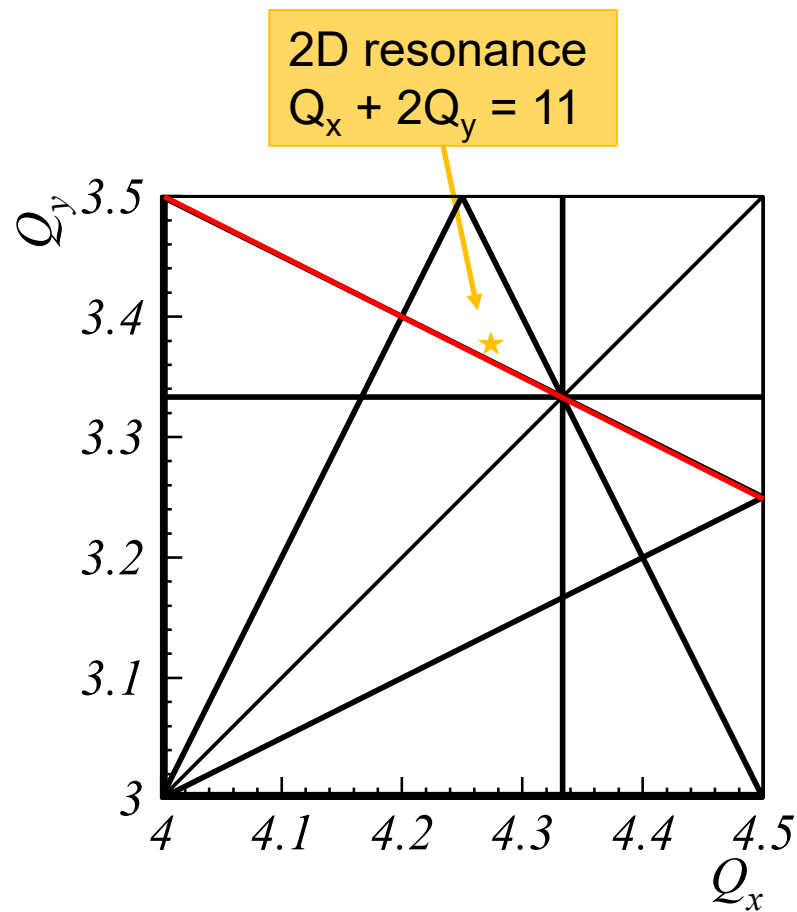
Halo size is determined  
by the outer position  
of islands

Longitudinal motion and space charge drives islands away too fast



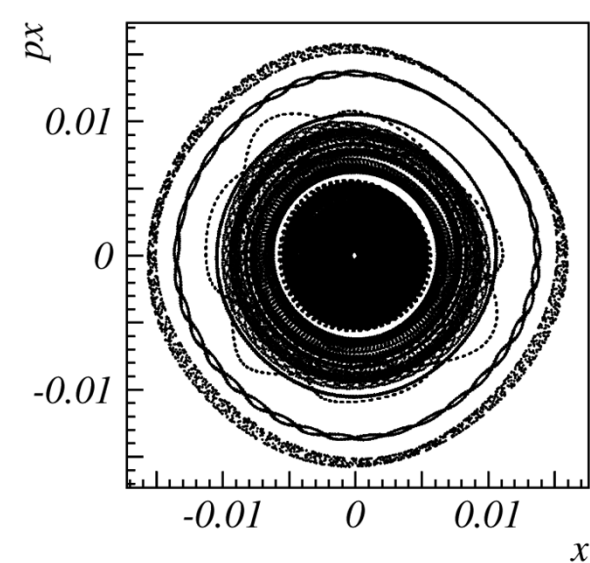
“scattering regime”

# The difficulty of the coupled dynamics

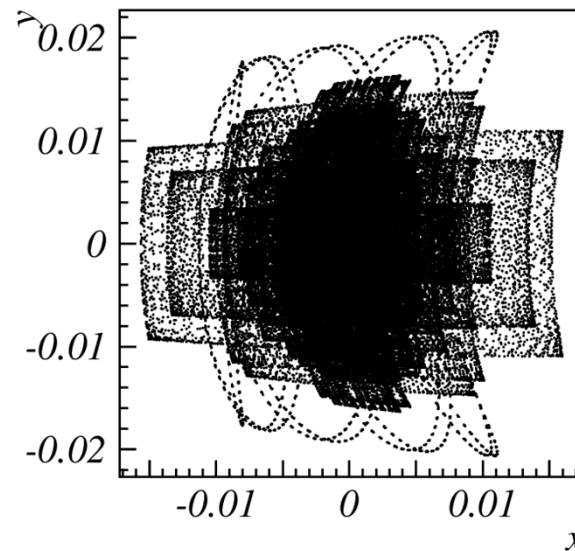
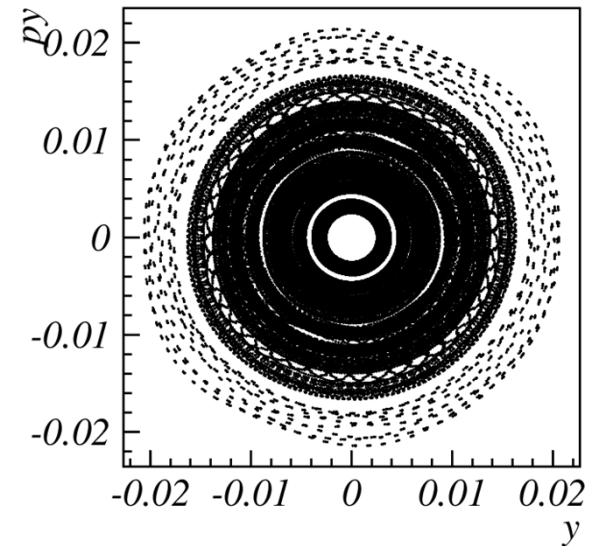


# The difficulty of the coupled dynamics

Near the resonance  $Q_x + 2 Q_y = 11$   $Q_x = 4.27, Q_y = 3.3575$

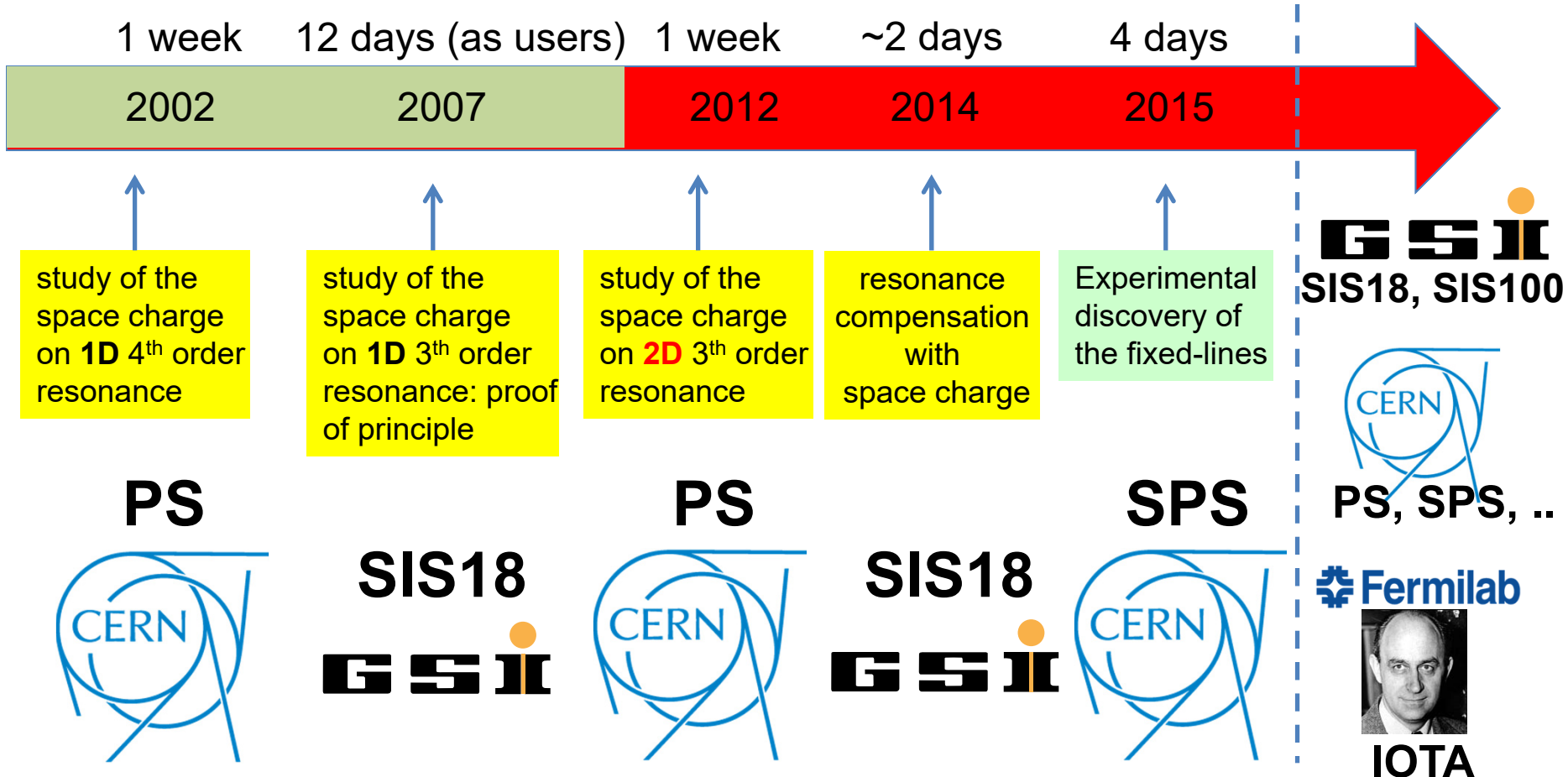


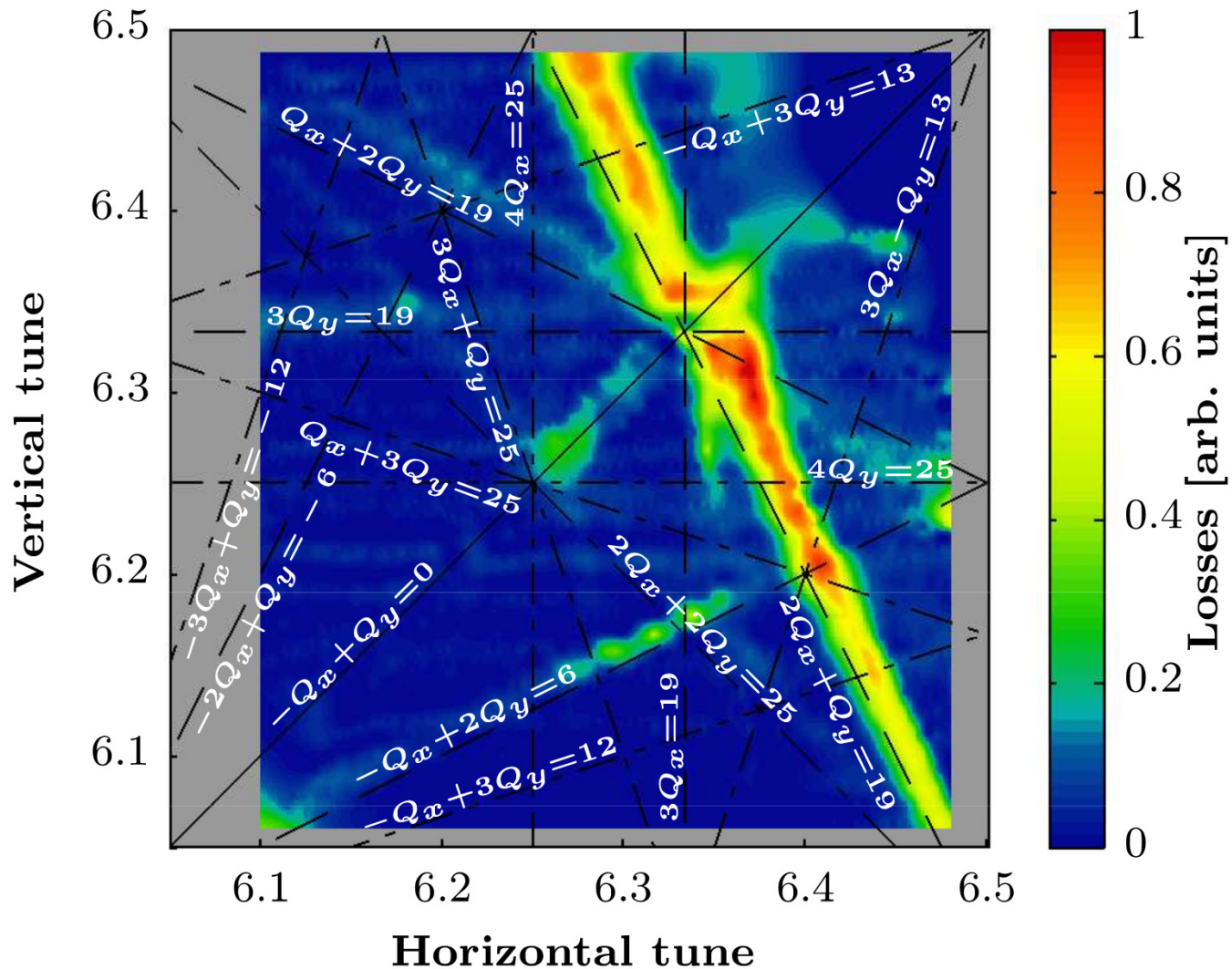
← Orbits become fuzzy →



Very difficult to understand what is going on.

