A CODE FOR OPTIMISING TRIPLET LAYOUT *

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Abstract

One of the main challenges when designing final focus systems of particle accelerators is maximising the beam stay clear in the strong quadrupole magnets of the inner triplet. Moreover it is desirable to keep the quadrupoles in the inner triplet as short as possible for space and costs reasons but also to reduce chromaticity and simplify corrections schemes. An algorithm that explores the triplet parameter space to optimise both these aspects was written. It uses thin lenses as a first approximation for a broad parameter scan and MADX for more precise calculations. The thin lens algorithm is significantly faster than a full scan using MADX and relatively precise at indicating the approximate area where the optimum solution lies.

INTRODUCTION

In order to maximise the luminosity of a particle collider, it is essential to minimise the beam size at the interaction point (IP) by focusing it using quadrupoles. For high energy proton colliders, the beams have a large rigidity, meaning that high gradient quadrupoles are required for the focusing. The strength of these quadrupoles is limited by both magnet technology as well as aperture available to the beam or beam stay clear (BSC), which is normally measured in standard deviations, σ, from the peak of the particle distribution.

On top of this, the total length of the focusing section is also a major constraint. Not just for cost and space reasons in a circular collider but also because the longer off-momentum particles are exposed to strong quadrupole fields the more they tend to disperse from the main beam, thereby increasing the chromaticity. Further constraints include, among others, how much the beam deviates from the reference orbit; the space, shielding and magnetic fields required by the experiments in the interaction regions; as well as beam-beam effects that can also be manipulated to increase the luminosity.

The goal of this research was to come up with an algorithm that scans a large parameter space to determine a layout for an triplet that best fulfill the constraints outlined above. Normally aperture simulations are computationally demanding so this algorithm uses the thin lens approximations to quickly and efficiently scan through a large parameter space and identify where more detailed scans should take place. This idea could be eventually be applied to a wider range of colliders.

FAST THIN LENS CALCULATIONS

Simplified Lens Solution

The triplet can be modeled by two common transfer matrices – the one for a drift D of length l and the matrix of a thin lens quadrupole Q of focal strength $g = kL$ where

$$D = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}, \qquad Q = \begin{pmatrix} 1 & 0 \\ g & 1 \end{pmatrix}. \qquad (1)$$

Thin lenses have been previously used in other algorithms to approximate the quadrupoles and drifts required for a final focus telescope with set transfer properties [1]. One of the main methods used for this was developed by Alex Chao [2].

However, this approach was different as it simplifies the system by realising that the exact magnifications of the final focus are irrelevant – the key property of an triplet is to focus a particle originating at $x = 0$ with and arbitrary x' far away from the triplet to $x = 0$ at the interaction point. This property is illustrated in Fig. 1.
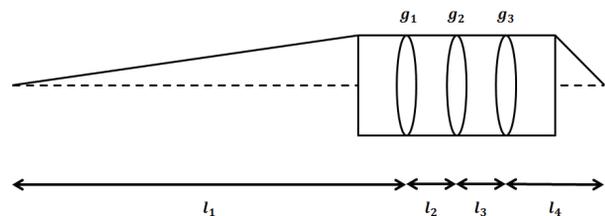

Figure 1: Schematic Layout of Triplet Modeled as Thin Lenses

By considering the overall transfer matrix and initial and final canonical positions and momenta,

$$\begin{pmatrix} 0 \\ x'_{IP} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} 0 \\ x'_{exit} \end{pmatrix}, \qquad (2)$$

one can derive that this property can be summarised as $m_{x12} = m_{y12} = 0$. Matrix multiplication and analytical methods were used to derive formulas giving the strength of g_1 and g_2 to meet this requirement for fixed lengths and g_3.

Figure of Merit

Apart from simplifying the triplet solution, one key advantage of the thin lens matrices is that it allows one to track the position of particles in every lens. In order to track a 1σ particle, one can set its initial position to be $x_{IP} = 0$ and $x'_{IP} = \sqrt{\epsilon/\beta^*}$, where ϵ is the beam emittance and β^* is the β function at the IP. This position can then be used to estimate the BSC, N, in each quadrupole using

$$N = \frac{r}{\sigma} = \frac{r}{x}. \qquad (3)$$

Where x is the position in each lens and r is the radius of the magnet aperture. The radius of the aperture depends on the field gradient k required by this lens and is limited by the maximum magnetic field, ΔB that can be achieved with the available technology and the finite length of the realistic quadrupole, l_q. A simple approximation for the radius is

$$r = \frac{\Delta B l_q}{g B \rho}, \tag{4}$$

where $B\rho$ is the beam rigidity and g is the focal strength. Next one can substitute Eq. 4 into 3 and ignore to the constants to get a figure of merit (FOM) proportional to the BSC, which we will define as

$$FOM = \frac{l_q}{g x}. \tag{5}$$

FOM Scans

All of the above can be brought together to create a code that varies g_3 and the drift lengths between the lenses and uses the thin lens solution to work out the strengths of the other two lenses. For each layout it can then work out the trajectory of a 1σ particle in the horizontal and vertical plane. Next l_q for each quadrupole can be computed from the space available between the lenses.

