HÉNON-HEILES SINGLE PARTICLE DYNAMICS AT IOTA

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Overview

The Hénon-Heilses system

Creating the Hénon-Heiles potential in a particle accelerator

Measuring the Poincare cross section

Numerical tracking simulations

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The Applicability of the Third Integral Of Motion: Some Numerical Experiments

MICHEL HÉNON* AND CARL HEILES

Princeton University Observatory, Princeton, New Jersey

Henon and Heiles set out to prove if the should be a 3rd isolating integral of stellar motion

Galactic potential was assumed to be

- Time-indendent
- Axially symmetric

$$V = V(R, z)$$

Only 2 isolating integrals were known

- Total energy
- Angular momentum

Observations of nearby stars behaved as if they had a 3rd integral of motion

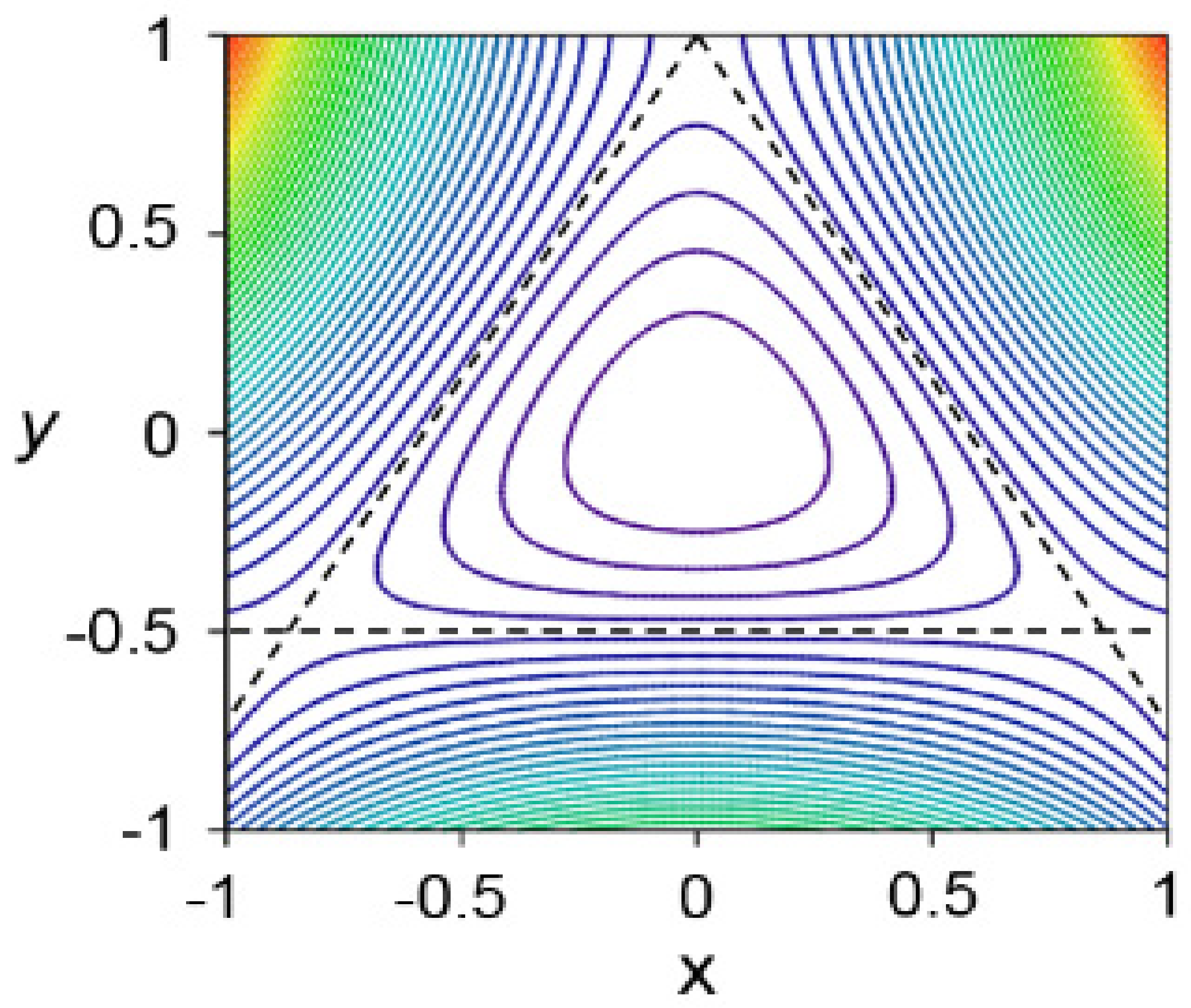
Image credit: NASA

Henon-Heiles System

Going to Cartesian coordinates (R, z) → (x, y)