# The quest of the incoherent effects of space charge







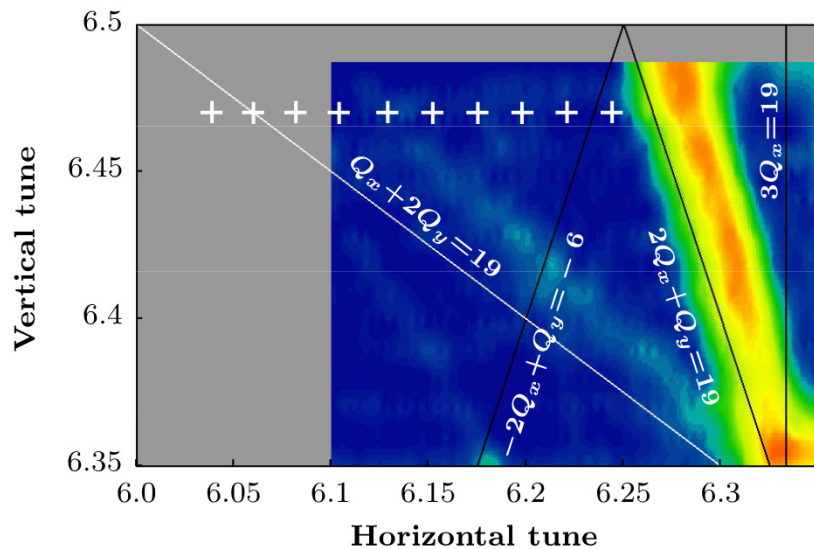
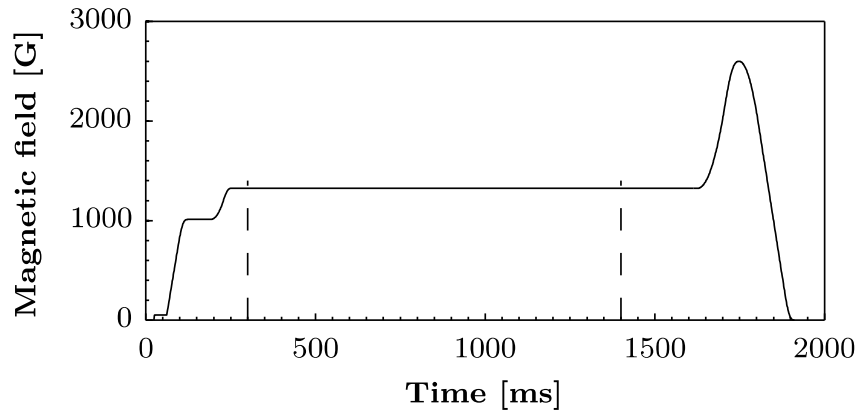
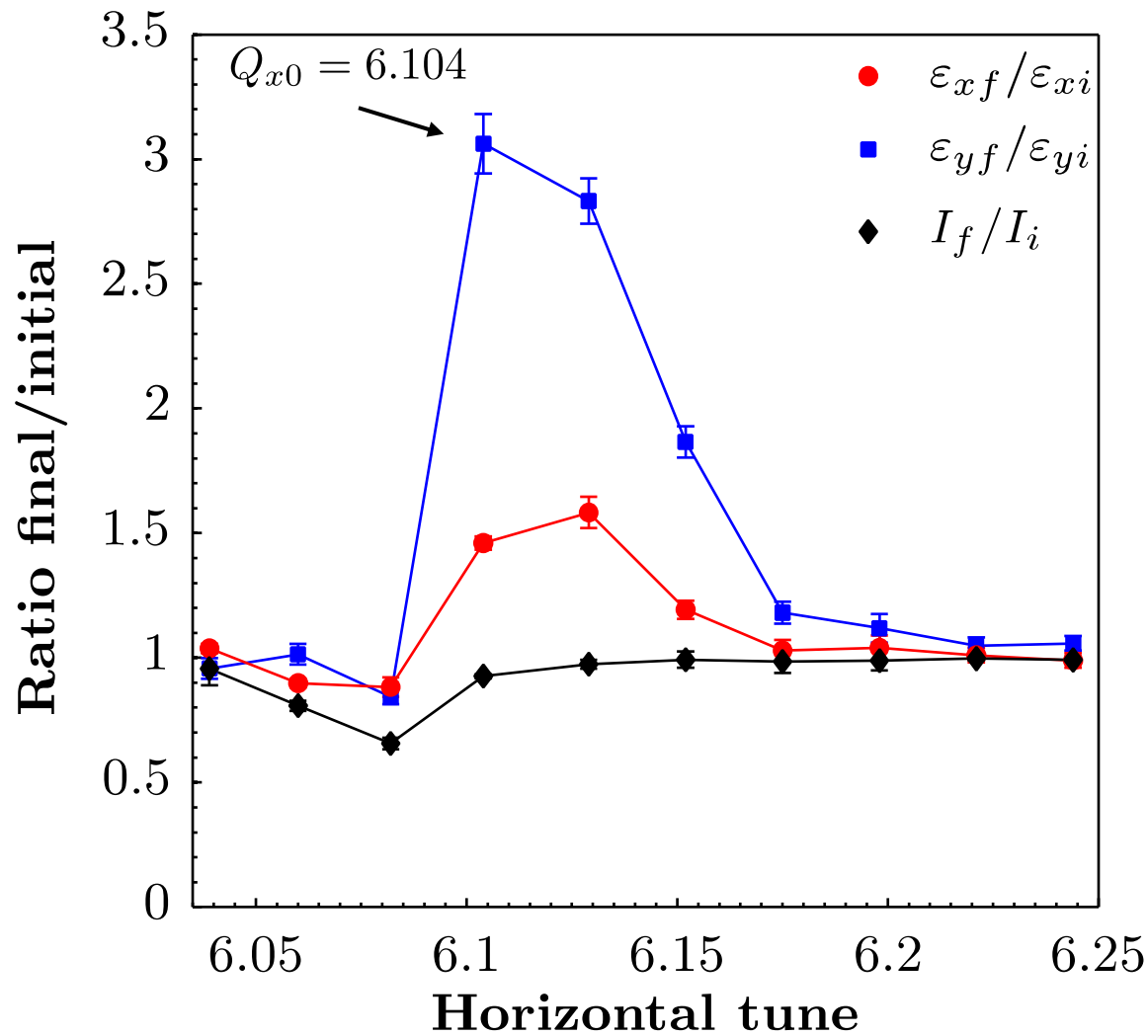


TABLE I. Beam and machine parameters.

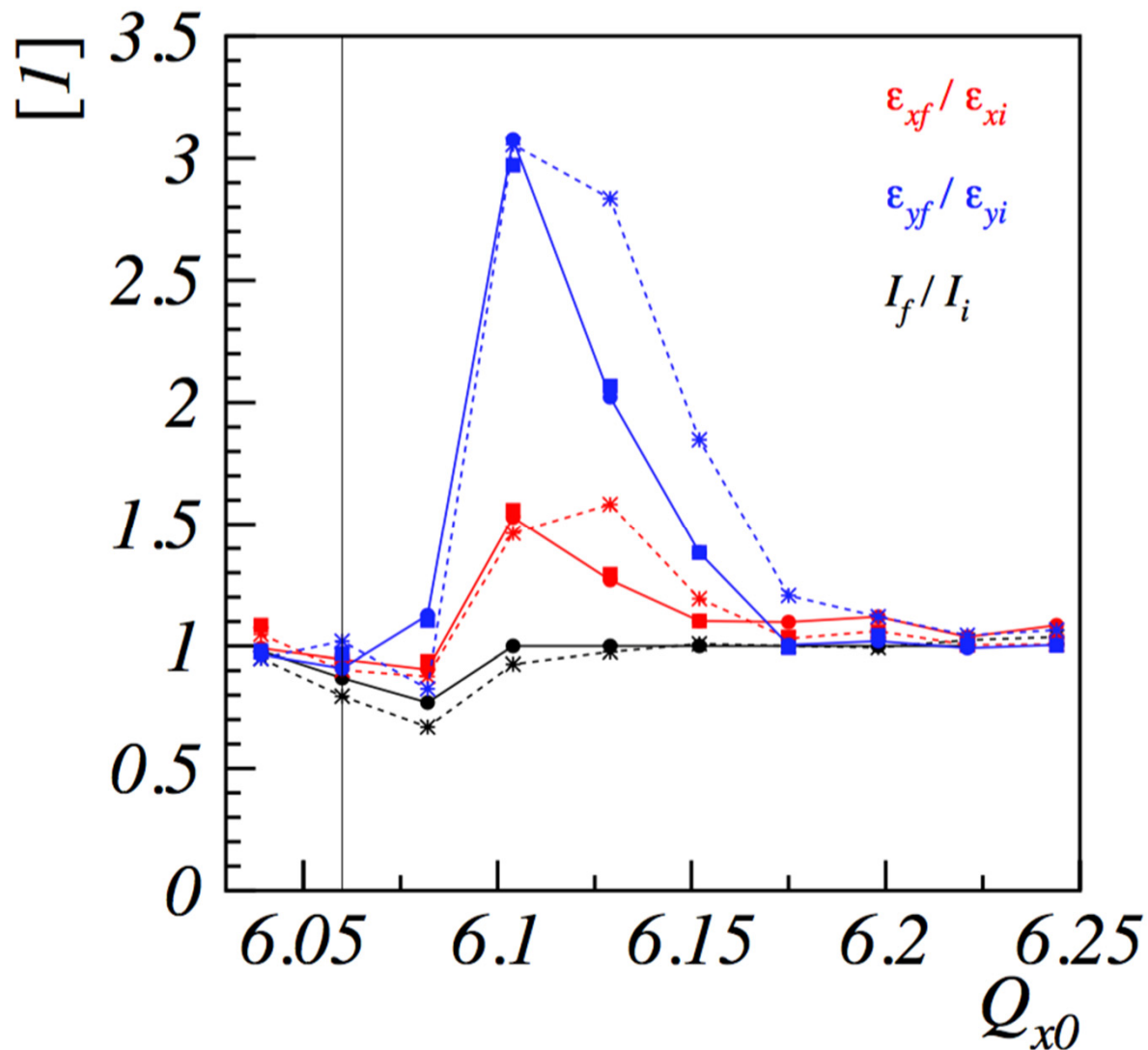
Parameter	Value
Intensity $N_p$ [ $10^{10}$ p]	55
Normalized horizontal rms emittance $\varepsilon_x^n$ [mm mrad]	3.6
Normalized vertical rms emittance $\varepsilon_y^n$ [mm mrad]	2.2
Rms bunch length $\sigma_t$ [ns]	33
Rms momentum spread $\frac{\Delta p}{p}$ [ $10^{-3}$ ]	0.95
Horizontal maximum tune spread $\Delta Q_{x,\max}^a$	-0.05
Vertical maximum tune spread $\Delta Q_{y,\max}^a$	-0.071
Sextupole current $I_{SX}$ [A]	2
Harmonic number $h$	8
RF voltage $V_{RF}$ [kV]	20.5
Horizontal linear chromaticity $\xi_x^b$	-0.83
Vertical linear chromaticity $\xi_y^b$	-1.12
Energy of stored beam [GeV]	2
Turns stored	497646
Storage time [s]	1.1
Relativistic $\beta$	0.948
Relativistic $\gamma$	3.14
Synchrotron tune	$1163^{-1}$
Horizontal flying w. (SS68 at 422.8 m) $\beta_x$ [m]	12.40
Vertical flying w. (SS64 at 397.7 m) $\beta_y$ [m]	21.75

<sup>a</sup> The tune spread is calculated according to Ref. [18].

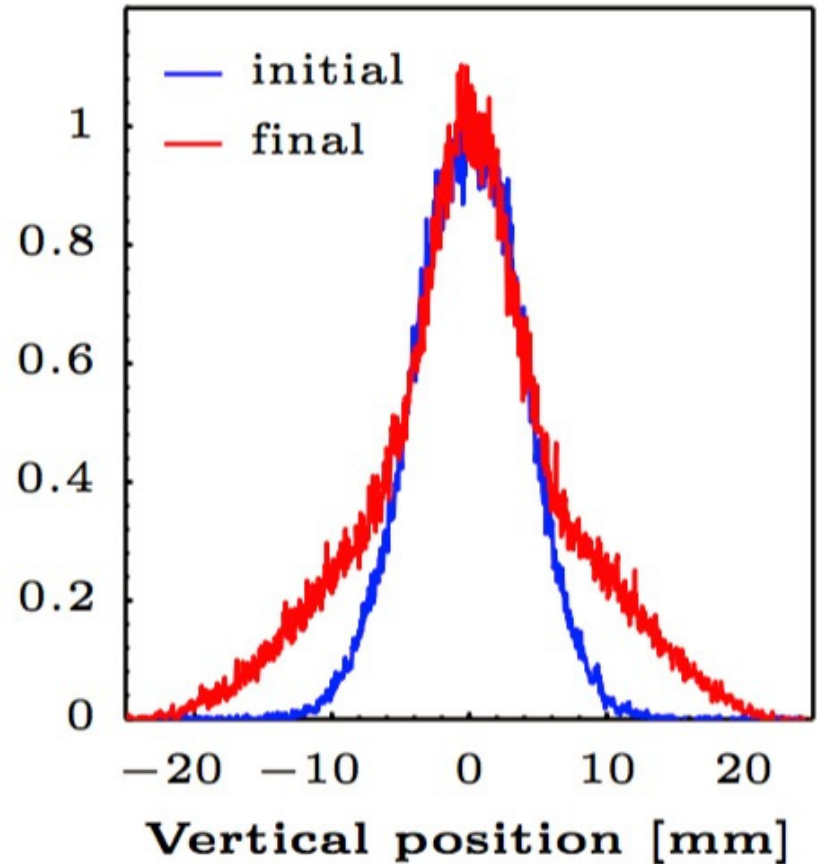
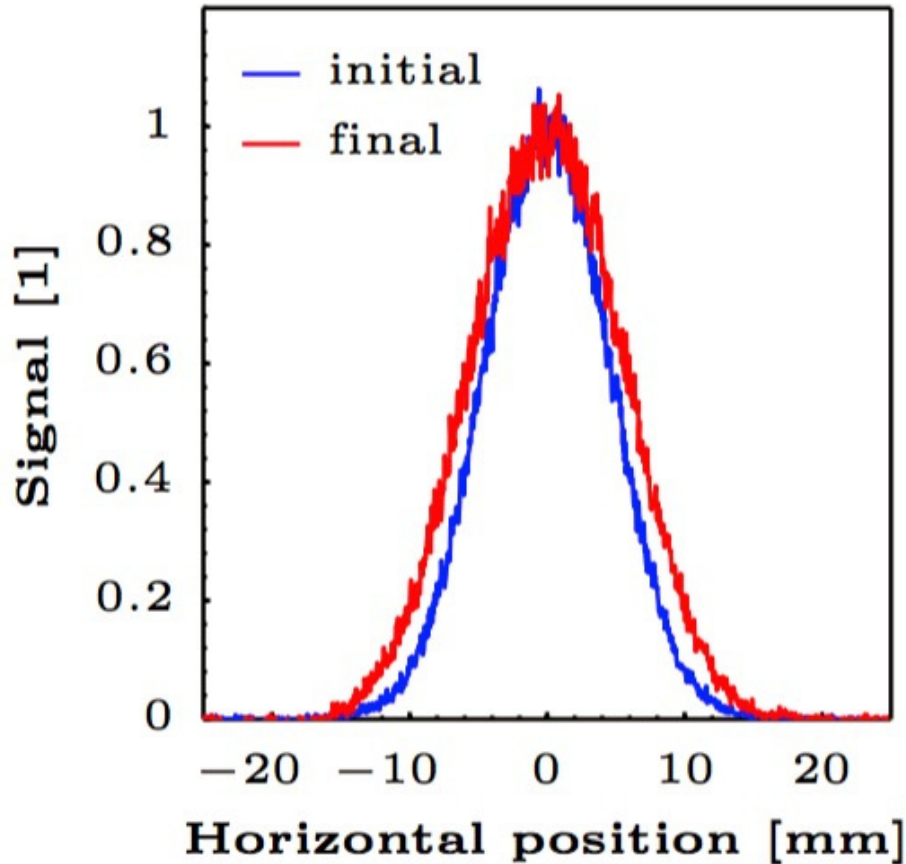
$$^b \xi_{x,y} = \frac{Q'_{x,y}}{Q_{x,y}} = \frac{\Delta Q_{x,y}/Q_{x,y}}{\Delta p/p}$$