Using this, one can work out the FOM from Eq. 5 for each configuration to get an estimate of the beam stay clear at each configuration. Since this entire process is done using simple mathematical operations, one can work out the FOM for a large amount of parameters in a very short time.

PRECISE SCANS

In order to compute the accurate BSC of a triplet, one needs to trace the beam size through realistic finite quadrupoles. This can be done using simulation programs such as MADX [3] and is computationally much more intensive than the thin lens FOM. Therefore, this scan can be limited to a small range around the optimum point identified by the thin lens FOM.

MADX Aperture Module

The first step required in the finite lens analysis is to work out the length of the individual quadrupoles. Next, the exact strengths of the finite quadrupoles have to be computed. The thin lens strength $g = kL$ can be used as a first approximation, however the MADX matching module has to be used to work out the exact strengths.

In order to test the accuracy of the analytical method, a FOM scan was performed to find a maximum FOM triplet. Next PyMADX [4] was used to write a script to compare the predicted and matched strengths of the quadrupoles in triplets with the spatial configuration of the maximum FOM triplet with different outer quadrupole strengths. The script wrote and executed MADX files of triplets, matched them and recorded the strengths of the quadrupoles. The matched strengths and predicted strengths were then plotted and are shown in Fig. 2.
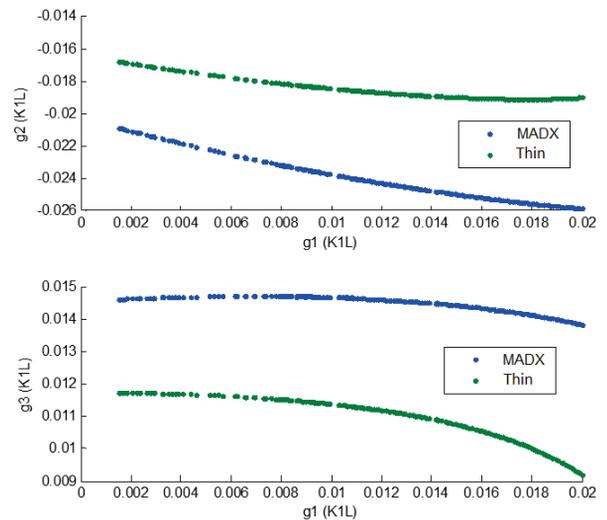
Figure 2: Predicted and Matched Quadrupole Strengths for Maximum FOM Configurations with Different Strength Outer Quadrupole.

From Fig. 2 it can be seen that whilst there is a discrepancy between the actual and predicted strengths of the quadrupoles, these two quantities follow similar patterns. Next, the new strength can be used to work out the available coil aperture using Eq. 4 and the MADX aperture module can work out the beam stay clear at every point, returning the minimum. An example plot for the maximum FOM triplet was produced using the aperture module is shown in Fig. 3.
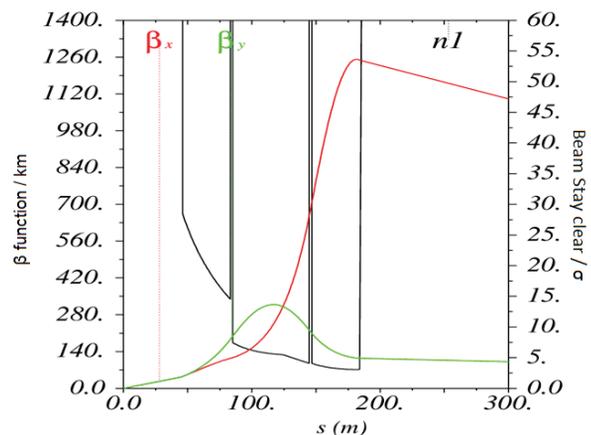

Figure 3: Example Plot Showing β Functions and Beam Stay Clear for Maximum FOM Triplet.