Consider a model potential

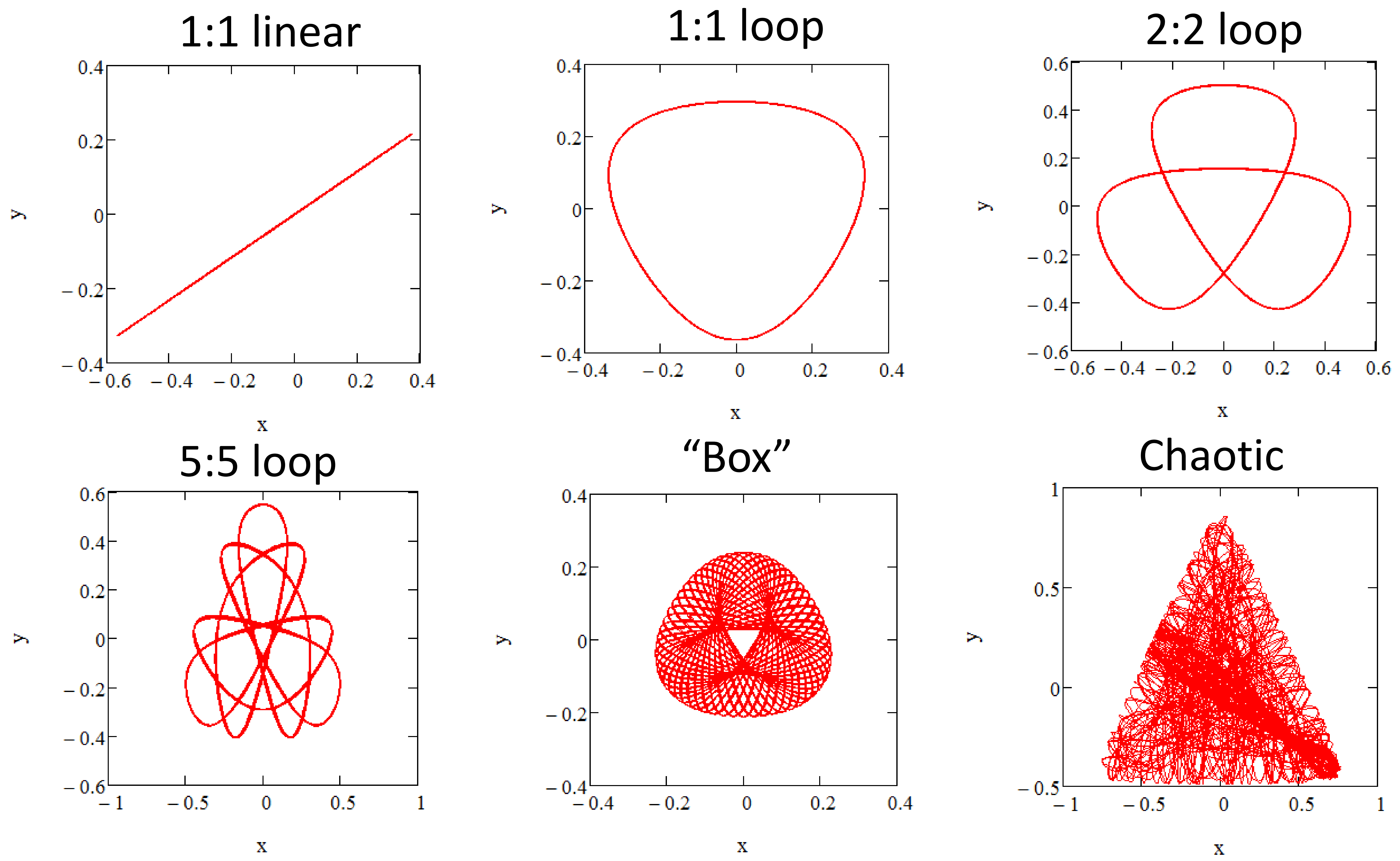
$$V(x,y) = \frac{1}{2}(x^2 + y^2) + x^2 y - \frac{1}{3} y^3$$



"… is analytically simple; this makes the computation of the trajectory easy; at the same time, it is sufficiently complicated to give trajectories which are far from trivial…"

"It seems probable that the potential is a typical representative of the general case, and that nothing would be fundamentally changed by the addition of higher-order terms."

The motion depends on initial conditions

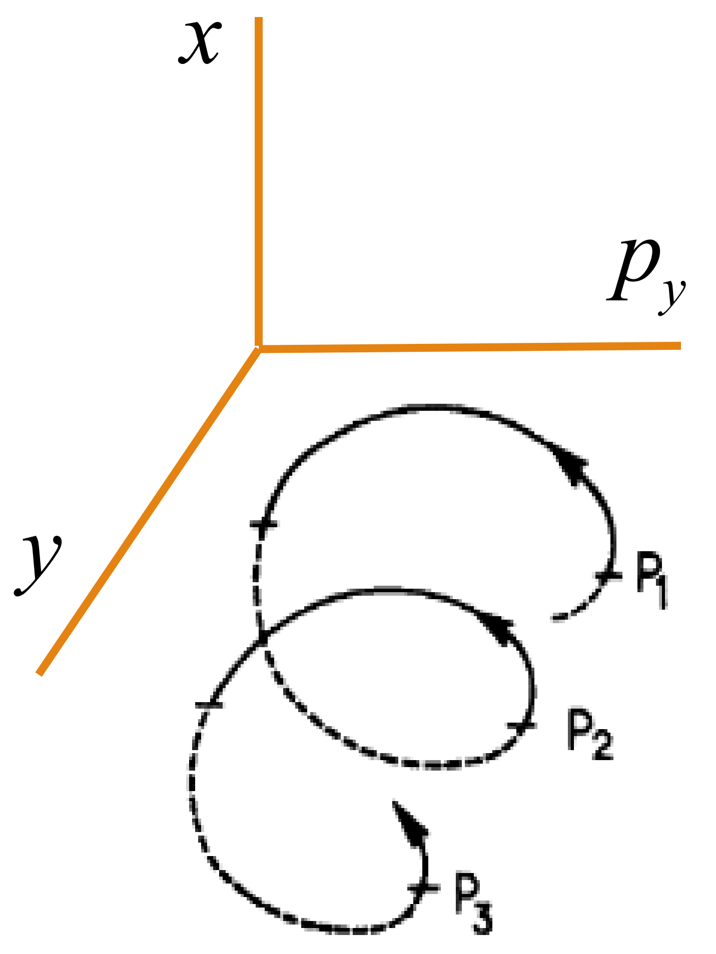


Orbit classification according to: E. Zotos, *Nonlinear Dynamics*, vol. 79, pp. 1665-1677, Feb. 2015

The system is governed by one parameter

$$H = \frac{1}{2}(p_x^2 + p_y^2 + x^2 + y^2) + x^2 y - \frac{1}{3}y^3 = E$$

Poincare cross section – trajectory cuts the (y, p_y) plane. Points: $P_i : x = 0, p_x > 0$



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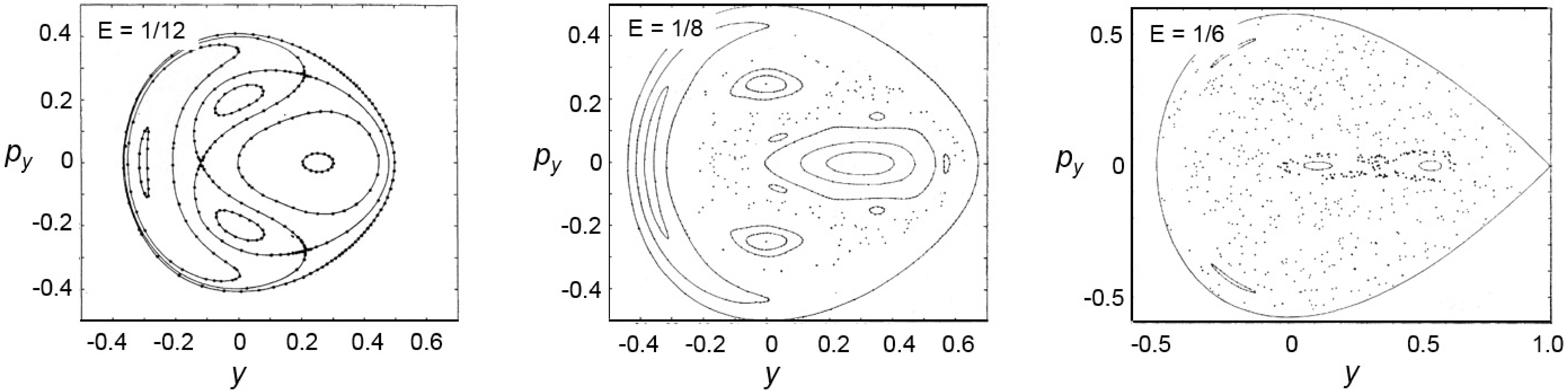
Low energies - different initial conditions lead to closed curves in the Poincare map
Higher energies - the trajectories fill the plane