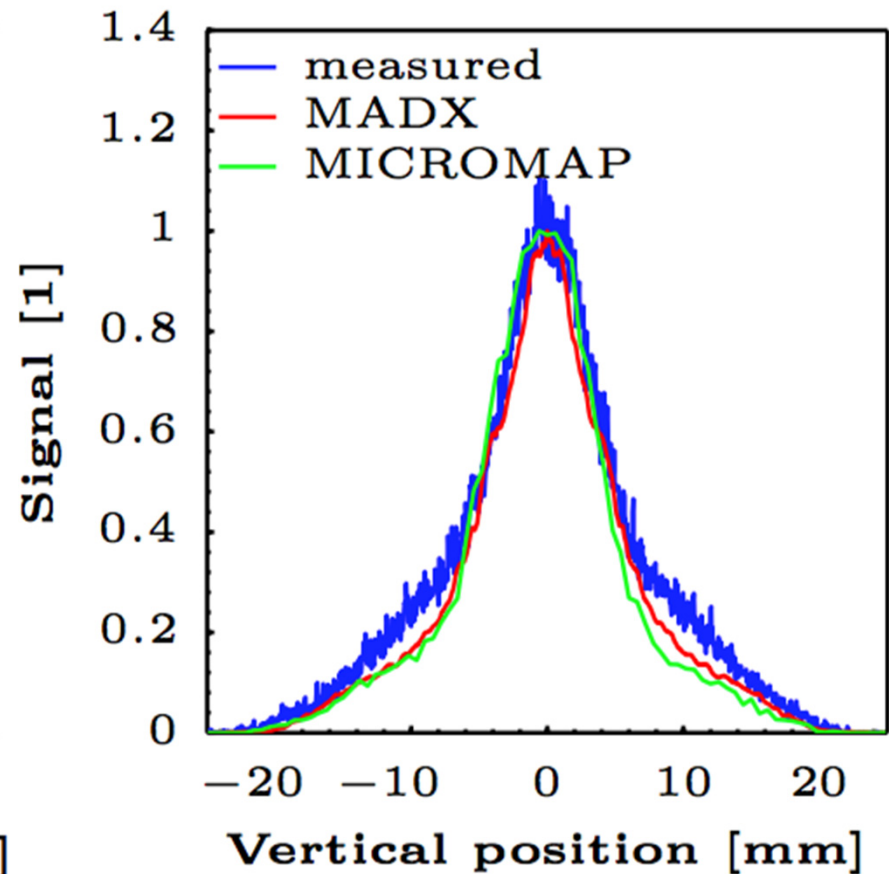
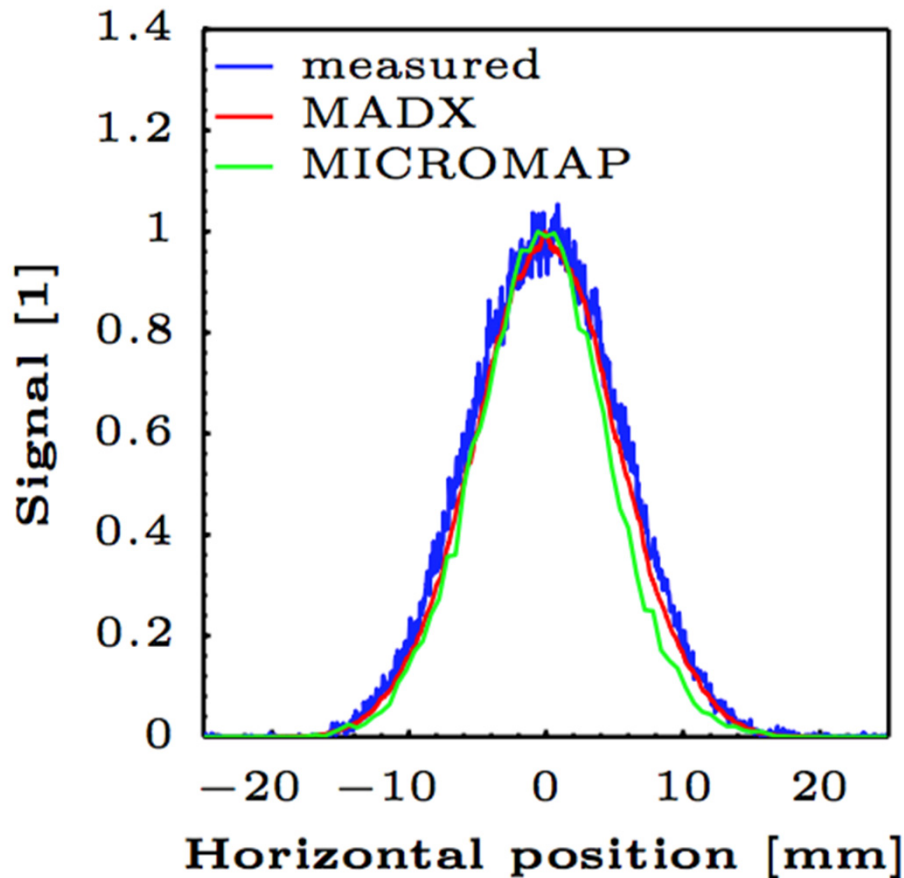


# Comparison with simulations



# Beam Profiles for $Q_{x0} = 6.104$





## No space charge

Distance of the resonance

$$\Delta_{r0} = Q_{x0} + 2Q_{y0} - 19$$

Resonance condition

$$\Delta_{r0} = 0$$

## With space charge

Distance from the resonance for one particle at amplitudes X,Y

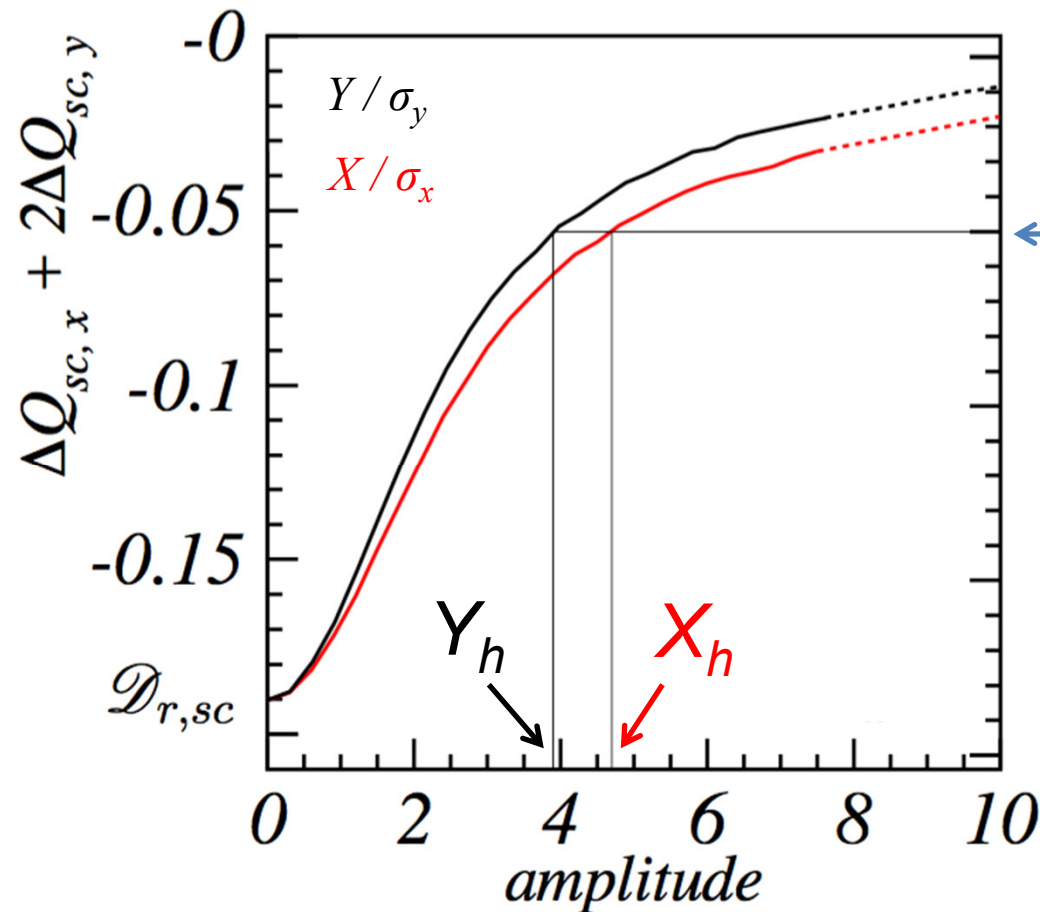
$$\Delta_r = \Delta_{r0} + \Delta Q_{sc,x}(X, Y) + 2\Delta Q_{sc,y}(X, Y)$$

Resonance condition

$$\Delta_r = 0$$

$\Delta_{r0}$  may be different from zero

$$Q_{x0} = 6.104 \longrightarrow \Delta_{r0} = 0.056$$

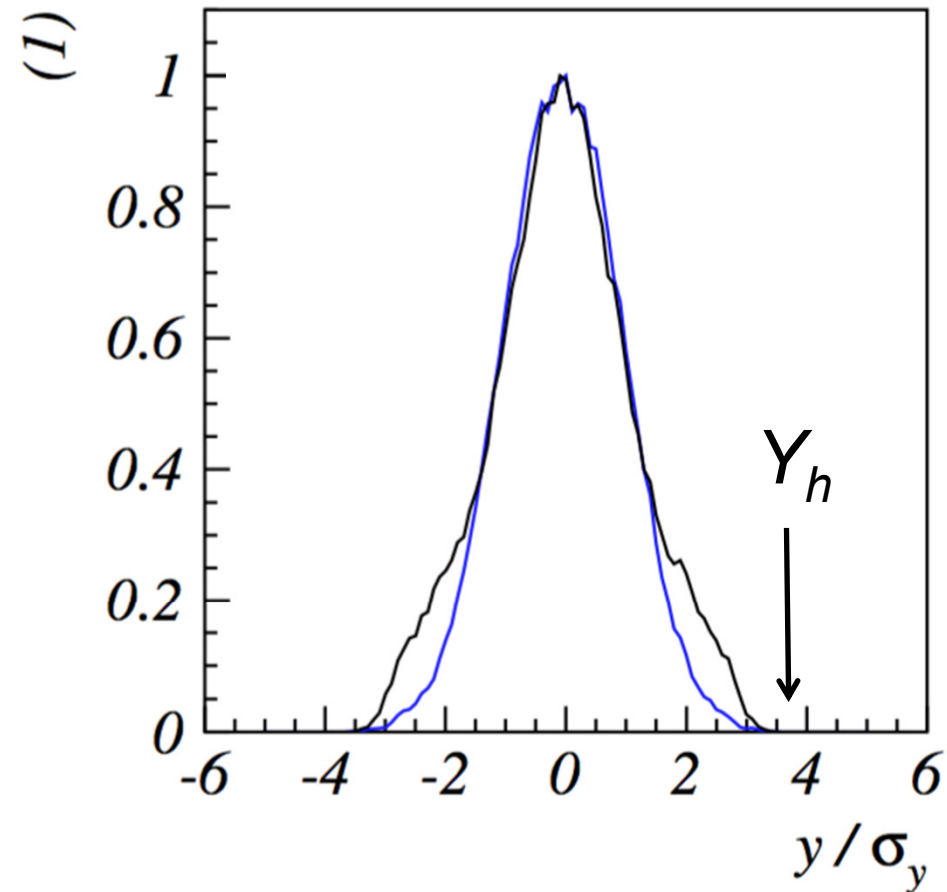
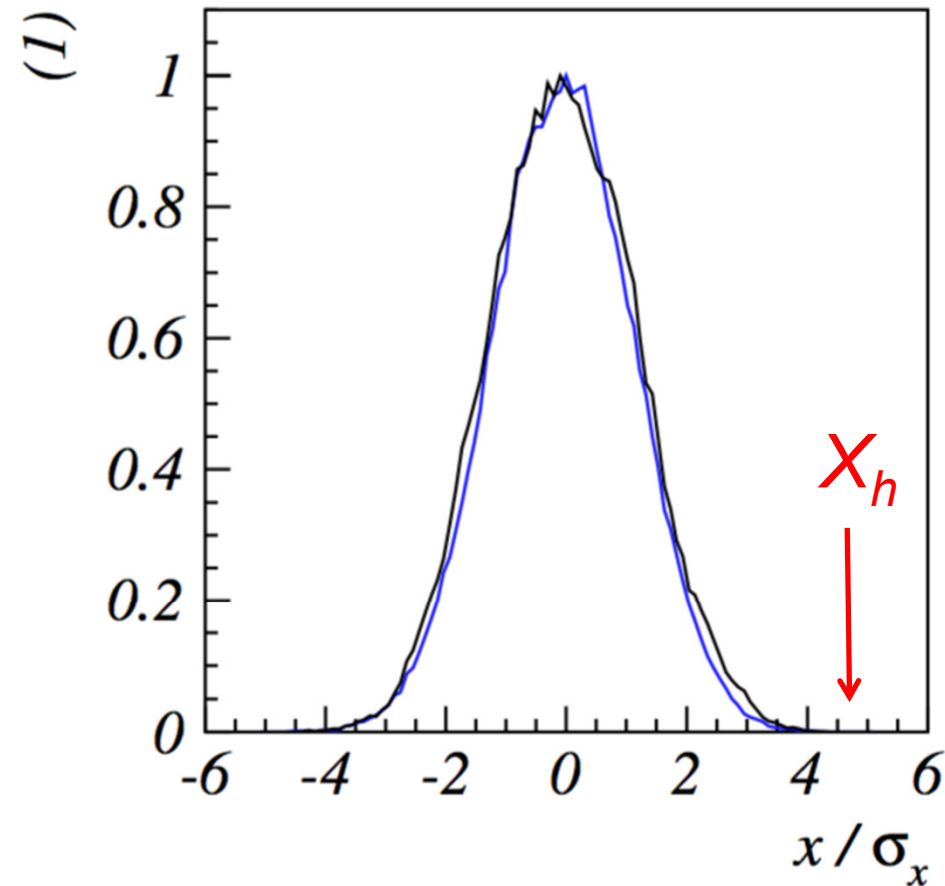


$$\Delta_{r0} = 0.056$$

“resonant detuning cancel  $\Delta_{r0}$  and makes particle resonant”

# Comparison with simulations without chromaticity

$$Q_{x0} = 6.104$$



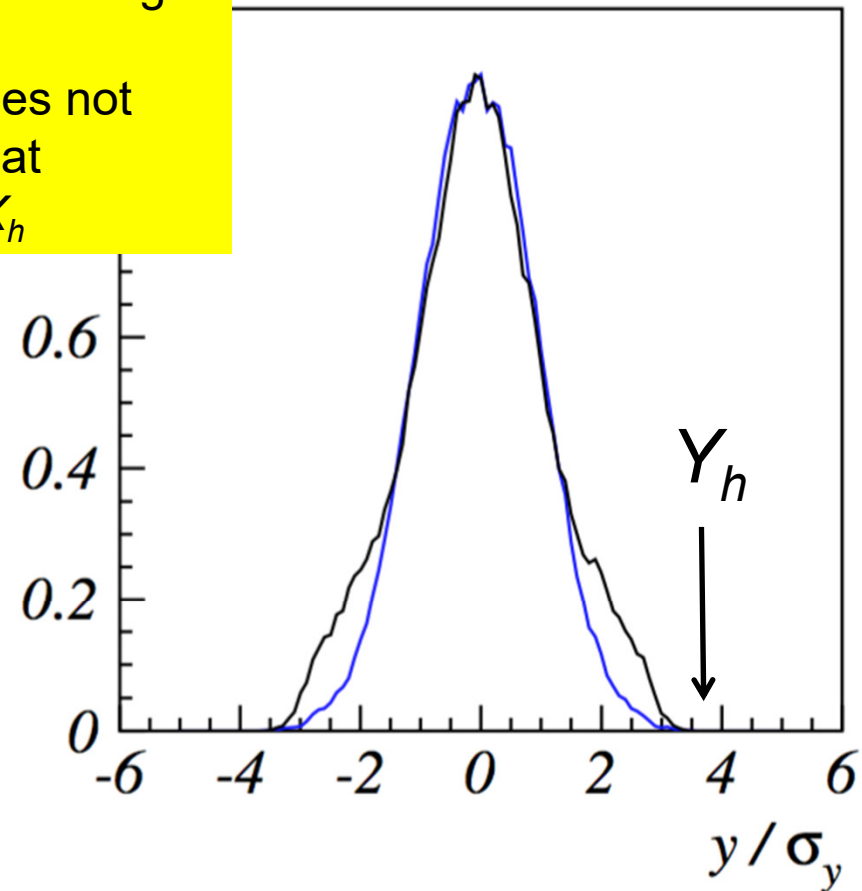
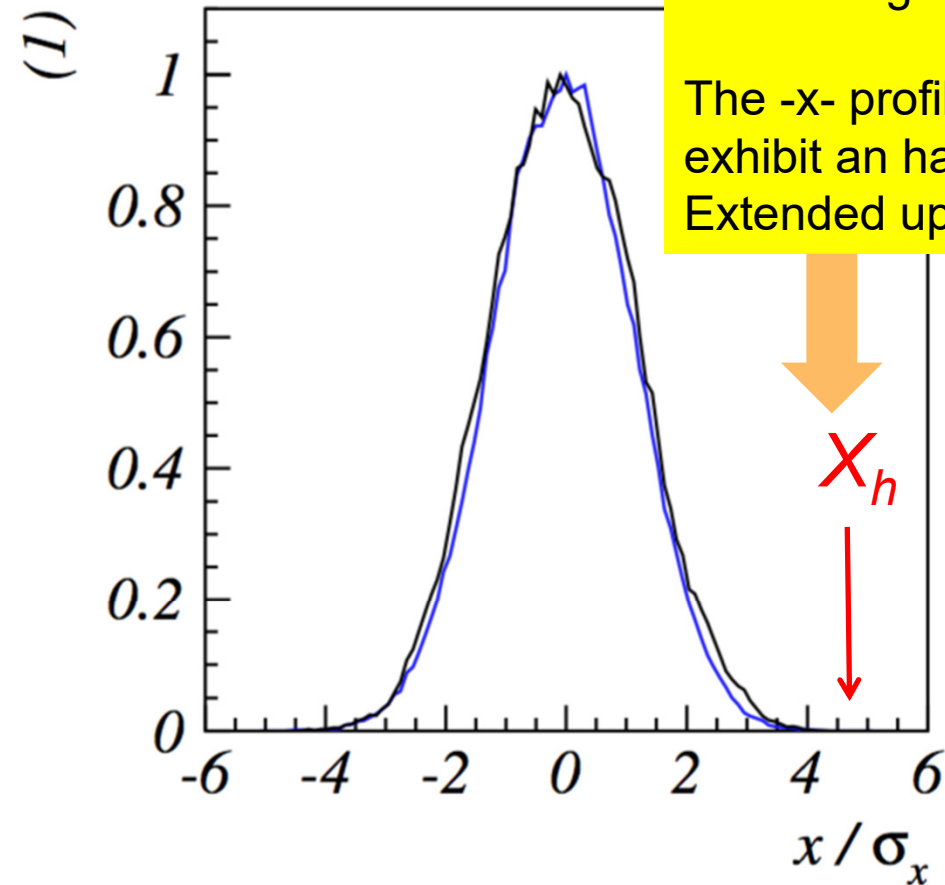


