GROUP VELOCITY MATCHING IN DIELECTRIC-LINED WAVEGUIDES AND ITS ROLE IN ELECTRON-TERAHERTZ INTERACTION

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Abstract

Terahertz(THz)-driven dielectric-lined waveguides have applications in electron manipulation, particularly acceleration, as the use of dielectric allows for phase velocities below the speed of light. However matching a single frequency to the correct velocity does not maximise electron-THz interaction; waveguide dispersion typically results in an unmatched group velocity and so the pulse envelope of a short THz pulse changes along the length of the structure. This reduces field amplitude and therefore accelerating gradient as the envelope propagates at a different velocity to the electron. Presented here is an analysis of the effect of waveguide dispersion on THz-electron interaction and its influence on structure dimensions and choice of THz pulse generation. This effect on net acceleration is demonstrated via an example of a structure excited by a single-cycle THz pulse, optimised for high field strength but short interaction length, and a structure with a focus on high interaction length. This is combined with a comparison of multi-cycle, lower intensity THz pulses on net acceleration.

INTRODUCTION

Terahertz frequencies are considered as an alternative to radio frequencies due to increased accelerating field gradients. This is a result of higher breakdown threshold, which scales with surface electric field, E_s, as $E_s \propto f^{1/2}\tau^{-1/4}$, where f is the operating frequency and τ is the pulse length [1]. Therefore short pulse durations and high frequencies are desirable. The use of THz over higher frequencies allows for larger structures which can be conventionally machined, and an electron bunch of higher charge can be confined within a single acceleration period. Dielectric-lined waveguides (DLWs) have been experimentally verified for THz-driven electron acceleration at 60 keV [2], in which an accelerating gradient of 2.5 MeV m^{-1} was achieved for a 10 μJ THz pulse energy. Recent relativistic experiments of beam-driven wakefield structures have found accelerating gradients of 320 MeV m^{-1} [3].

DISPERSION IN DIELECTRIC-LINED WAVEGUIDES

The accelerating modes, LSM$_{m,n=odd}$, of a rectangular DLW, such as in Fig. 1 are described by the dispersion relation [4]
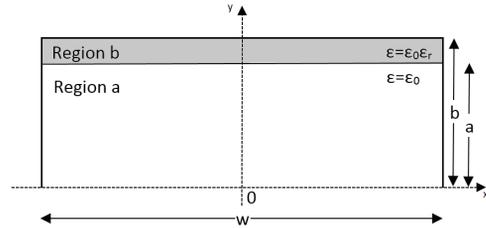
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Figure 1: Half cross-section of the waveguide design. The waveguide is symmetric about the x-axis.
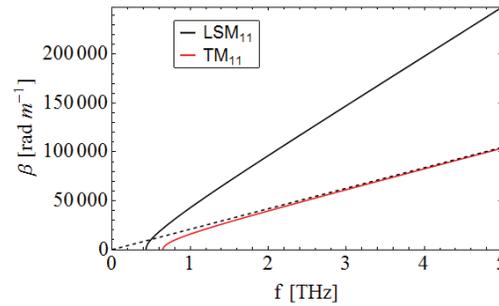

Figure 2: Dispersion relation for the TM$_{11}$ mode of a hollow rectangular waveguide and the LSM$_{11}$ mode of a dielectric-lined waveguide. The dashed line correspond to $\omega = c\beta$, the speed-of-light line.