The Henon-Heiles system can be created in an accelerator

V. Danilov, S. Nagaitsev, "Nonlinear accelerator lattices with one and two analytic invariants", *Phys. Rev. ST AB* **13**, 084002 (2010)

0. Start with the Hill's equation:

$$x'' + K_x(s)x = 0$$
$$K_x(s+C) = K_x(s),$$

In normalized coordinates

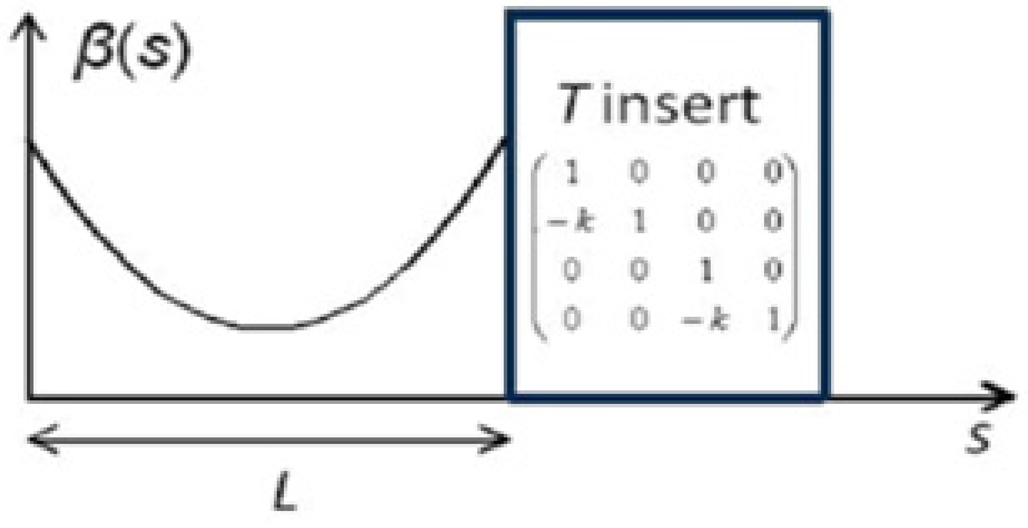
$$x_N = x/\sqrt{\beta_x(s)}$$
$$p_{x,N} = p\sqrt{\beta_x(s)} - \beta_x'(s)x/2\sqrt{\beta_x(s)}$$

Hamiltonian of a harmonic oscillator

- New time is different for each plane

$$H_{x,N} = \frac{1}{2}(p_{x,N}^2 + x_N^2) \qquad d\mu_x = ds/\beta_x(s)$$
$$H_{y,N} = \frac{1}{2}(p_{y,N}^2 + y_N^2) \qquad d\mu_y = ds/\beta_y(s)$$

1. Equal β-functions
   - The "time" flows equally fast in x and y planes



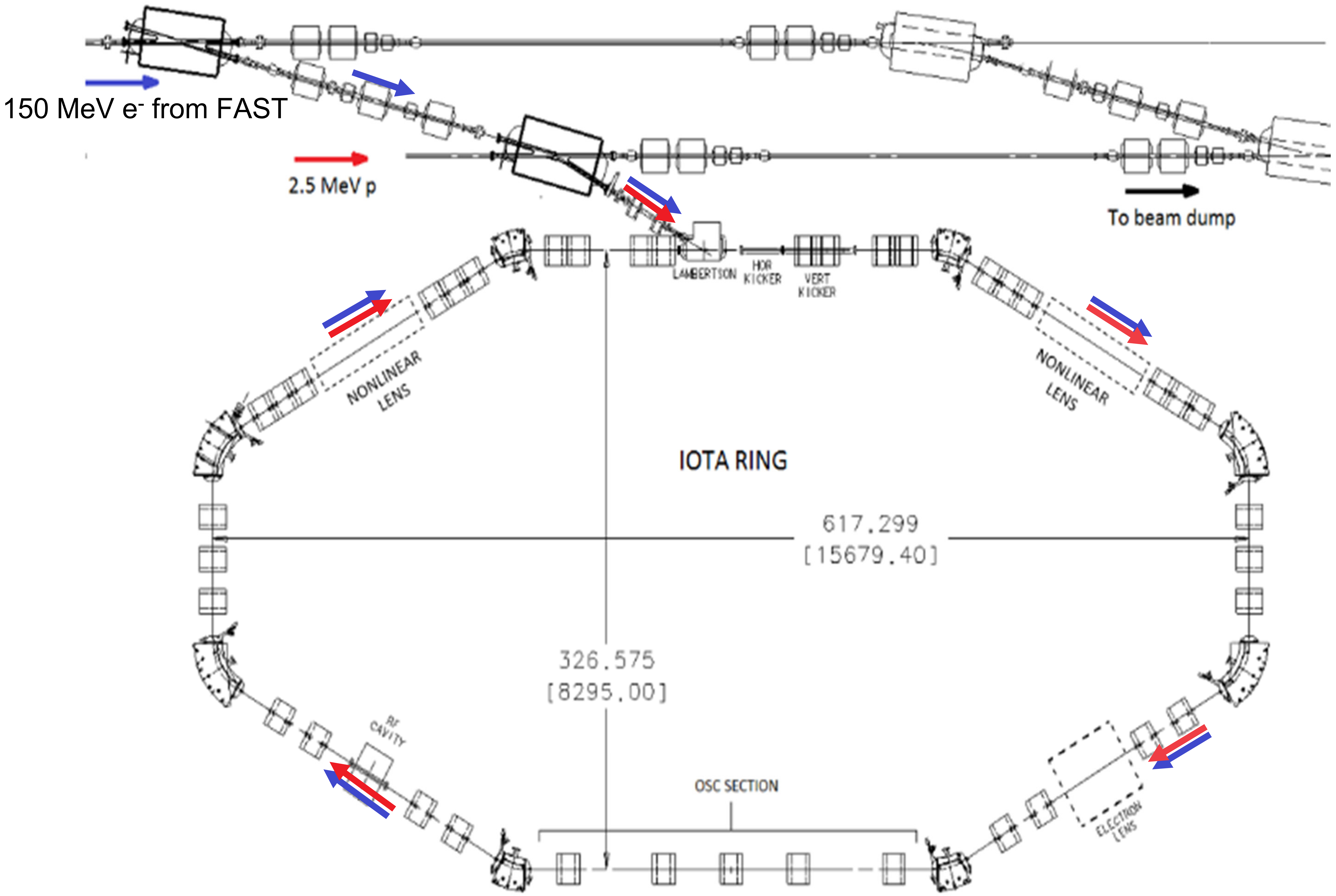
2. Scale the potential with β: $V(x,y;s) = \frac{\alpha}{\beta(s)^{5/2}}(xy^2 - \frac{1}{3}y^3)$