# Comparison with simulations without chromaticity

$$Q_{x0} = 6.104$$

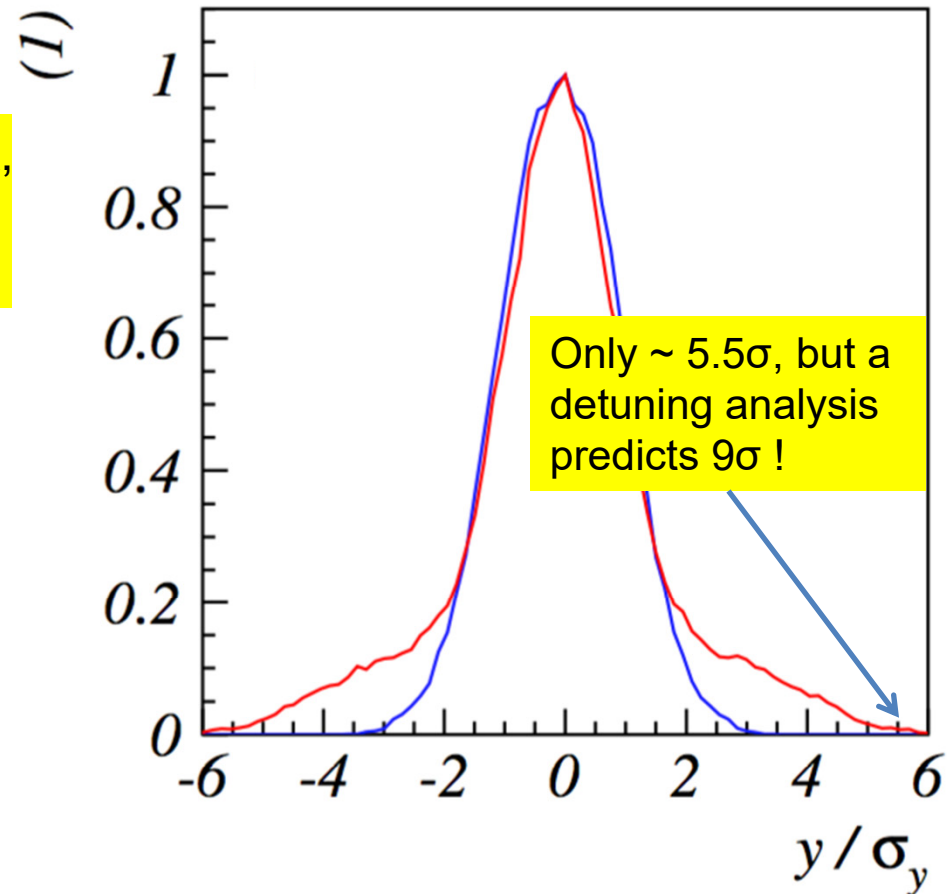
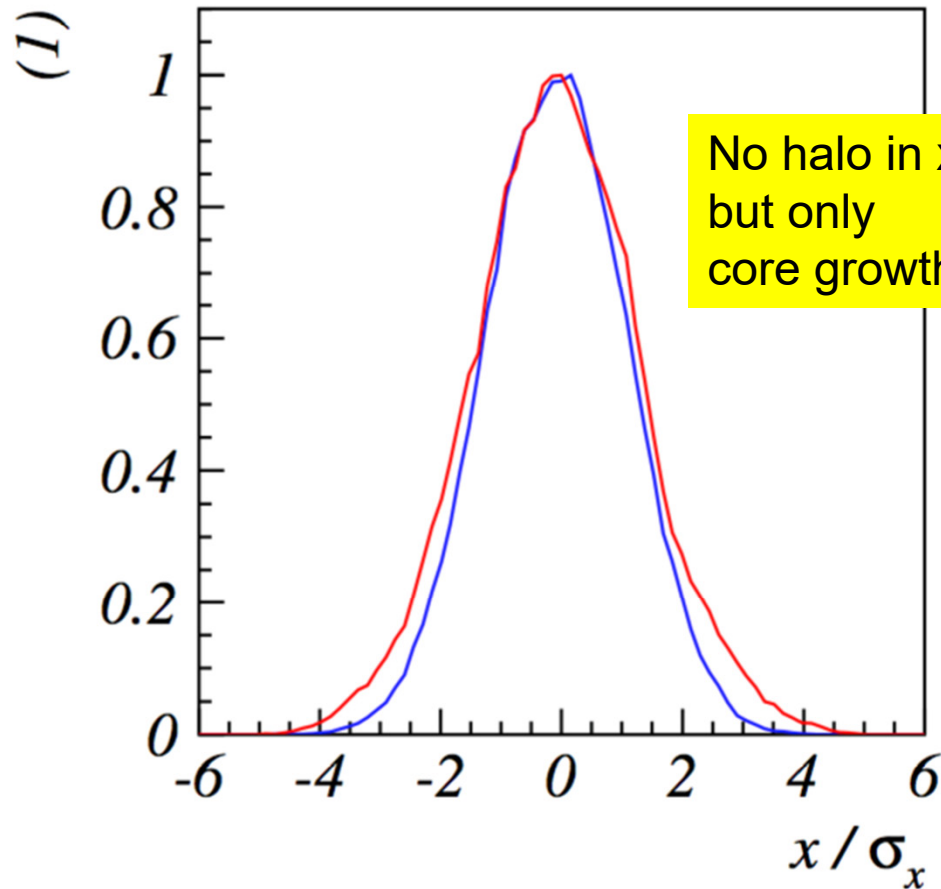
Something seems wrong !!

The -x- profile does not exhibit an halo that Extended up to  $X_h$



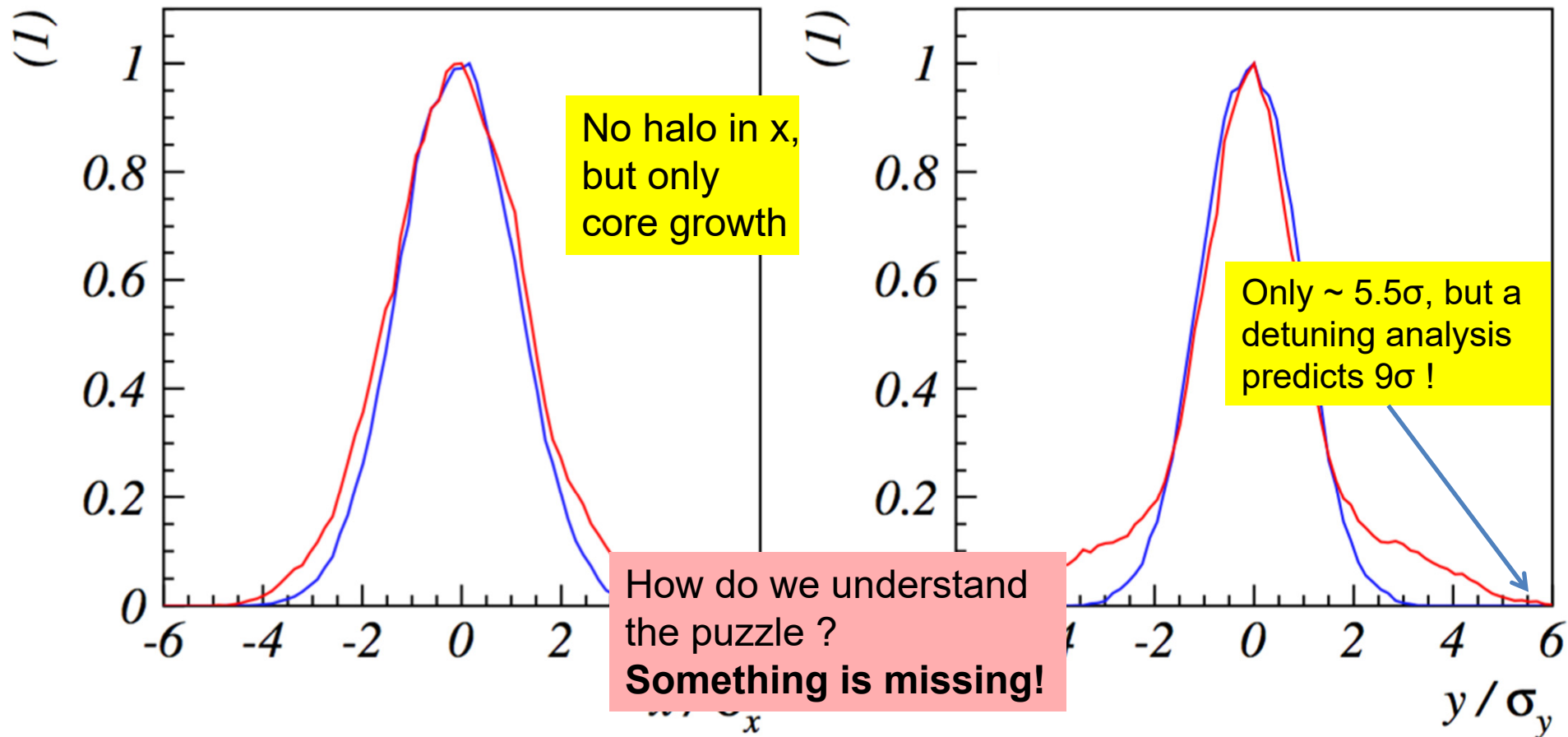
# Comparison with simulations including chromaticity

$$Q_{x0} = 6.104$$



# Comparison with simulations including chromaticity

$$Q_{x0} = 6.104$$



# Missing: the coupled dynamics on the resonance

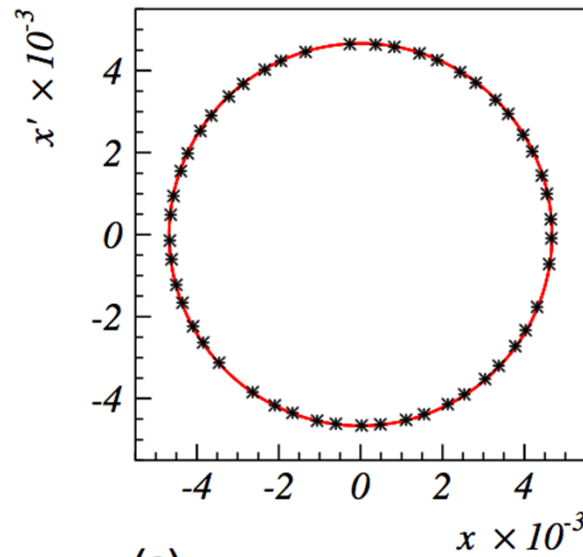


**DANGER!**  
**Fixed-lines**

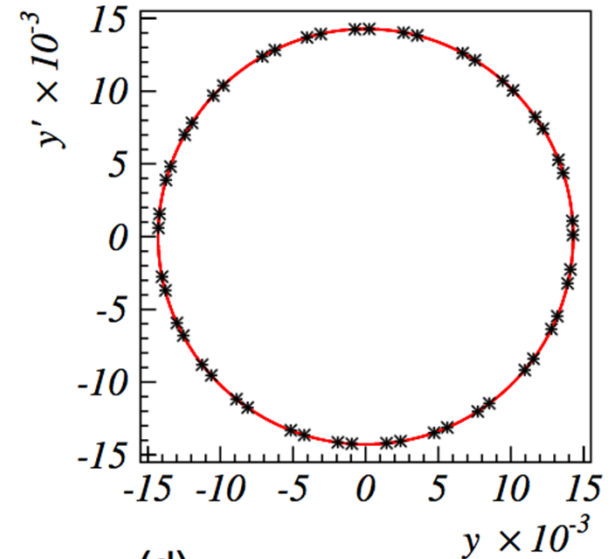
F. Schmidt PhD thesis, *and others*

G. Franchetti and F. Schmidt  
Phys. Rev. Lett. **114**, 234801 (2015).

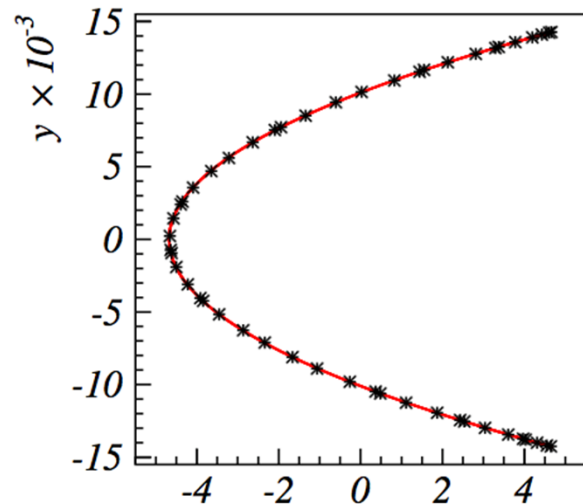
G. Franchetti and F. Schmidt  
<http://arxiv.org/abs/1504.04389>



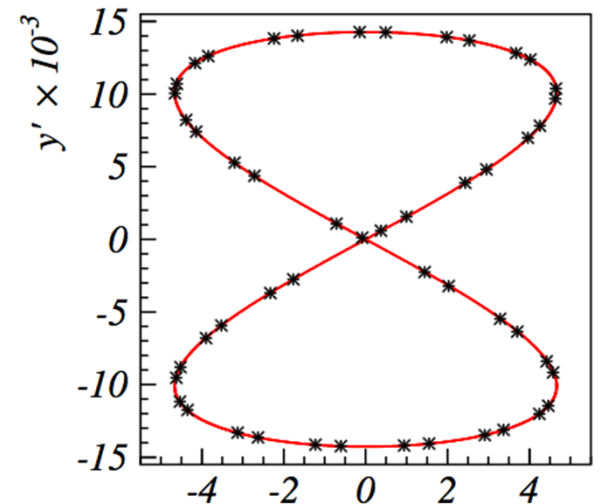
(c)



(d)