$$k_y^1 \tan\left(k_y^1 (b-a)\right) = \epsilon_r k_y^0 \cot\left(k_y^0 a\right) , \qquad (1)$$

where $k_y^0 = \sqrt{\left(\frac{\omega_0}{c}\right)^2 - \beta^2 - \left(\frac{m\pi}{w}\right)^2}$ and $k_y^1 = \sqrt{\epsilon_r \left(\frac{\omega_0}{c}\right)^2 - \beta^2 - \left(\frac{m\pi}{w}\right)^2}$. ω_0 is the free-space frequency, β is the propagation constant inside the DLW and c is the speed of light. b, a, and w are defined in Fig. 1, and ϵ_r is the relative permittivity of the dielectric. The modes are described as longitudinal section magnetic/electric (LSM/LSE), with LSM_{11} being the first accelerating mode. These are a hybrid of TM/TE modes due to the dielectric-vacuum interface. Throughout the paper parameters $a = 100$ μm, $w = 500$ μm, $\epsilon_r = 5.68$ (corresponding to CVD diamond) and dielectric thickness $t = b - a = 60$ μm will be used. The dispersion relation is shown in Fig. 2 and compared to a corresponding hollow rectangular waveguide. The phase velocity v_p and group velocity v_g are given by

$$v_p = \frac{\omega}{\beta(\omega)},$$
$$v_g = \left(\frac{d\beta(\omega)}{d\omega}\right)^{-1}. \qquad (2)$$

The use of dielectric reduces v_p to below the speed of light, making continuous acceleration of charged particles possible.

v_p and v_g are shown in Fig. 3 for the DLW. v_p can be matched to electron velocity, v_e, which is only possible for the synchronous frequency f_s. It is assumed $v_e = c$.
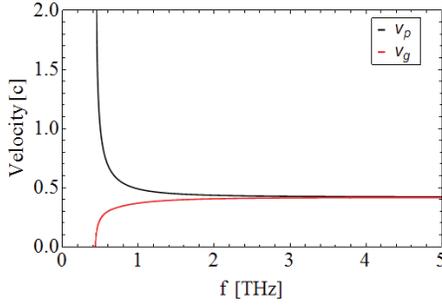

Figure 3: v_p and v_g for the LSM_{11} DLW. At $v_p = v_e c$, $v_g = 0.24$ c.

Pulse Slippage

The THz pulse envelope propagates at v_g. If $v_g \neq v_e$, f_s (and the electron) propagates at a different velocity to the pulse and so the field amplitude experienced by a co-propagating electron decreases along the length of the structure. For a given interaction length L the temporal pulse length necessary for continuous acceleration is described by

$$\tau_p \geq \left(\frac{1}{v_g} - \frac{1}{v_e} \right) L . \tag{3}$$

For a transform-limited Gaussian pulse, the bandwidth $\Delta \nu \approx 0.44/\tau_p$, corresponding to

$$\Delta \nu \leq \frac{0.44}{L \left(\frac{1}{v_g} - \frac{1}{v_e} \right)} . \tag{4}$$

This assumes that v_g is frequency-independent. As $v_g = v_g(\omega)$, as the pulse propagates the envelope is temporally broadened due to group velocity dispersion. If $v_g < v_e$ this broadening occurs behind the electron.

Phase Slippage

Frequencies other than f_s accelerate for a certain length before slipping out of phase with a co-propagating electron. At a phase difference of $\pm \pi / 2$ these frequencies cause deceleration. The range of frequencies which will constantly accelerate over the length of the structure is described as the *accelerating bandwidth*, Δf. The phase slippage is defined by

$$\beta(\omega) L - \beta_e L \leq \pm \frac{\pi}{2} , \tag{5}$$

where β_e is the electron propagation constant. Δf is calculated by taking the limits of Equation (5) and approximating $\beta(\omega)$ using a Taylor series expansion. Taking the second approximation, Δf is given as

$$\Delta f = \frac{\omega_2 - \omega_1}{2\pi} = \frac{1}{2L \left(\frac{1}{v_g(\omega_s)} - \frac{1}{v_e} \right)} . \tag{6}$$

Maximising Δf is achieved by decreasing structure length or by $v_g(\omega_s) \approx v_e$. Comparing Eqs. (4) and (6), $\Delta \nu$ and Δf are approximately equal for a Gaussian pulse. The requirement to phase match over a range of frequencies is equivalent to matching the group velocity over the same range. Provided the bandwidth of the THz pulse is less than Δf, all frequencies in the pulse will accelerate a co-propagating electron.