$$H_N = \frac{1}{2}(p_{x,N}^2 + p_{y,N}^2 + x_N^2 + y_N^2) + \beta \times V(x_N\sqrt{\beta}, y_N\sqrt{\beta}; s) = const$$

3. Renormalize: $x_N\alpha \to x \quad y_N\alpha \to y$

$$H = \frac{1}{2}(p_x^2 + p_y^2 + x^2 + y^2) + x^2y - \frac{1}{3}y^3 = E$$

Integrable Optics Test Accelerator

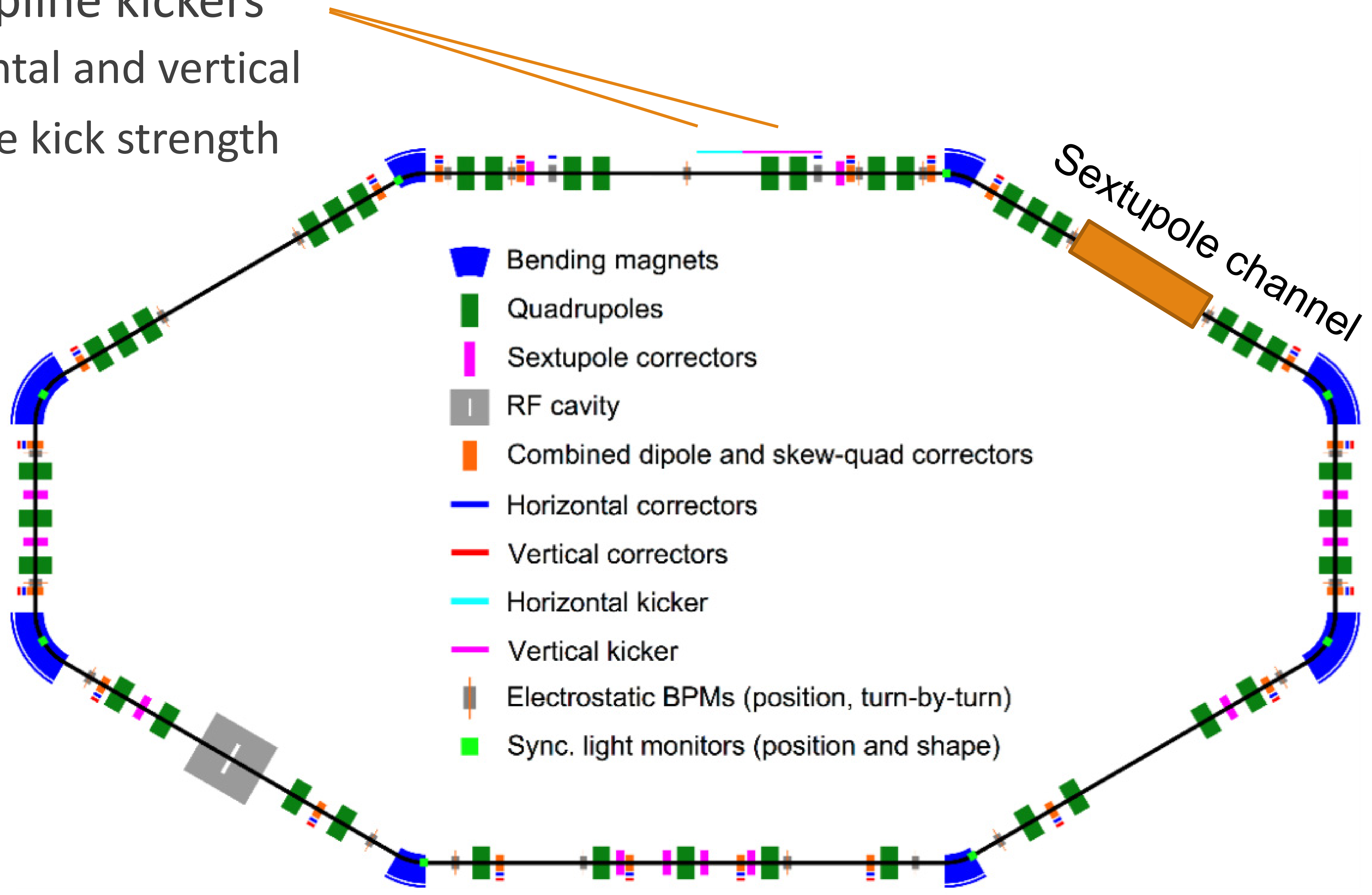


Electron energy	150 MeV
Number of bunches, particles	1, 10^9
Ring circumference	40 m
Synchrotron radiation damping time	1 s
Equilibrium emittance, x & y, rms	0.04 mm-mrad
Betatron tunes, x & y	5.3, 5.3
Natural chromaticities, x & y	-11.4, -7.1
Synchrotron tune	5.3×10^{-4}
Harmonic number and voltage	4, 1 kV
Energy spread, rms	1.35×10^{-4}
Bunch length, rms	10.8 cm

Can put pencil electron beam at an arbitrary position in the transverse plane and measure its trajectory