G. Franchetti  $x \times 10^{-3}$



17.05.2017  $x' \times 10^{-3}$

# SPS campaign on May 2015

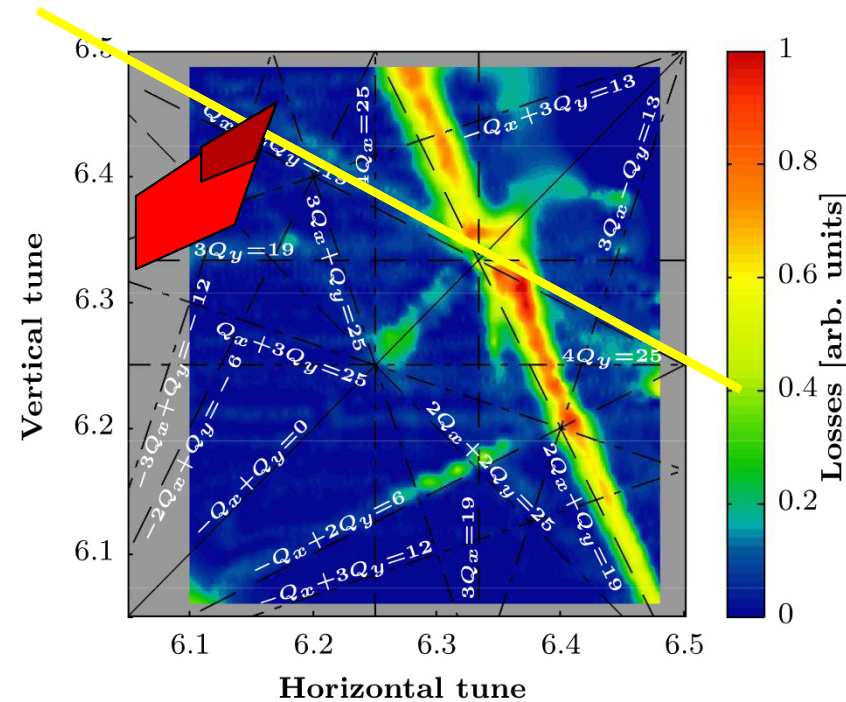
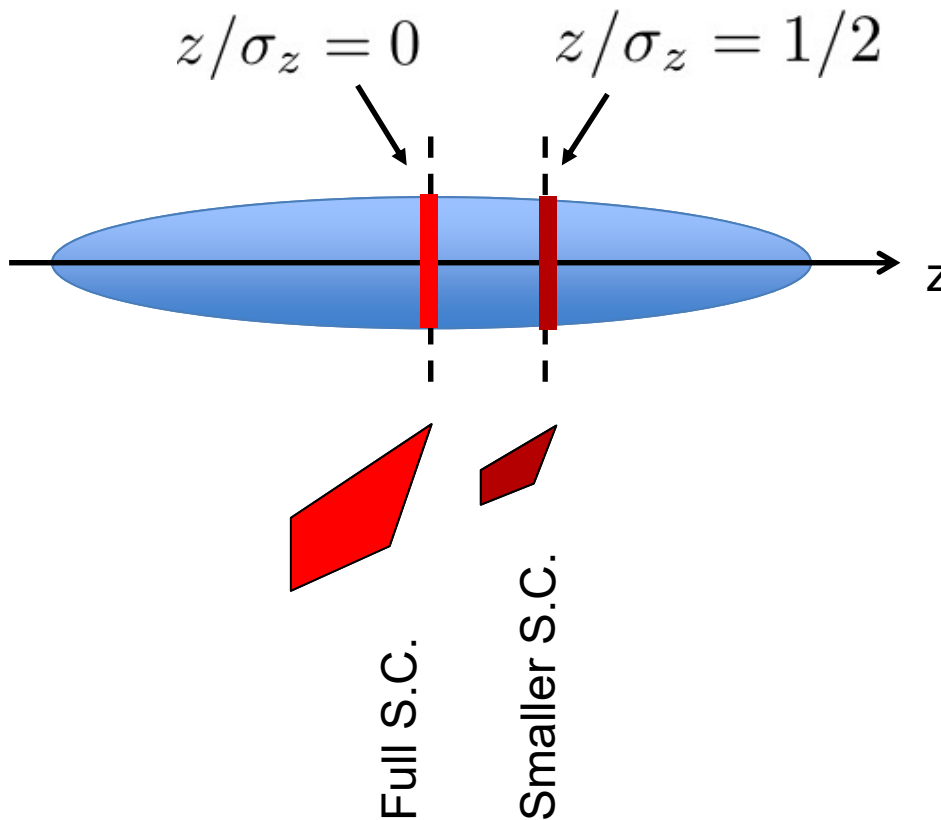
Experiment organized by F. Schmidt



*Fixed-lines do exist*

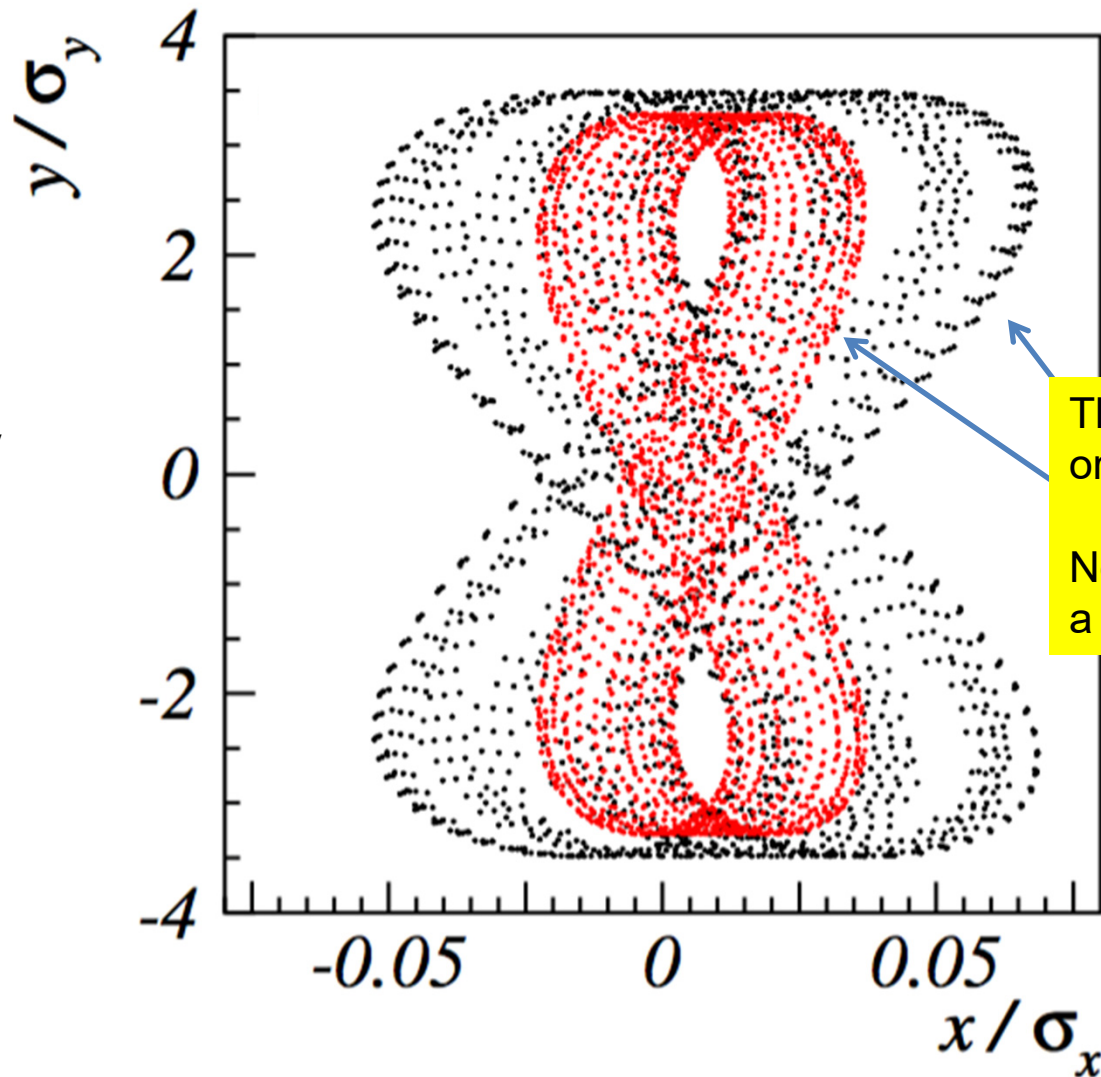
# Does fixed-lines play a role with space charge ?

longitudinal motion is kept frozen, so to retrieve Poincare' section orbits



# Largest resonant orbits at $z/\sigma_z = 0$

No  
chromaticity

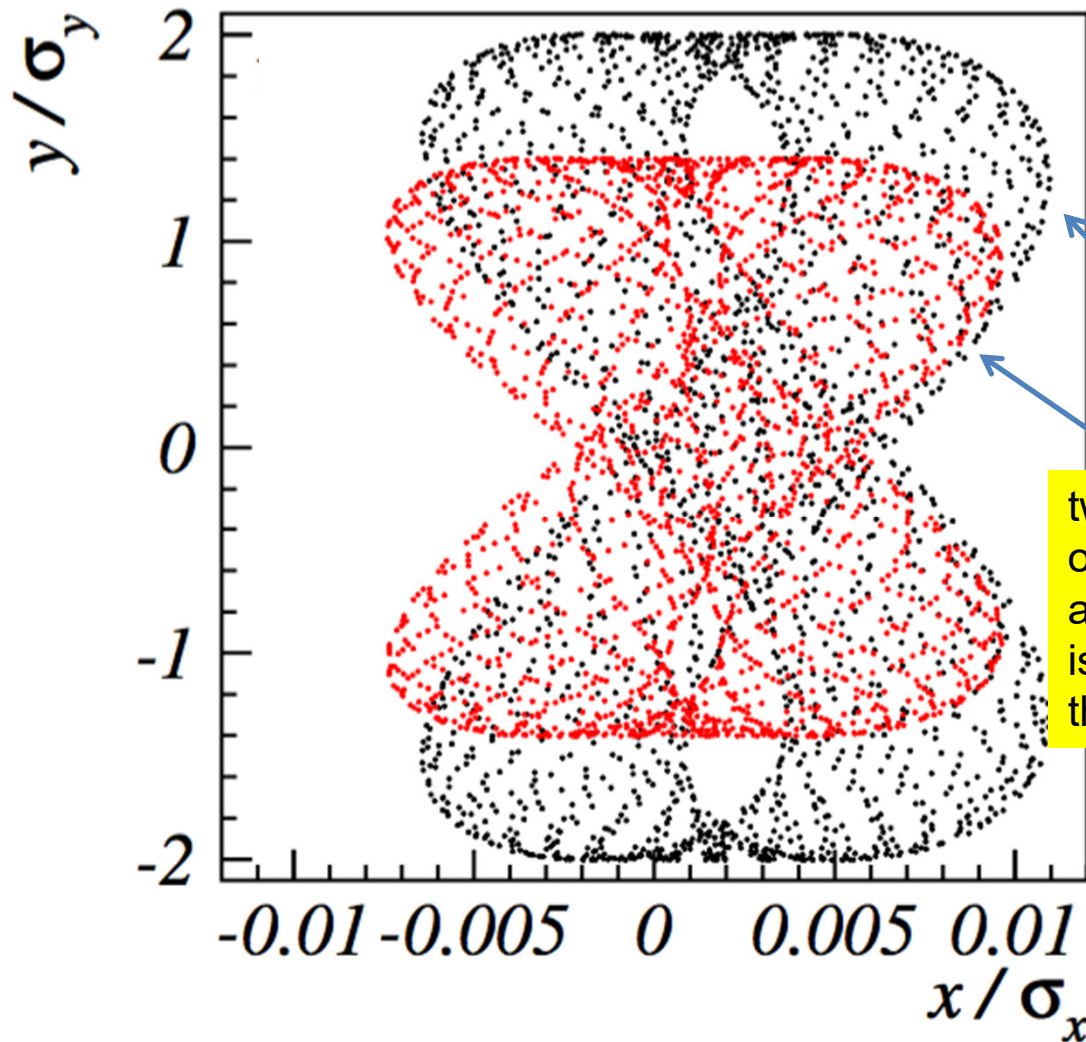


The two larger resonant orbits...

No doubt they have a structure of fixed-lines

# Largest resonant orbits at $z/\sigma_z = 1/2$

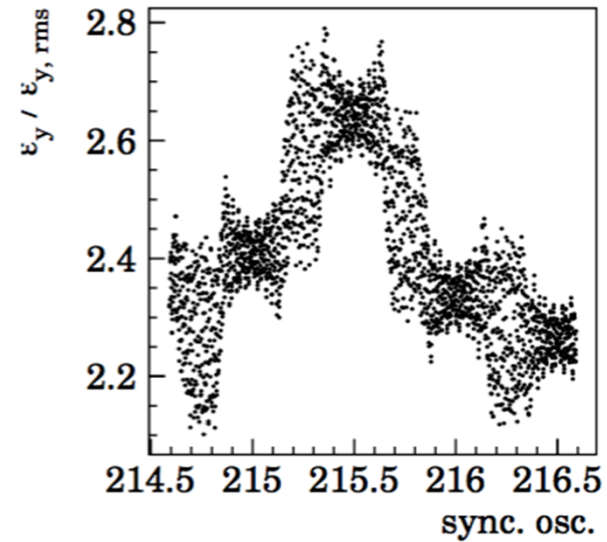
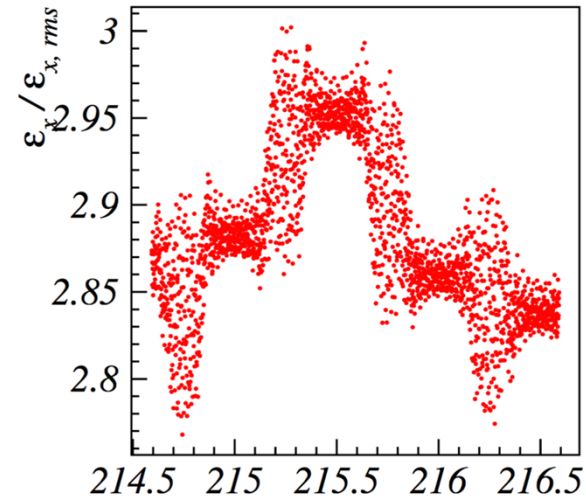
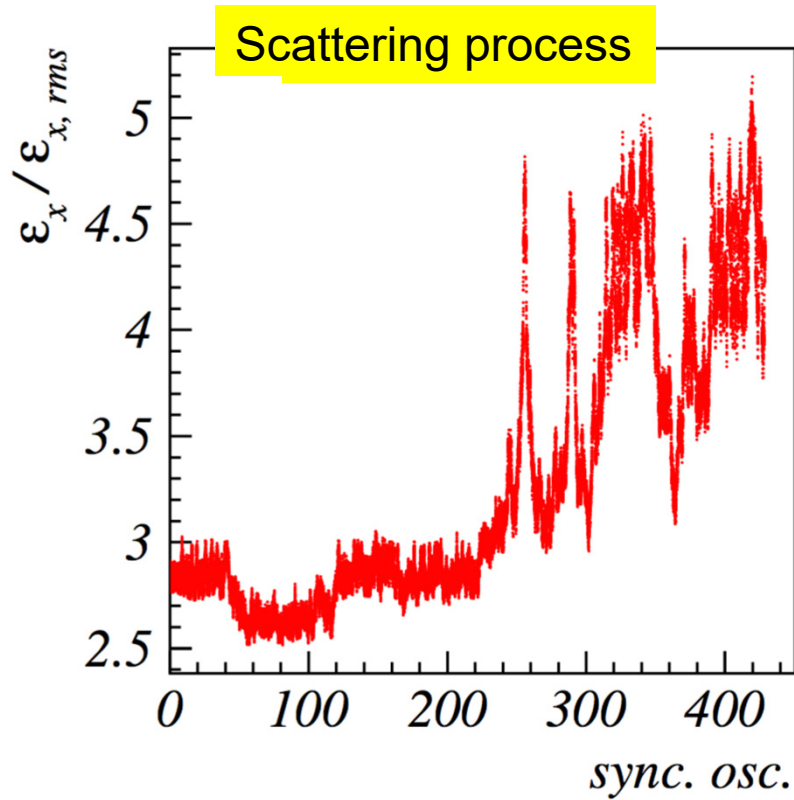
No  
chromaticity



two larger resonant  
orbits: now the  
amplitude of the orbits  
is smaller, but still have  
the structure of fixed-lines

