AN OPTIMIZATION TOOL TO DESIGN A CORELESS NON-LINEAR INJECTION KICKER MAGNET

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Abstract

Top-up injection into low emittance light sources is challenging due to their inherent small dynamic apertures. The use of a multipole-magnet injection kicker (e.g. a pulsed quadrupole or sextupole) enables to kick the injected bunch while the circulating beam remains undisturbed. However, the injected bunch is mismatched due to the focusing effect and the effective phase space required to capture it increases. Non-linear injection kicker magnets can produce a transverse step-like magnetic field distribution which prevents this mismatch. It is crucial to maximize the spatial derivative of this field distribution in order to kick the injected bunch inside the dynamic aperture. For the standard configurations of straight conductors the resulting clear aperture is typically too small, so we have developed an optimization tool to determine constrained current distributions required to generate a desired magnetic field. With it we obtained new design solutions for possible coreless injection kicker magnets that overcome the clear aperture limitation of the standard designs. We present an example for the injection into SLS-2.

INTRODUCTION

The new generation of low emittance light sources have dynamic apertures (DA) on the order of a few millimeters. While top-up injection into such light sources is desired for the stability of the photon beam, it is also challenging due to the inherently small DA. Multipole-magnet injection kickers, e.g. a pulsed quadrupole or sextupole, have been successfully used to kick injected bunches while keeping the circulating beam undisturbed [1,2]. However, the kicked bunch is mismatched due to the focusing effect caused by the linear field profile when a pulsed quadrupole is used. In the case of a pulsed sextupole it is mismatched due to the filamentation caused by the nonlinear field profile. As a result the effective phase space required to capture it increases. The ideal magnetic field distribution which prevents the mismatch would have no field at the position of the circulating beam ($\vec{B}|_{r=0} = 0$, $\partial\vec{B}/\partial r|_{r=0} = 0$) and a constant field, or *field plateau*, B_y at the position of the injected bunch. Such a field can be generated by a coreless non-linear injection kicker magnet with the use of different configurations of straight conductors, such as the bipolar 8-conductor configuration designed and tested at Bessy [3], or the multi-conductor approach like a unipolar *massless septum* proposed at CERN [4].

We tested these configurations in simulations for the injection into SLS-2, the planned upgrade of the Swiss Light Source. We placed the kicker magnet at the end of a straight section and set the origin of coordinates at the position of the circulating beam. The magnet has to provide a 2 mrad kick at $x = 5.5$ mm to keep the betatron oscillations within the DA until damped. For a 2.4 GeV light source, and assuming a length of the pulsed magnet of 0.5 m, a field amplitude $B_y = 32$ mT is needed.

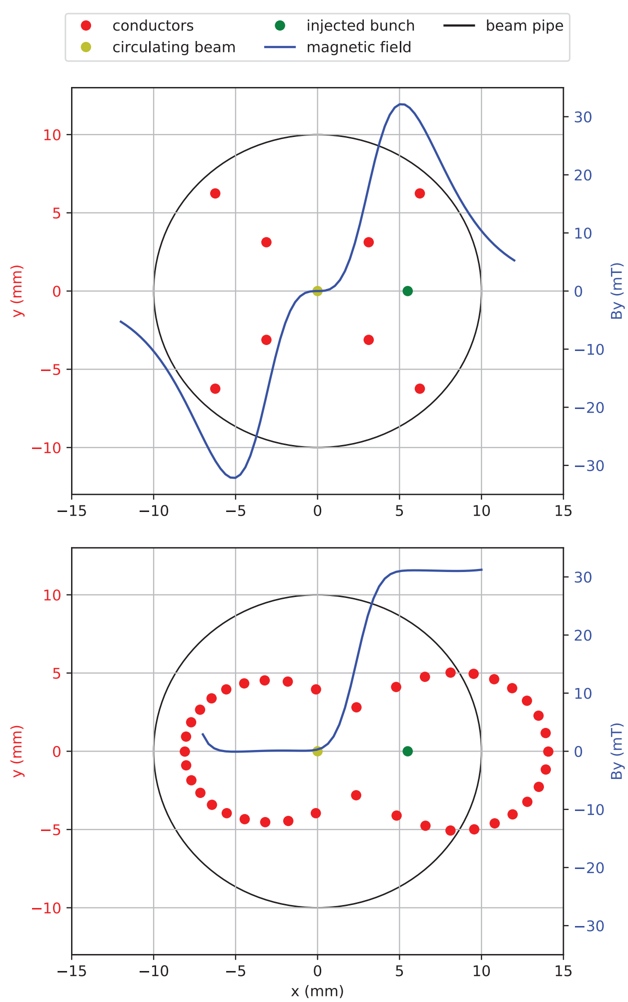

Figure 1: Different possible configurations of straight conductors for injection into SLS-2: *8-conductor* (up) and *massless septum* (down). In both cases the conductors would need to be placed inside the beam pipe in order to create a field spatial derivative high enough to have a zero-field and a field plateau at the positions of the circulating and injected beam, respectively.