2 fast stripline kickers

- Horizontal and vertical
- Variable kick strength



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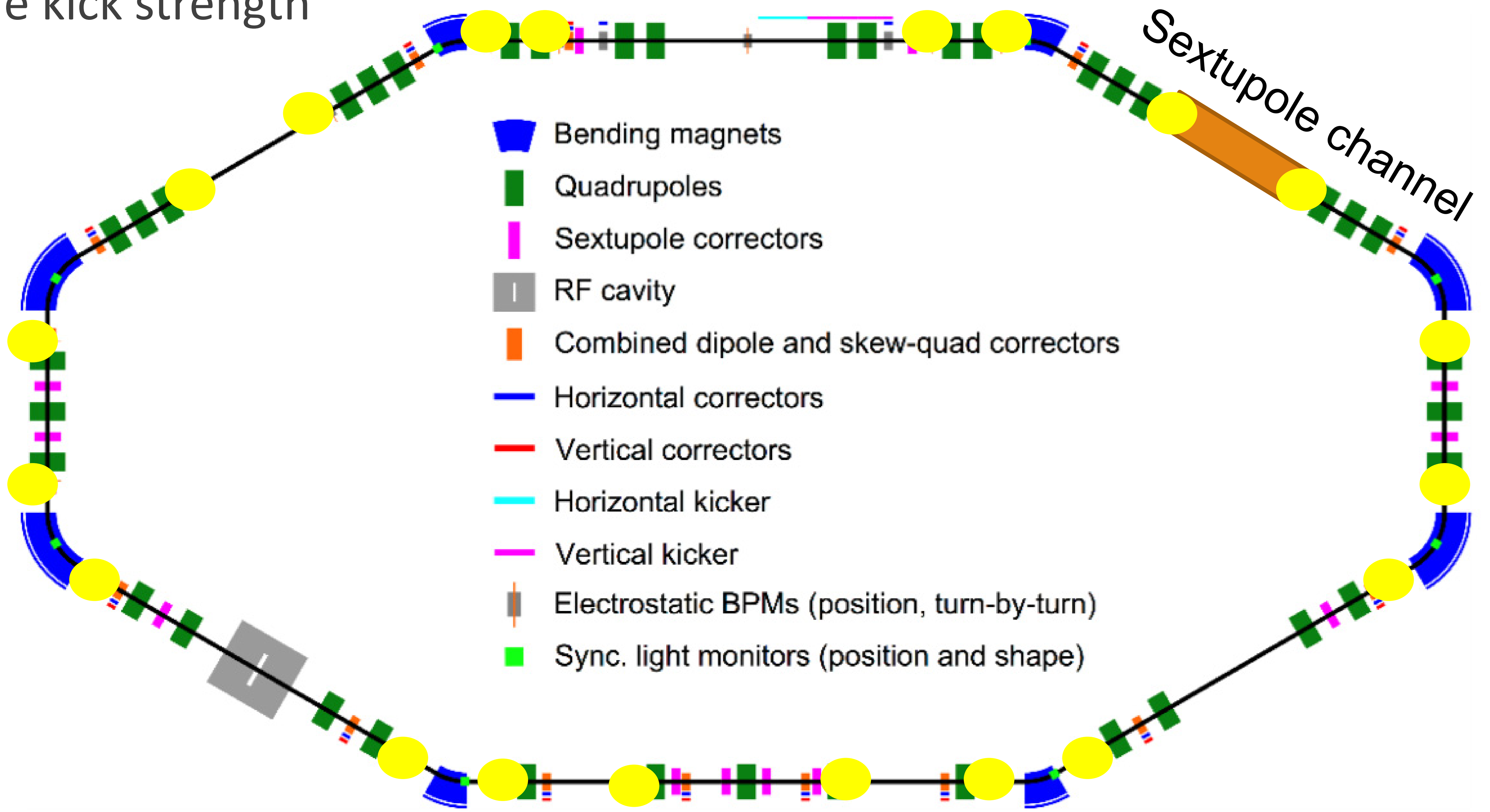


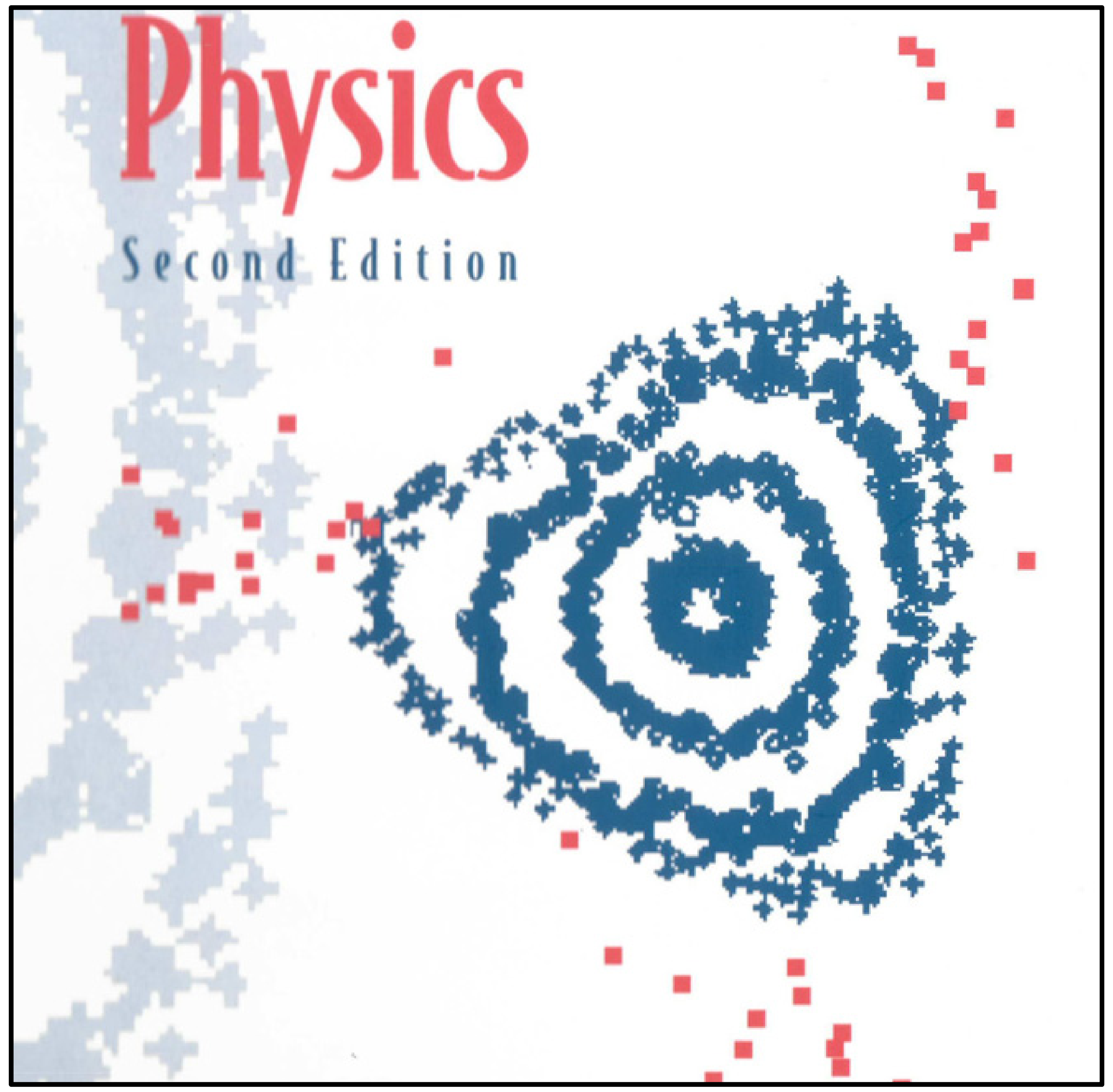
20 Beam position monitors
- Distribute around the ring
- 50 µm turn-by-turn resolution
- Horizontal + Vertical