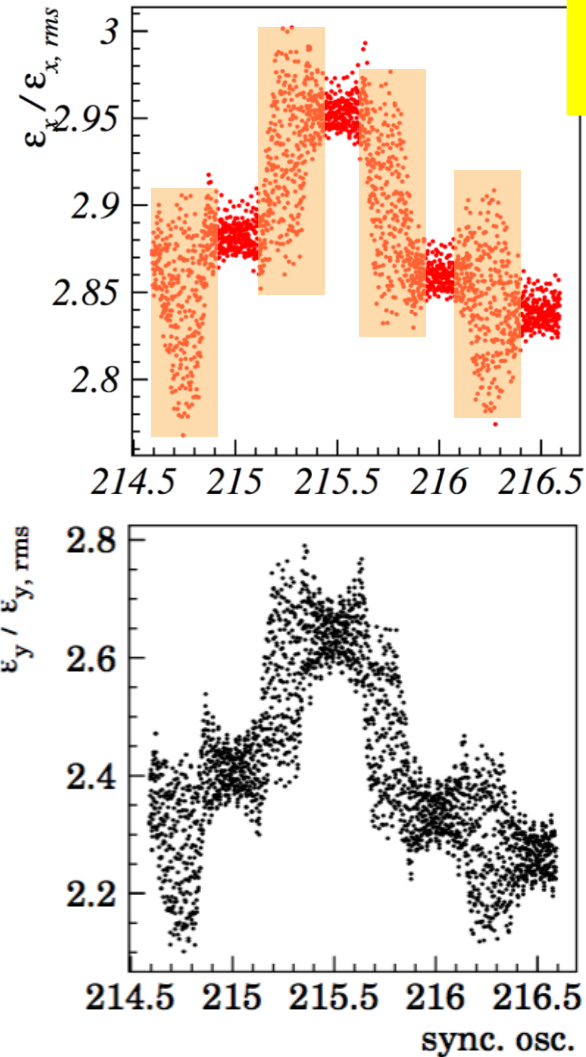
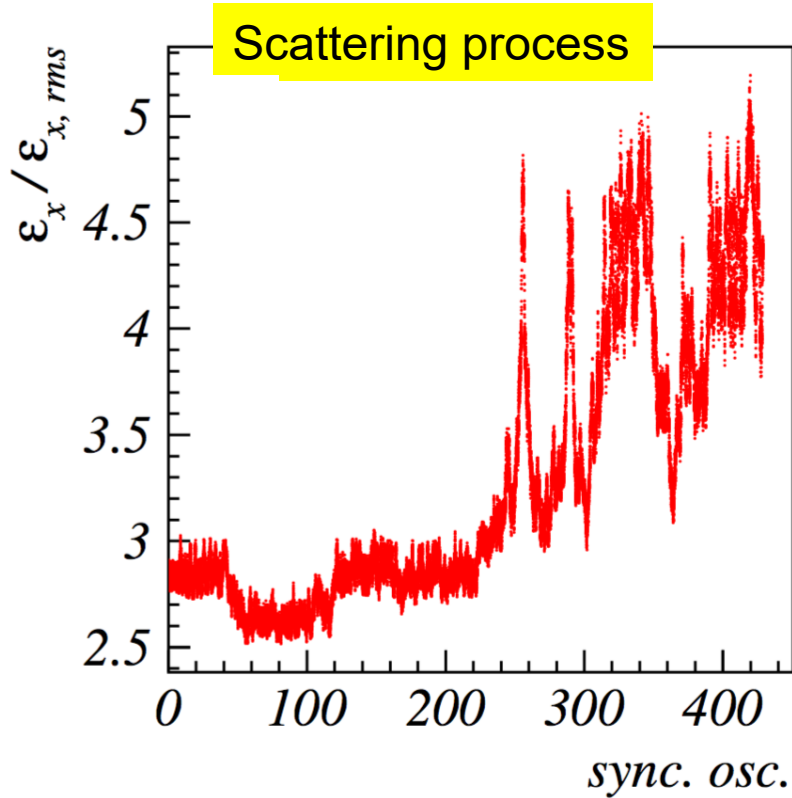


# Periodic crossing of fixed-lines



Extended paper to appear in PRAB

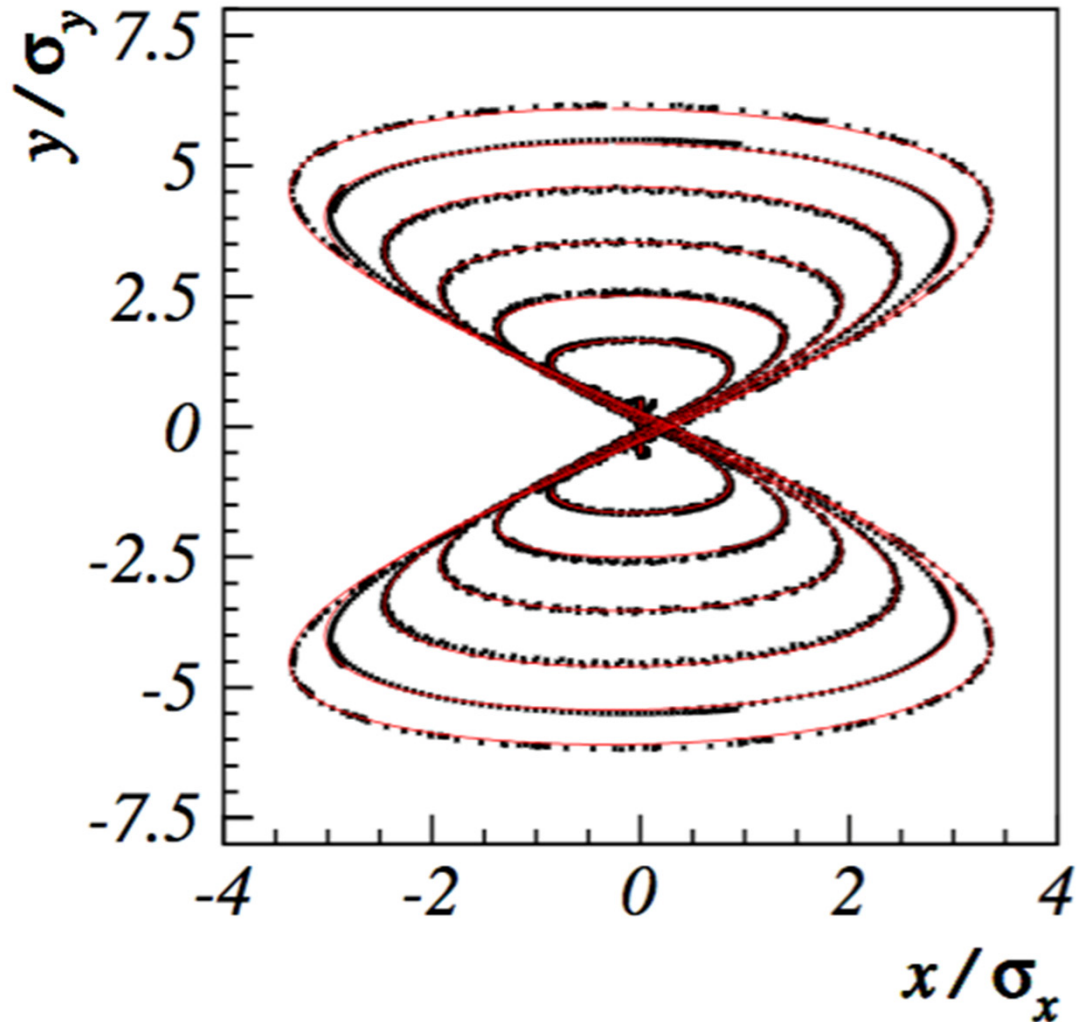
# Periodic crossing of fixed-lines



4 kicks per synchrotron oscillation

Extended paper to appear in PRAB

# Prediction of the halo size: the adiabatic limit

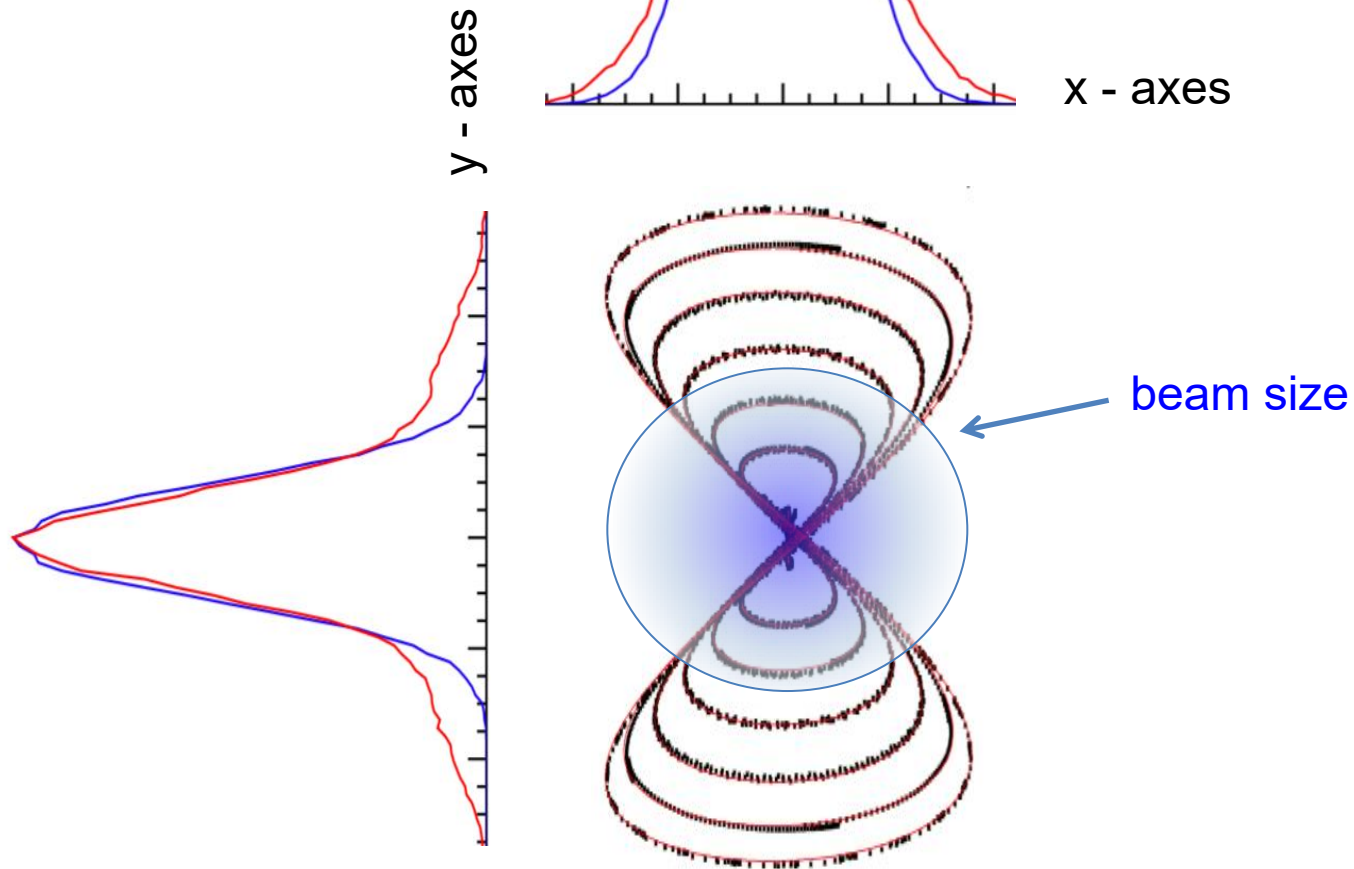


$$x = \sqrt{\beta_x a_x} \cos(-2t - \alpha + \pi M)$$
$$y = \sqrt{\beta_y a_y} \cos(t)$$

For adiabatic synchrotron motion  
all particles trapped are transported  
to the “same” fixed-line

The sizes of this fixed-line  
characterize the halo/core  
formation

# Halo asymmetry explained with fixed lines



- A successful experiment-code benchmarking of the beam dynamics on the 3<sup>rd</sup> order coupled resonance is carried out for the full PS structure.
- Outstanding asymmetric halo is formed well retrieved by the simulations
- Thinking in terms of resonance detuning leads paradoxes
- The “fixed-lines” or tori are the new objects that explain the dynamics of diffusion in a high intensity bunch subject to a coupled resonance
- **“Fixed lines” are experimentally measured in the SPS**
- Simulations show that the periodic crossing of the fixed-lines causes the asymmetric halo as result of fixed lines geometry.
- *Particle seems to diffuse to “one” fixed-line → adiabatic limit*
- **The doors are open for massive studies of all coupled resonances and space charge**
- **→ Strategies to mitigate particle diffusion**

# Outlook

Open problems:

- Estimating the diffusion time
- Mitigation strategies:
  - 1) Resonance compensation
  - 2) E-lenses ?
- Coherent vs. incoherent...



# SPACE CHARGE 2017

Chairs: O.Boine-Frankenheim, G. Franchetti  
Secretary: P. Lindenberg  
4-6 October 2017, TUD, Darmstadt

## International Advisory Committee

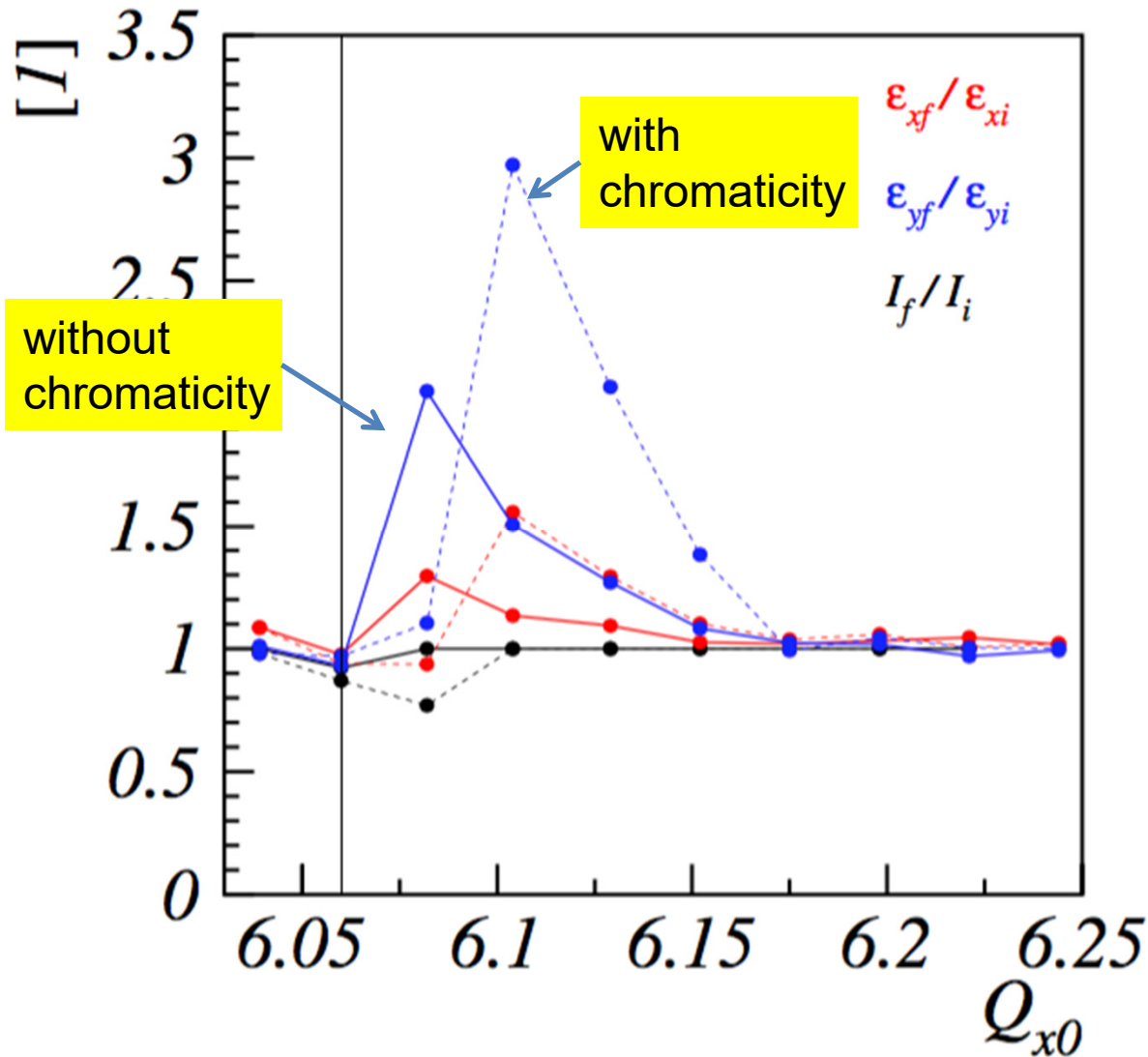
M. Bai	GSI
B. Beaudoin	UMD
Y-Ho Chin	KEK
I. Hofmann	TUDa/GSI
J. Holmes	ORNL
A. Lombardi	CERN
D. O Jeon	IBS
S. Machida	RAL
F. Schmidt	CERN
J-L. Vay	LBNL
S. Webb	Radiasoft
H. Zhao	IMP/ADSR/HIAF