We found that to get a sufficiently high field spatial derivative, $\Delta B_y/\Delta x$, the conductors would need to be located inside the beam pipe, in positions where it is technically impossible

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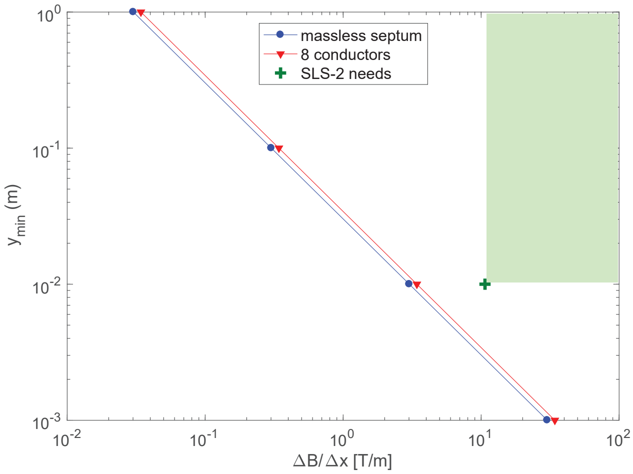

Figure 2: Vertical position of the closest conductor (located between the injected and circulating beams) as a function of the transverse field spatial derivative for different conductor configurations. The SLS-2 minimum requirement to ensure a clear aperture larger than the beampipe is marked with a cross, larger field gradients with larger clear apertures (i.e. values within the green area) would also be valid.

to place them, as shown in Fig. 1. Thus, for the injection into a low emittance ring it is crucial to maximize the spatial derivative of the transverse field. However, comparing the results of the studied unipolar and bipolar configurations the observed dependence between the vertical position of the closest conductor (located between the injected and circulating beams) and the maximum transverse field spatial derivative, shown in Fig. 2, constrains the design of coreless non-linear kicker magnets for low emittance rings. At MAX IV an 8-conductor kicker, of the type shown in Fig. 1 (up) is used, and injection takes place in the linear part of the field profile [5]. In that case this configuration can be used since the bunches are injected directly from a Linac and their horizontal emittance is only 1.7 nm·rad, for which the resulting mismatch is acceptable. For a bunch with a larger horizontal emittance (typically any bunch injected from a booster ring) the mismatch would cause non-negligible losses.

OPTIMIZATION TOOL

We have developed an optimization tool to determine constrained current distributions required to generate a desired magnetic field [6]. The optimization of the conductor arrangements is formulated as a least squares minimization problem using an analytic description of the magnetic field. We define one or several points $p_1 \dots p_m$, with $p_j = (x_{p_j}, y_{p_j})$, at which target values for the magnetic field components are specified (e.g. a given field plateau at a given transverse position):

$$T_{u,1}, \dots, T_{u,m}, \text{ with } u = x, y. \tag{1}$$

We then choose a number of conductors n with a current I and calculate the magnetic field at each field point p_j as:

$$\begin{aligned} B_{x,j} &= -\sum_{i=1}^{n} \frac{\mu_0}{2\pi} \frac{I_i (y_{p_j} - y_i)}{(x_{p_j} - x_i)^2 + (y_{p_j} - y_i)^2} \\ B_{y,j} &= \sum_{i=1}^{n} \frac{\mu_0}{2\pi} \frac{I_i (x_{p_j} - x_i)}{(x_{p_j} - x_i)^2 + (y_{p_j} - y_i)^2} \end{aligned} \tag{2}$$

At this point it might be tempting to define an objective function:

$$f_0(X) = \sum_{j=1}^{n} [(B_x - T_x)^2 + (B_y - T_y)^2] \tag{3}$$

for the optimization variables $X = (x_1, \dots, x_n, y_1, \dots, y_n, I_1, \dots, I_n)$ and solve a constrained optimization problem consisting of minimizing f_0 subject to equality and inequality constraints. However, this problem is nonlinear and non-convex, making it difficult to solve for large numbers of variables, and also making it difficult to find a global minimum since there may be many local minima. In contrast, convex problems can always be solved efficiently [7]. It is not generally possible to guarantee convexity for problems in which variables are multiplied. We note that the equations of the magnetic field generated by straight conductors are nonlinear in the position of the filament, but not in the current. If we fix the position and only vary the current we can formulate a problem in which optimization variables are not multiplied within the objective function. And if we fill a region of space where we can accept to place wires in our magnet design with a grid of conductors, then we can obtain essentially the same information as from (3); if the current in the solution is localized in certain regions, then we know precisely where the wires should be positioned.