Poincare maps in accelerators have been measured only in special cases

Betatron motion in x and y planes is uncoupled

Near a resonance

- The system is essentially 1-D
- Any phase-space trajectory is a Poincare map



Poincare map of the betatron motion near a 3rd order resonance $3Q_x = 11$ at IUCF on the cover of S. Y. Lee, *Accelerator Physics*
D. D. Caussyn, *et. al*, *Phys.Rev.* A46, pp. 7942-7952, 1992

Time independence of the system allows measuring the Poincare map

IOTA offers a precise control of optics and the strength of the nonlinearity

The system does not depend on time

$$H = \frac{1}{2}(p_x^2 + p_y^2 + x^2 + y^2) + x^2 y - \frac{1}{3} y^3 = E$$

Can use all the BPMs in the ring to measure (x, p_x, y, p_y) before and after the nonlinear section

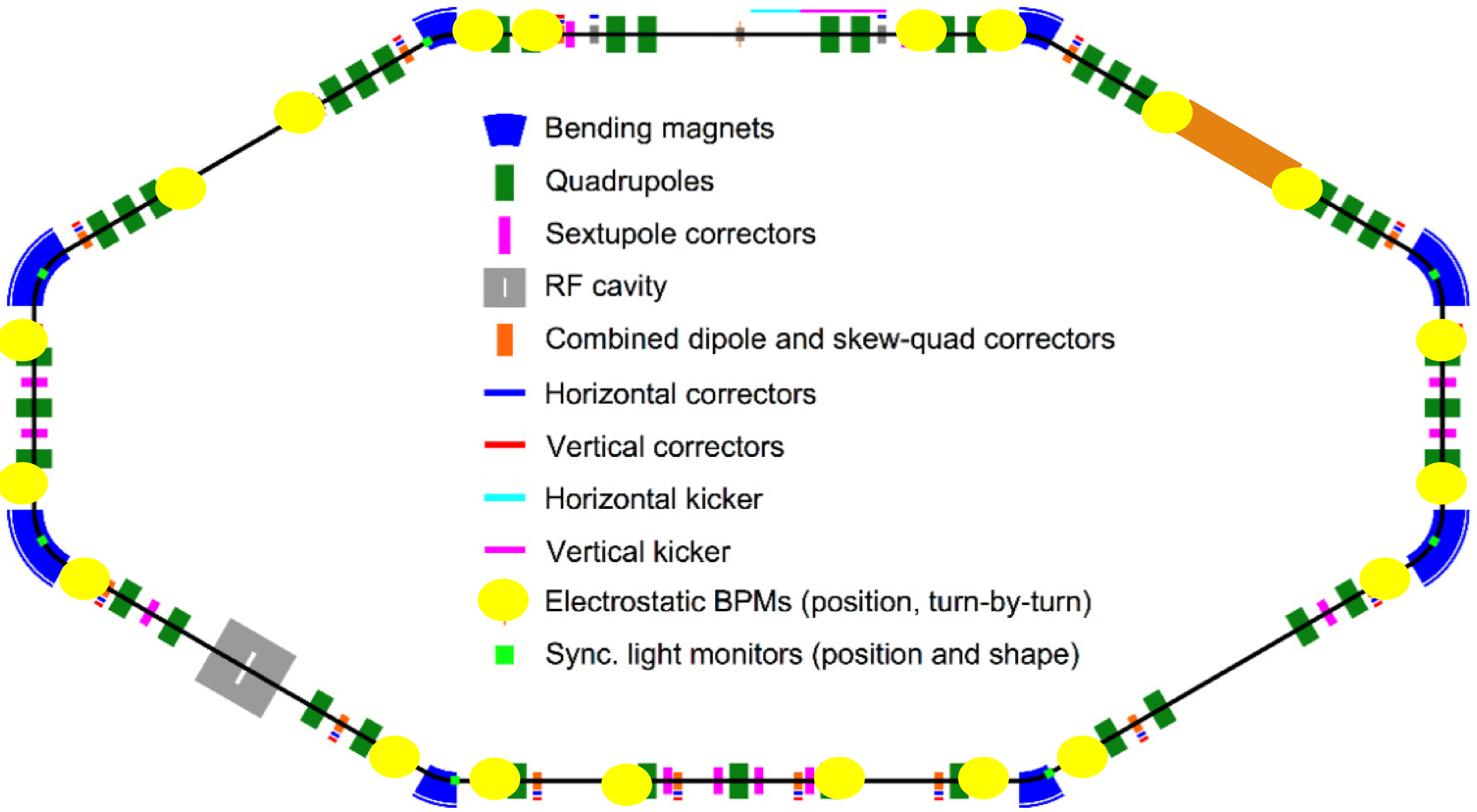
- E can be calculated from the position

Extrapolate the trajectory into the channel

- The strength of the sextupole is known

Look for the crossing of $x = 0$ plane with $p_x > 0$

- (y, p_y) gives the point in the cross section



4D Tracking

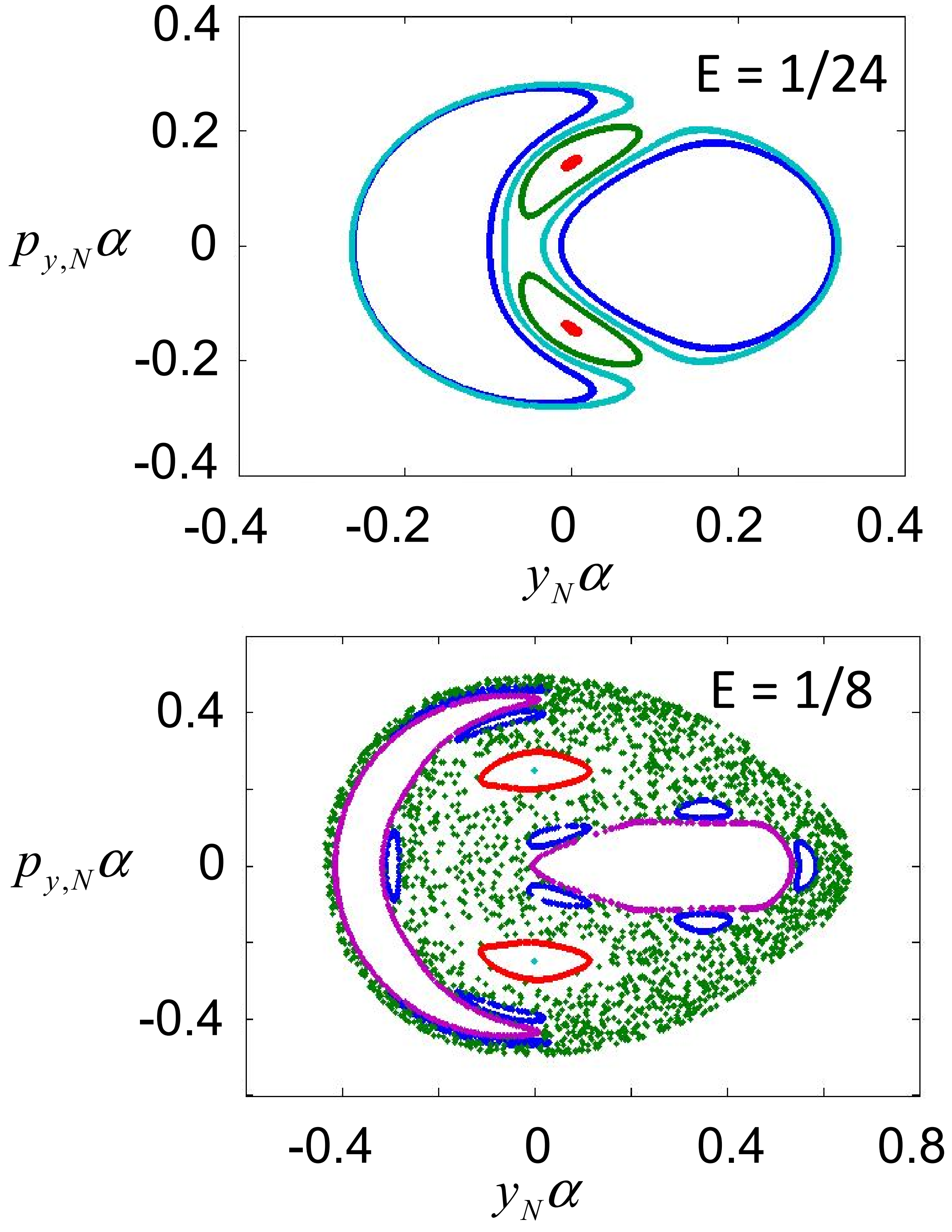
Channel:
- 18 thin sextupoles
- Strength α = 800 $m^{-1/2}$
- Spaced with equal phase advance

Ring:
- Linear transfer matrix

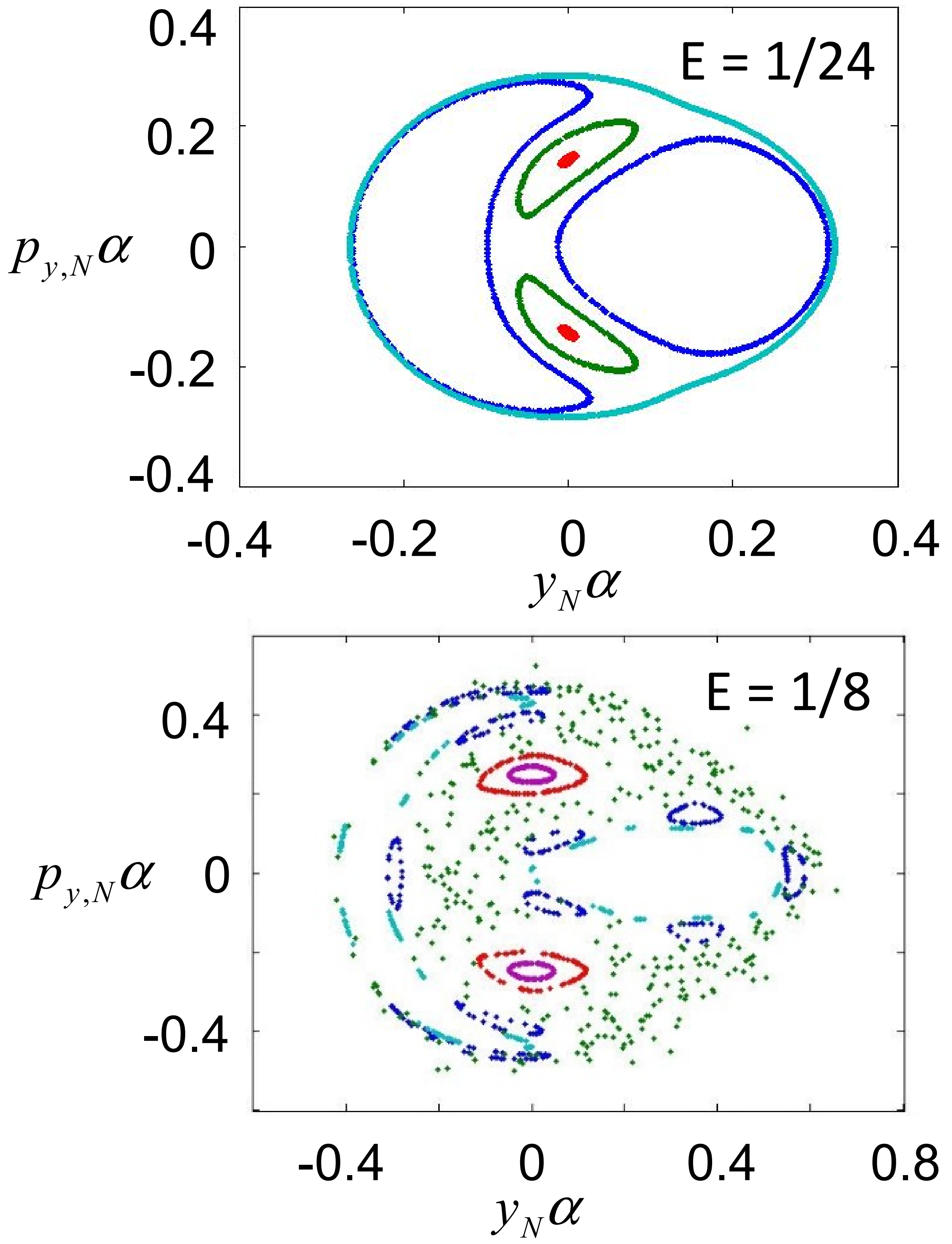
Tracking:
- 10^6 turns – ideal lattice
- 10^5 turns – with errors