<https://indico.gsi.de/conferenceDisplay.py?confId=5600>

With permission of O.Dolinsky



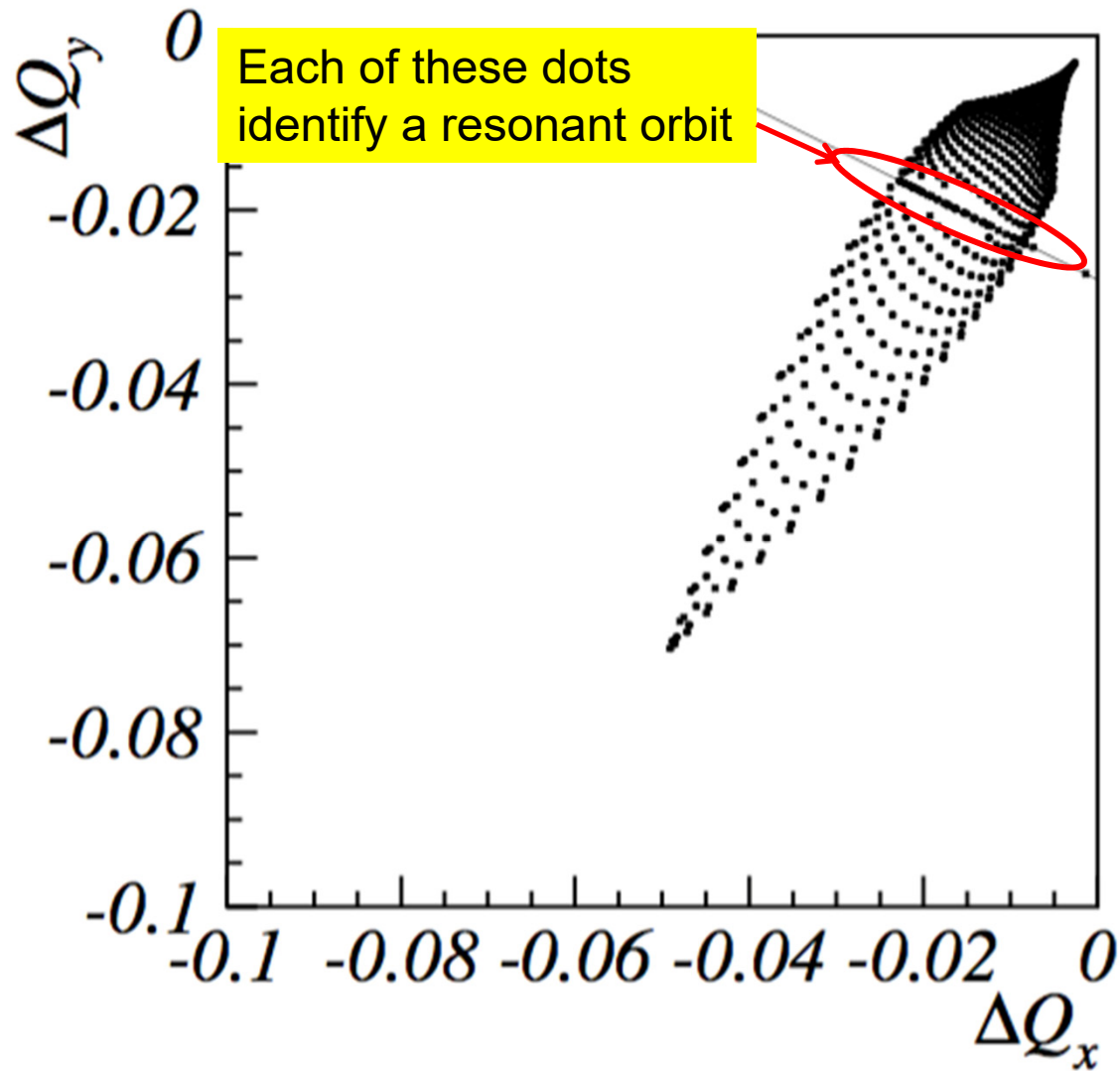
# Simulations: the effect of chromaticity



How do we understand the puzzle ?

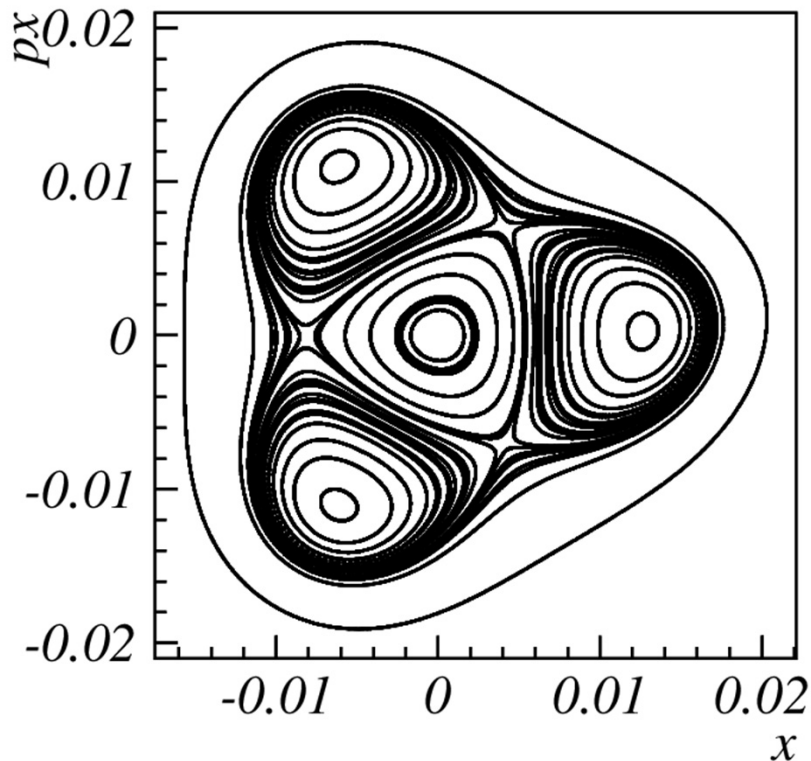
**Something is missing!**



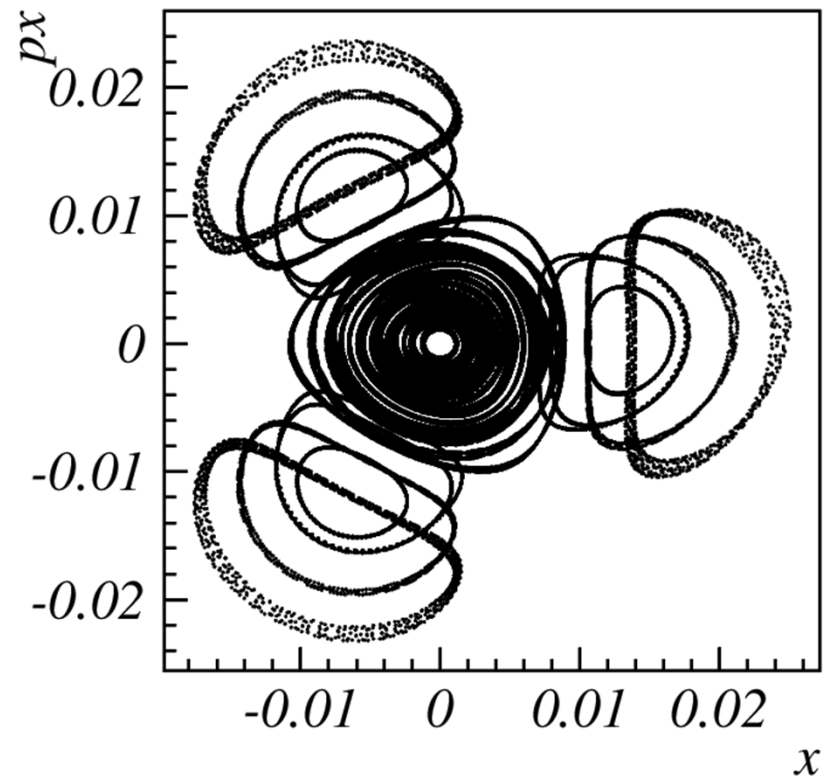


# The difficulty of the coupled dynamics

Near the resonance  $3 Q_x = 13$   $Q_x = 4.335$ ,  $Q_y = 3.27$



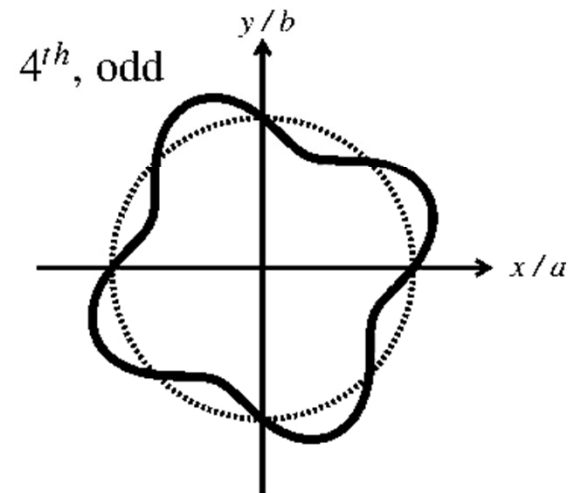
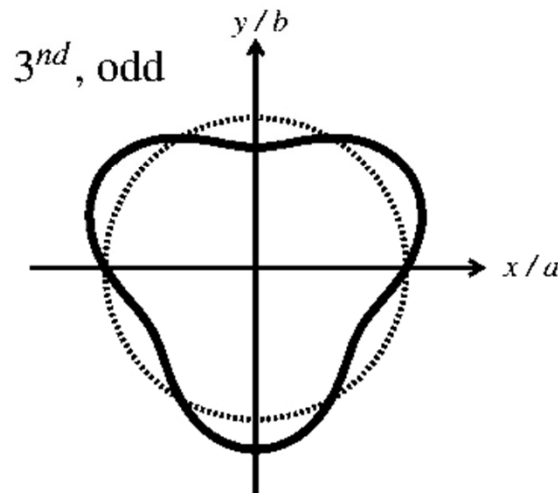
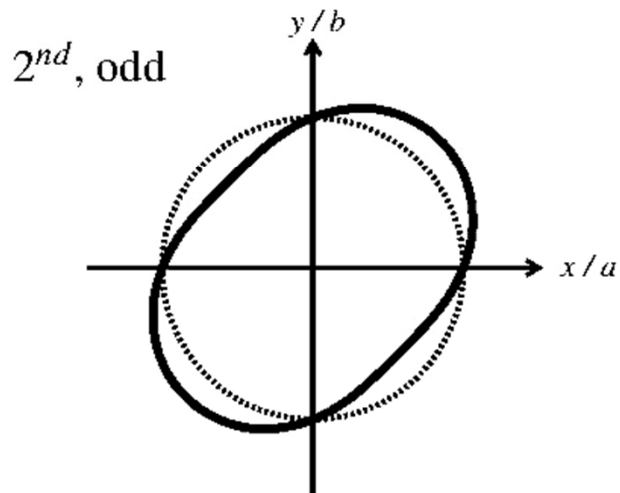
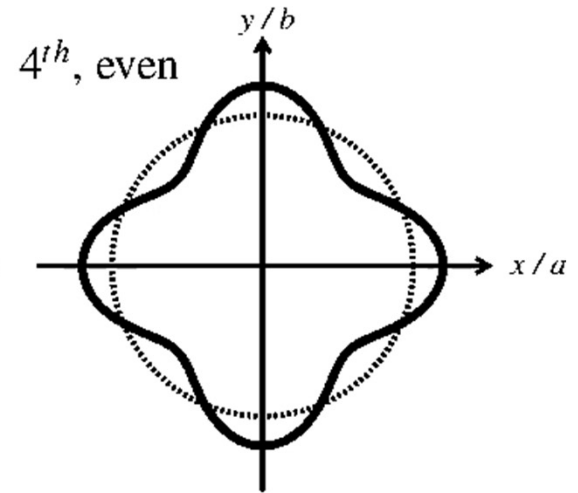
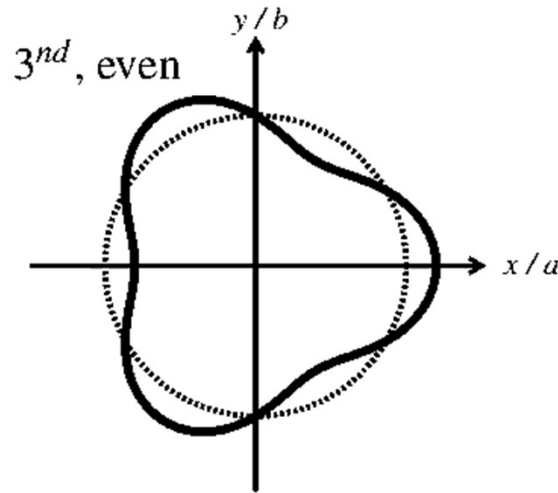
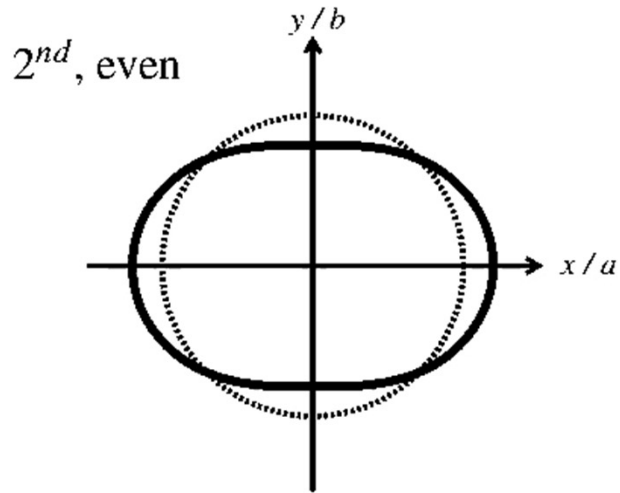
$$y = y' = 0$$