Effect of DLW Cut-off Reqency

Frequencies below the cut-off of the DLW decay evanescently at the waveguide entrance. This causes further temporal broadening. If these frequencies are phase-matched then energy gain by an electron is limited as pulse energy is lost. It is therefore preferable to have a pulse with a minimum frequency above cut-off and a bandwidth that aligns with the accelerating bandwidth of the waveguide. The cut-off frequency is dependent on choice of waveguide dimensions.

THz-ELECTRONINTERACTION

Maximising interaction between an input THz pulse and an electron requires a balance between a long interaction length and a high accelerating field. Shorter pulses have a larger field amplitude for the same input energy, with a $1/\sqrt{\tau_p}$ relationship. Axial electric field at f_s is given by [4]

$$E_z(x, y = 0) = A \frac{2\pi f_s}{c} \sqrt{\left(\frac{2\pi f_s}{c} \right)^2 \left(1 - \left(\frac{c}{v_e} \right)^2 \right) - \left(\frac{\pi}{w} \right)^2} , \tag{7}$$

where A is a constant dependent on the input THz pulse energy (normalised to 1 J in all simulations) and waveguide cross-section. Three parameters were optimised: frequency, group velocity, and accelerating electric field. a was fixed to $100 \, \mu m$, the minimum aperture required to allow for electron propagation, and $\epsilon_r = 5.68$. Fig. 4 shows the effect of w and t on the three parameters. v_g is maximised by $t \rightarrow 0$ with w having little effect. Frequency decreases as t and w increases. $E_z(x, y = 0)$ is maximised for $t = 20 \, \mu m$ and $w = 400 \, \mu m$, and decreases slightly with increasing t and w. As $t, w \rightarrow 0$, $E_z(x, y = 0) \rightarrow 0$. Therefore achieving $v_g = c$ is only possible with no dielectric resulting in $E_z(x, y = 0) = 0$. Considering $f_s = 0.5$ THz, the low group velocity results in $\Delta f < 5$ GeV for an interaction length $L = 10$ mm. For a single-cycle pulse of bandwidth 1 THz, $L = 0.05$ mm and for a 10-cycle pulse $L = 0.5$ mm.

SIMULATION RESULTS

A PIC (Particle-In-Cell) simulation was performed using CST MWS to investigate the effect of an input THz pulse on a Gaussian electron bunch. The timing of launch of both THZ pulse and electron bunch affects acceleration of the electrons. Launching the electrons after the pulse increases interaction as the pulse is broadened (more cycles). The maximum energy gain due to a nominal input pulse of 1-, 10- or 20-cycles with the same input pulse energy is shown
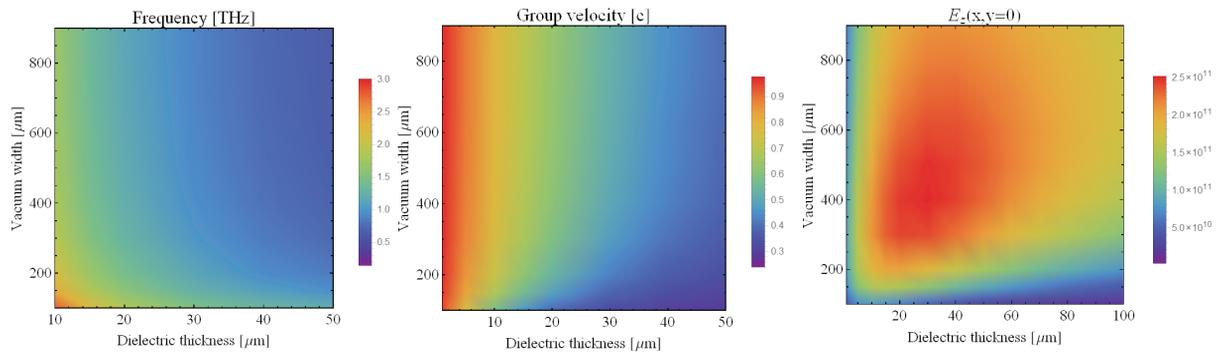
Figure 4: Peformance parameters of the DLW as a function of *w* and *t* for fixed *a* and ϵ_r.
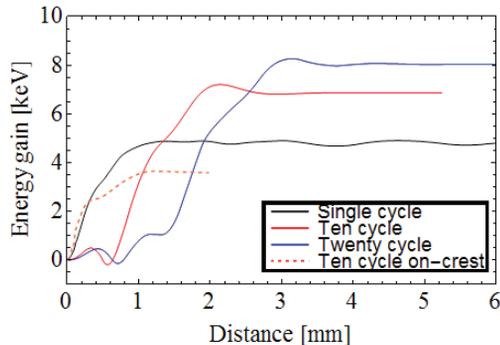