Comparison of Thin and Precise Scan

Again, PyMADX was used to write a python script that iterated through several different triplet configurations. The script produced MADX scripts for each configuration, matched them and ran the aperture module. The minimum beam stay clear was recorded for each configuration and is plotted in Fig. 4. For comparison, the FOM was calculated for each of these configurations and is also shown in

Fig. 4. A low resolution for this was chosen since the MADX matching and aperture calculations take long to compute.
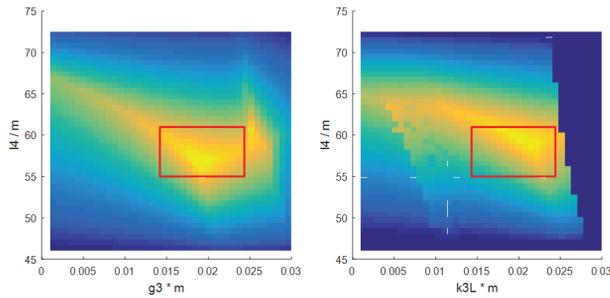

Figure 4: FOM from Thin Lens (Left) and BSC (right) for Different Configurations of Triplet - Yellow Corresponds to a Large FOM or BSC Whilst Blue Corresponds to a Low One.

From Fig. 4 one can see that whilst the maximum points identified by the FOM and precise scans are not found at the same exact points in parameter space, the thin lens FOM provides a good approximation of the general behaviour of the BSC. Moreover, the two points are not very far apart and one can select a sensible margin around the FOM maximum for where precise scans could be performed. An example of such a margin is shown by the region surrounded by a red box in both plots. One can optimise the size of these margins by monitoring the BSC scans and adjusting them if needed.

APPLICATIONS

FCC-hh

The triplet optimisation code was initially written with the aim of optimising an alternative final focus of the Future Circular hadron-hadron Collider (FCC-hh) [5]. The unique problem of the FCC-hh is that large amount of collision debris requires the triplet quadrupoles to have a significant amount of shielding. Therefore, the shielding was added when performing the precise aperture scans.

The code was further modified to constrain the length of the first and third quadrupoles to be identical and let all three quadrupoles have the same coil aperture to ease manufacturing. Moreover, the aperture radius was worked out using an empirical formula based on data presented by the FCC magnet team [6]. The code was used to iterate over triplet of increasing lengths, finding the first one that offered a BSC of 15.5σ. An outline of this algorithm is shown in Fig. 5.

It is very important to have just enough shielding to protect the triplet from debris but not unnecessarily limit the BSC. Therefore, once an adequate triplet was found using the algorithm, radiation studies were performed to see if more or less shielding would be required [7]. If the shielding had to be changed, this was done and the algorithm would be run again. This was repeated until a short triplet with enough BSC and shielding was found.
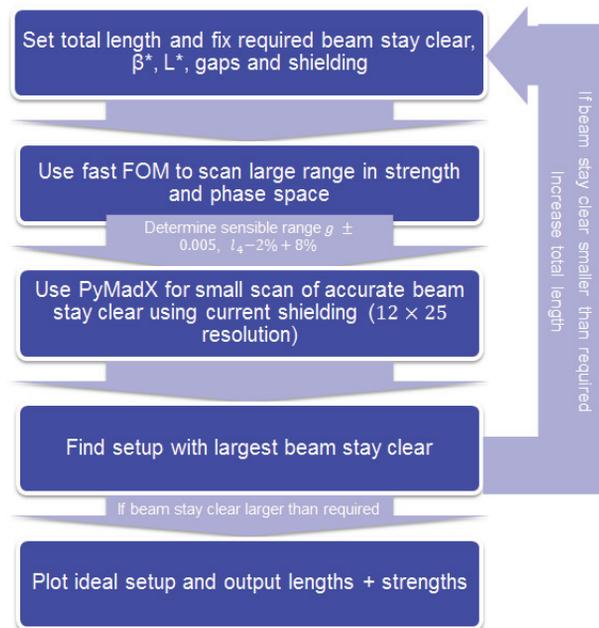

Figure 5: Outline of Triplet Optimisation Code used for an Alternative Triplet in FCC-hh.

Future Application and Variations

By modifying the optimisation algorithm in a similar way as for the FCC-hh, one could use it to optimise the triplet of other high energy proton machines such as the High Energy LHC. Depending on the stage and nature of the accelerator design one can add or relax constrains.

The code could also be changed to optimise other properties such as the dynamic aperture instead or on top of BSC. In this case the magnets could be simplified to thin lens again. If that representation would not be sufficient, the thin lens solution can be used as a good starting point for matching finite lenses.

CONCLUSION

The optimisation of triplets was tackled by approximating the quadrupoles as thin lenses and simplifying the focal properties of an triplet. This made it possible to analytically work out the strengths for a thin lens triplet, which can also be used as a starting point when matching finite lenses. A thin lens FOM was developed that shows a similar behaviour as more precise scans. This allows one to greatly decrease the parameter space range over which precise scans should be performed in order to find an optimum triplet. This method has been applied to the FCC-hh and possible further application have also been discussed.

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