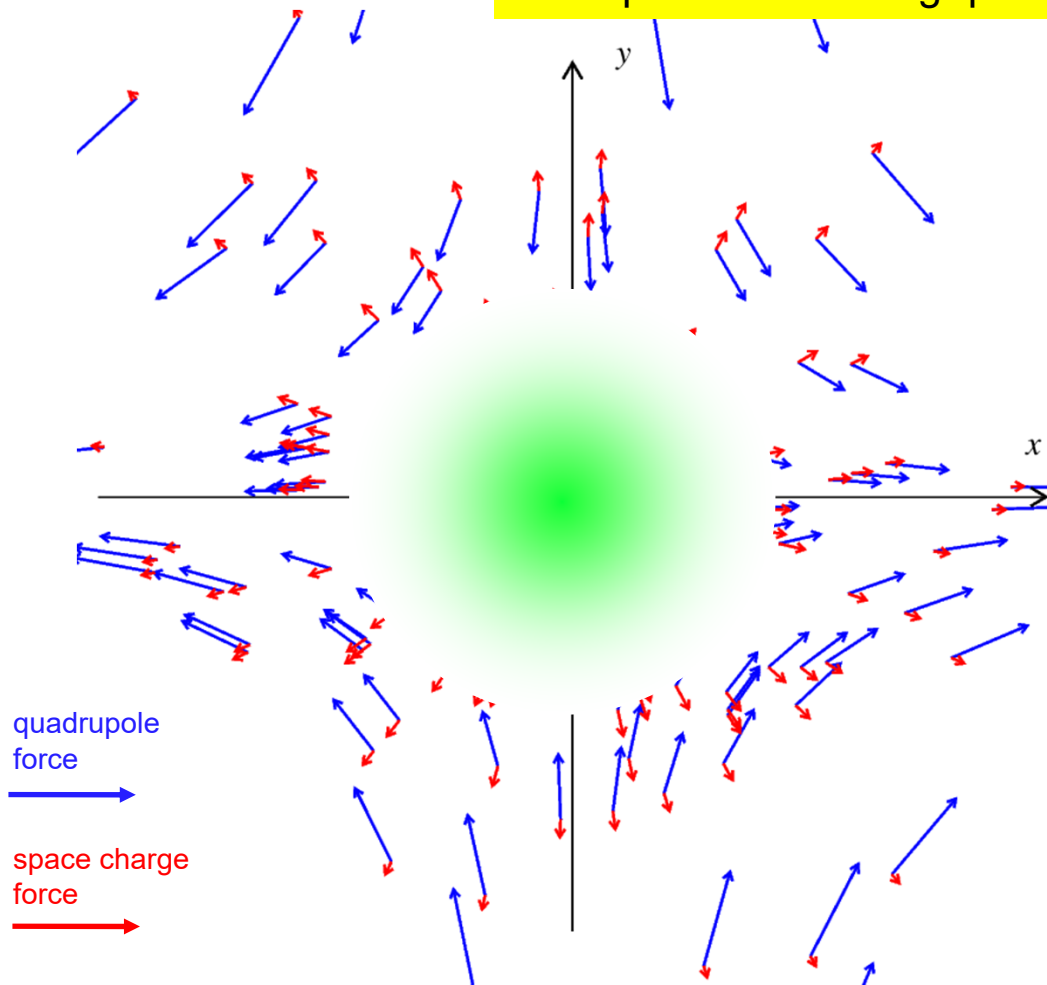
$$y \neq y' \neq 0$$

X - Y coupling

# Modes of oscillation



## Example in a focusing quadrupole



Quadrupole forces

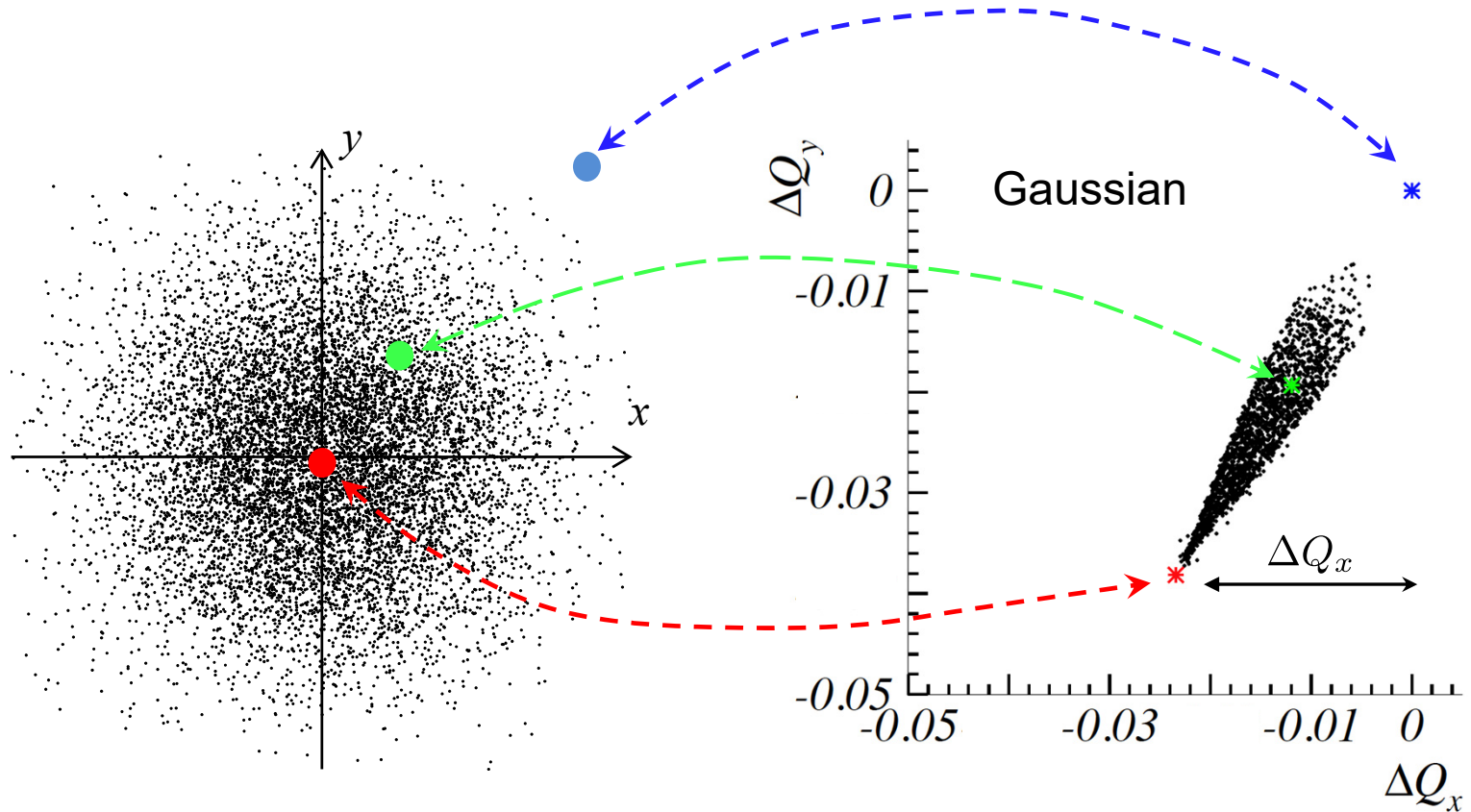
$$F_x = kx$$

$$F_y = -ky$$

Space charge forces

$$F_x = K \frac{x}{r^2} \left( 1 - e^{-\frac{1}{2} \frac{x^2 + y^2}{\sigma^2}} \right)$$

$$F_y = K \frac{y}{r^2} \left( 1 - e^{-\frac{1}{2} \frac{x^2 + y^2}{\sigma^2}} \right)$$



For a Gaussian distribution

$$\Delta Q_x = -\frac{R^2 K}{Q_x} \frac{1}{2 \sqrt{\tilde{\epsilon}_x \langle \beta_x \rangle_s} (\sqrt{\tilde{\epsilon}_x \langle \beta_x \rangle_s} + \sqrt{\tilde{\epsilon}_y \langle \beta_y \rangle_s})}$$

## Lattice induced nonlinear resonances

$$n_x Q_{x0} + n_y Q_{y0} = m$$

G. Guignard, CERN 78-11, (1978);  
A. Bazzani et al., CERN94-02 (1994).

## Resonant dynamics



## Resonance driving terms

A combination of optics, and  
Magnets strength

$$\kappa = \frac{1}{2\pi(2R)^{(N/2)} |n_x|! |n_z|!} \int_0^{2\pi} d\theta \beta_x^{|n_x|/2} \beta_z^{|n_z|/2} \times$$

$$\times \exp \left\{ i \left[ n_x \mu_x + n_z \mu_z - (n_x Q_x + n_z Q_z - p) \theta \right] \right\} \begin{cases} (-1)^{(|n_z|+2)/2} K_z^{(N-1)} & \text{for } n_z \text{ even} \\ (-1)^{(|n_z|-1)/2} K_x^{(N-1)} & \text{for } n_z \text{ odd} \end{cases}$$

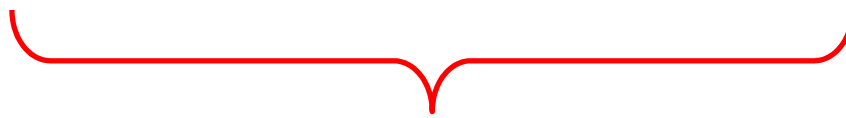
Magnets nonlinearities drives resonances

$$\Delta_r = \Delta_{r0} + \Delta Q_{sc,x}(X, Y) + 2\Delta Q_{sc,y}(X, Y) + Q'_x \delta p/p + 2Q'_y \delta p/p$$



**Bare tunes**

$$\Delta_{r0} = 0.056$$



effect of space charge  
**AMPLITUDE DEPENDENT**

incoherent tune-shift

$$\Delta Q_{x,max} \simeq -0.05,$$

$$\Delta Q_{y,max} \simeq -0.071$$



$$\mathcal{D}_{r,sc} \simeq -0.19$$

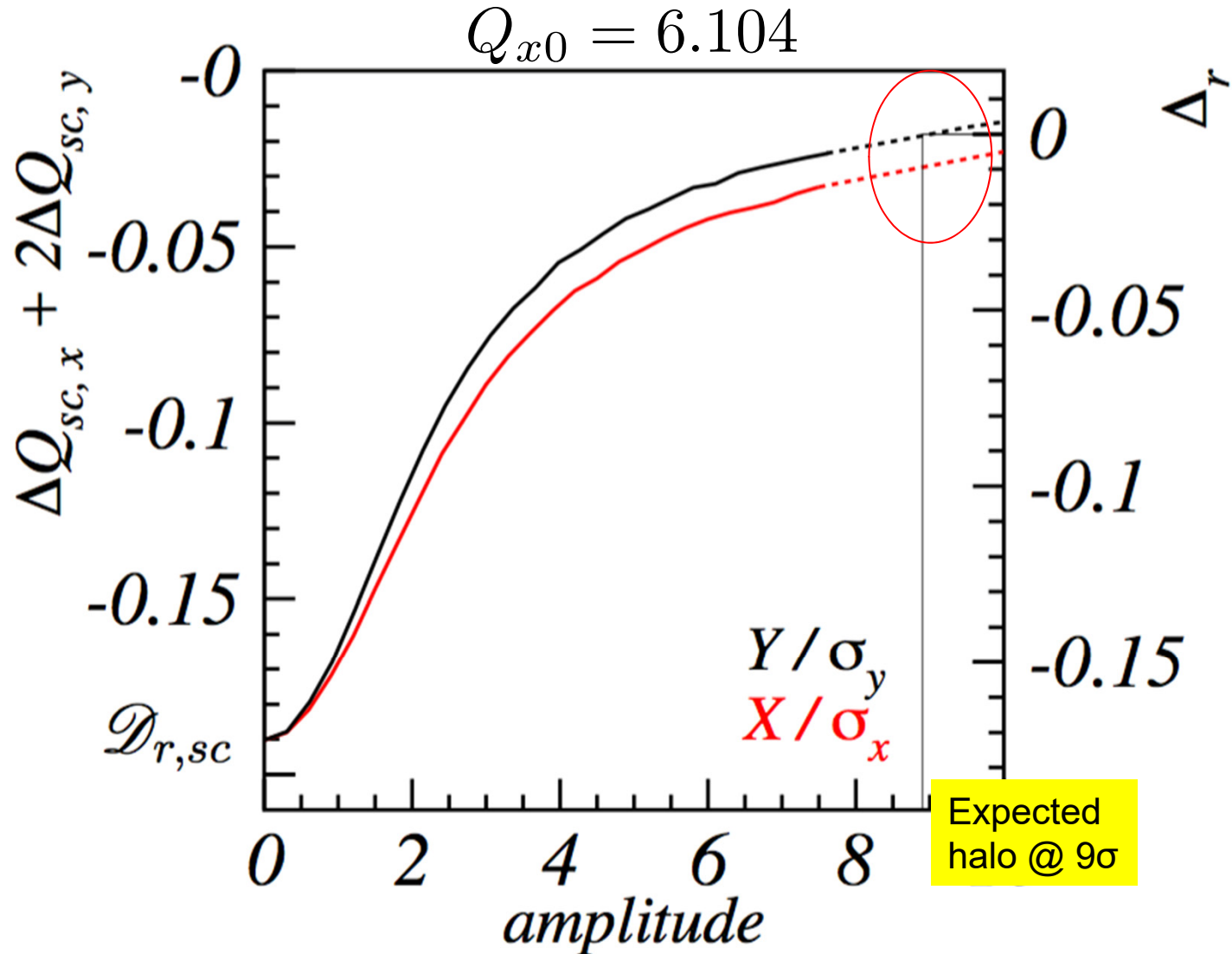


effect of chromaticity  
**AMPLITUDE INDEPENDENT**

consider a test particle  
with maximum  $dp/p$

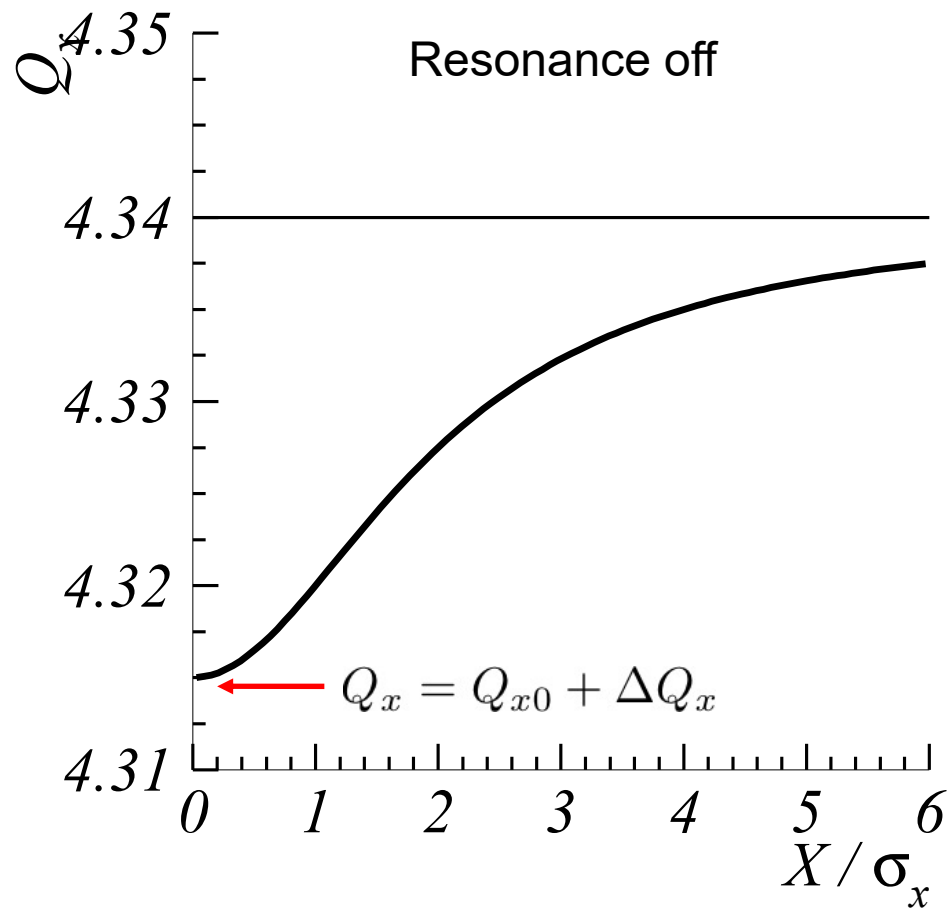


$$\pm 0.037$$





# Position of the islands (1D resonances)



# Position of the islands (1D resonances)