Figure 5: Energy gain of an electron co-propagating with a single cycle, 10-cycle and 20-cycle THz pulse of the energy. The solid curves represent the highest possible energy gain for a pulse with off-centre maximum. The dashed curve represents energy gain for an electron injected at the peak of the THz pulse, with pulse maximum in the centre.

in Fig. 5. Note that, unless stated, the THz pulse maximum is not in the centre of the pulse. A 20-cycle pulse is included as the bandwidth does not overlap with the cut-off frequency, which is 0.44 THz for the given DLW dimensions. There is an increase in energy gain with number of cycles for the same THz pulse energy. There is not constant acceleration for multiple cycles and the effect is more pronounced for increasing pulse length. An electron passing through several cycles experiences decelerating field. Included is the energy gain for an electron injected on the peak of a 10-cycle THz pulse where the pulse maximum is in the centre. This electron does not experience deceleration but net energy gain is lower. The temporal broadening of each pulse after 1 mm is shown in Figs. 6 and 7. The single-cycle pulse has been highly dispersed and is no longer recognisable, unlike the 20-cycle pulse.

CONCLUSIONS

Group velocity matching is not achievable for ultrarelativistic electrons in the range 0.2-0.8 THz. For a single-cycle input THz pulse it is also not possible to achieve interaction over the entire length of a structure, with a sub-mm interaction length. The interaction length is larger than predicted
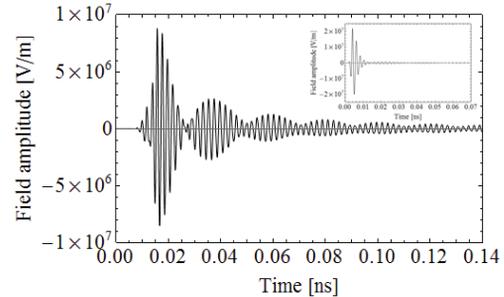

Figure 6: Field amplitude of a single-cycle pulse, 1 mm into the DLW. Inset is the pulse 1 μm into the DLW, showing broadening due to loss of lower-frequency components.
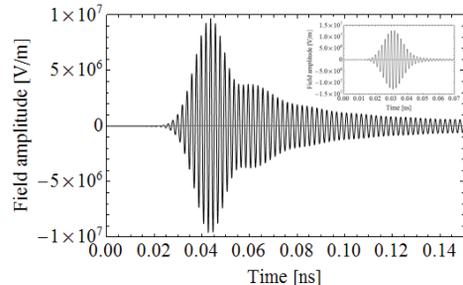

Figure 7: Field amplitude of a 20-cycle pulse, 1 mm into the DLW. Inset is the pulse 1 μm into the DLW, showing the broadening of the pulse due to loss of lower-frequency components.

due to the effect of the cut-off frequency of the waveguide, which causes temporal broadening, and the effect of group velocity dispersion. A longer pulse results in greater electron acceleration for the same pulse energy despite the lower field amplitude. This is a result of the rapid dispersion, which reduces field amplitude.

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