BPM resolution:
- 10^{-3} (norm. units)

IDEAL LATTICE
NO MAGNET ERRORS



10^{-3} ERROR IN TUNE
5% IN MAGNETIC FIELD



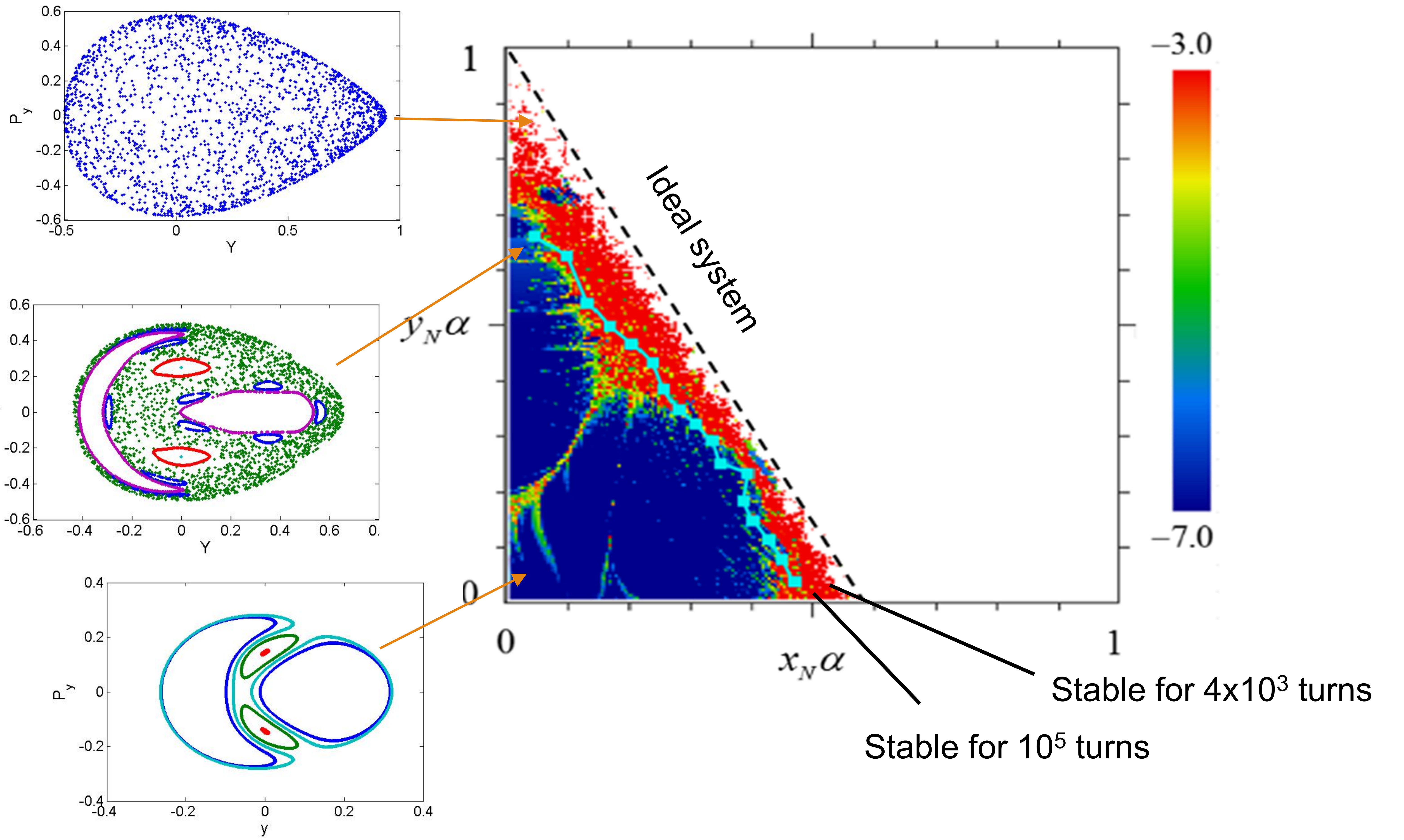
Frequency map analysis

6D ring model:

- Dipoles
- Quadrupoles
- Fringe fields
- RF

FMA phase scan – 2^{13} turns

Long-term tracking – 10^5 turns



IOTA is scheduled to go online in 2018

IOTA Construction and Research Timeline

	Electron Injector	Proton Injector	IOTA Ring
FY15	20 MeV *e-* commiss'd beam tests	Re-assembly began @MDB	50% IOTA parts ready
FY16	50 MeV *e-* commiss'd beam tests	50 keV *p+* commiss'd	IOTA parts 80+% ready
FY17	150-300 MeV *e-* beam commissioning/tests *	2.5 MeV *p+* commiss'd beam tests @ MDB	IOTA fully installed first beam ? *
FY18	*e-* injector for IOTA + other research	*p+* RFQ moved from MDB to FAST *	IOTA commiss'd with *e-* **Research starts (NL IO)**
FY19	*e-* injector for IOTA + other research	2.5 MeV *p+* commiss'd beam tests	**IOTA research with *e-*** IOTA commiss'd with *p+*
FY20	*e-* injector for IOTA + other research	*p+* injector for IOTA	**IOTA research with *p+****

beam operations

- contingent on $$: FY17-20 - under current budget scenario…together with OHEP GARD management we explore options to accelerate start of research by 1 year (supplemental)

Courtesy A. Valishev

Conclusion

Hénon-Heiles system is a classic example of a time-independent Hamiltonian system that is both computationally simple and generic in its basic properties.

The potential can be created and studied experimentally in a realistic accelerator setup

The system can be used to investigate building Poincare cross sections in an accelerator

The required tolerances of 5% in sextupole field strength and 10^{-3} can be achieved in the IOTA

Under these tolerances the electron beam will remain stable for at least 10^5 revolutions. This will allow observing the coexistence of regular and irregular motion in the system.

Thank you

PLEASE CHECK OUT OUR OTHER CONTRIBUTIONS

- S. Nagaitsev, “*A Concept of an Integrable Octupole-Based Accelerator Focusing Lattice*” WEPIK104
- T. Zolkin, “*Axially-Symmetric McMillan Lens for SC Compensation*” THPAB067
- J. Eldred, “*Space-Charge Simulation of Integrable Rapid Cycling Synchrotron*” THPVA032