To obtain a convex problem we start by defining the weights of the i^{th} filament at point p_j as

$$\begin{aligned} w_{x,i,j} &= -\frac{\mu_0}{2\pi} \frac{(y_{p_j} - y_i)}{(x_{p_j} - x_i)^2 + (y_{p_j} - y_i)^2} \\ w_{y,i,j} &= \frac{\mu_0}{2\pi} \frac{(x_{p_j} - x_i)}{(x_{p_j} - x_i)^2 + (y_{p_j} - y_i)^2} \end{aligned} \tag{4}$$

If we define the current and weight matrices as:

$$I = \begin{pmatrix} I_1 \\ \vdots \\ I_n \end{pmatrix}, \quad W_u = \begin{pmatrix} w_{u,1,1} & \dots & w_{u,1,n} \\ \vdots & \ddots & \vdots \\ w_{u,m,1} & \dots & w_{u,m,n} \end{pmatrix} \tag{5}$$

By writing also the target values from (1) as vectors, we can define an objective function:

$$f_0(I) = (W_x I - T_x)^2 + (W_y I - T_y)^2 \tag{6}$$

such that the optimization problem consists of minimizing f_0 subject to inequality and equality constraints $GI \leq h$ and

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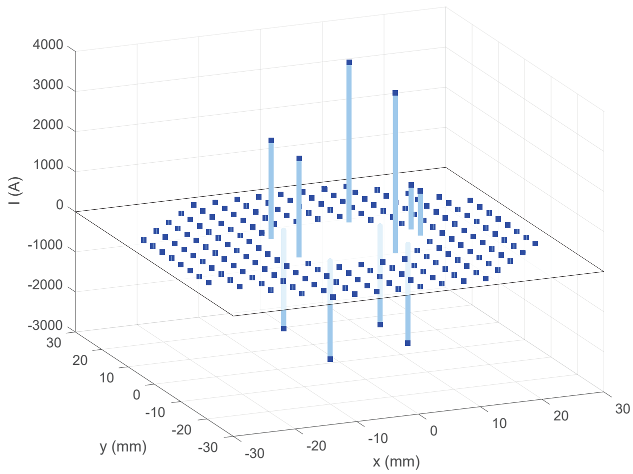

Figure 3: Optimized current for each conductor of the input grid.

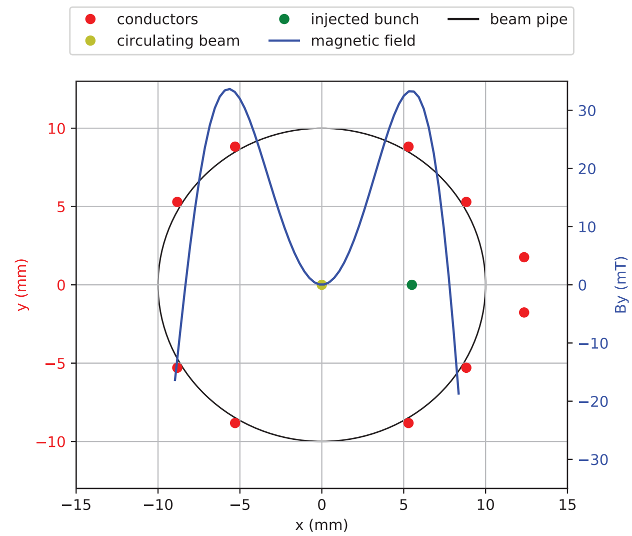

Figure 4: Optimized configuration of conductors to generate the magnetic field needed for injection in SLS-2.

$AI = b$, respectively. This enables to set constraints on the maximum total current, on the maximum current per wire, on the number of wires, etc. This problem was implemented in Python using open source solvers with the Python module `cvxpy` [8, 9].

EXAMPLE APPLICATION TO SLS-2

With the use of this tool we obtained new design solutions for conductor arrangements that go beyond the limitations of the standard designs. Some design solutions for SLS 2.0 subject to different constraints were presented in [6], where also the design for MAX IV was revisited for a lower current solution.

We present here just one example for the SLS-2 case. We first create a regularly spaced grid of conductors, as shown in Fig. 3. In this case we choose a grid with a circular clear aperture of 10 mm radius, a conductor spacing of 3.5 mm and an outer radius of 30 mm. We set the same target values for SLS-2 as the ones used in the introduction section. We also set the following constraints: maximum current per conductor of 5 kA and maximum total current of 25 kA. Then we solve the optimization problem and find a solution for ten conductors with five different currents, as shown in Fig. 3. All other conductors from the input grid are set to zero current and we can remove them. The resulting field profile and final conductor arrangement is shown in Fig. 4. A sufficient field spatial derivative for injection into SLS-2 is achieved with the use of five pairs of conductors, with five different currents, all placed outside the beam pipe.

CONCLUSIONS

None of the standard conductor configurations enables simultaneously a clear aperture larger than the beampipe and to fulfill the targets for a low emittance ring, namely a zero-field at the position of the circulating beam and a field plateau of the order of some tens of mT at a distance of a few mm to kick the injected bunch inside the DA. We have overcome this limitation by developing an optimization tool to determine constrained current distributions required to generate a desired magnetic field. It is easy to apply constraints to the currents and conductor positions, so that it should be possible to adapt to technical limitations while still meeting target values for the field. We can also find out for which constraints a solution is unlikely to be possible. With this tool we provided some preliminary design solutions for SLS-2, and an example that goes beyond the limitations of the standard designs has been presented here.

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