DISTRIBUTION AND EXTREME LOSS ANALYSIS IN THE ESS LINAC: A STATISTICAL PERSPECTIVE

Alexander Lauge Pedersen, Dragi Anevski, Lund University, Sweden
Mohammad Eshraqi, Ryoichi Miyamoto, ESS, Lund, Sweden

Abstract

The report takes a statistical approach in the study of distribution evolution of the proton beam within the ESS linac and reports a new technique of pinpointing the non-linear space-charge effect of the propagating proton beam. By using the test statistic from the nonparametric Kolmogorov-Smirnov test the author visualises the change in the normalised distributions by looking at the supremum distance between the cumulative distribution functions in comparison, and the propagation of the deviation throughout the ESS linac. This approach identifies changes in the distribution which may cause losses in the linac and highlights the parts where the space-charge has big impact on the beam distribution. Also, an Extreme Value Theory approach is adopted in order to quantify the effects of the non linear forces affecting the proton beam distribution.

INTRODUCTION: THE ESS PROJECT

The European Spallation Source (ESS), presently under construction, will be the highest brightness neutron source powered by a 5 MW proton linac located in Lund, Sweden. One of the major tasks of the Beam Physics team is simulations and design of the ESS linac within the ESS Accelerator Project. As seen in Figure 1, the accelerator consists of a normal conductive section and a superconductive section. The normal conductive section consists of five sections: Ion Source (IS), Low Energy Beam Transport (LEBT), Radio Frequency Quadrupole (RFQ), Medium Energy Beam Transport (MEBT) and Drift Tube Linac (DTL). This part of the linac is responsible for the initial acceleration of the proton beam up to ~ 90 MeV. The superconductive section consists of Spoke, medium β, high β elliptical cavities and brings the beam energy to 2 GeV. The High Energy Beam Transport (HEBT) section will transport the beam and by using a set of raster systems paint the tungsten target [1]. One big challenge in such a high power linac the ESS project is to identify and mitigate losses, both *fast* (short term, high power losses that may damage components in the linac) and *slow* (continuous and low power losses that can cause radioactivity in the components). This report will focus on the slow losses and how to quantify them by using a statistical EVT approach. Also statistical methods are used to study the beam distribution evolution. This may add to the understanding of where the non-linear forces are prominent and thus affecting the beam distribution and increase the risk of causing losses.

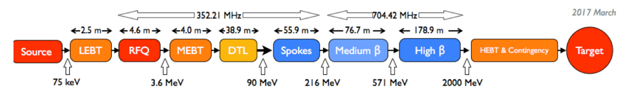

Figure 1: A schematic figure of the ESS linac.

BACKGROUND

Hypothesis Tests

A statistical hypothesis is a hypothesis for unknown parameters in a distribution, that is testable via a set of observations of a random variable with that distribution. When testing for differences between two parameters the test is normally conducted by testing two data sets against each other; the test is then called a two-sample test. By asserting an idealised null-hypothesis, one can conduct a statistical test that examines whether the null-hypothesis can be rejected or not for a certain statistical significance level [2]. For example, consider the null (H_0) and alternative (H_1) hypothesis

$$\begin{cases} H_0 : F_X = F_Y \\ H_1 : F_X \neq F_Y, \end{cases} \tag{1}$$

where X and Y are the random variables (r.vs). We would the like to test H_0 at significance level α. One approach that uses a quantity measure, the supremum norm, to study the differences between distributions is the two-sample Kolmogorov-Smirnov test.

The Two Sample Kolmogorov-Smirnov Test

The two-sample Kolmogorov-Smirnov (KS) test is a statistical test for testing for differences between theoretical distributions [3], as formulated in (1), so it is a test for whether they are the same or not. The test uses the so called KS test statistic

$$D_{n,n'} = \sup_{(x,y)\in\Re} \{|F_Y(y,n) - F_X(x,n')|\}, \tag{2}$$

where $F_Y(y,n)$ and $F_X(x,n')$ are the empirical n and n':th-sample cumulative distribution functions (c.d.fs) of the r.vs X and Y. The limiting distribution of $D_{n,n'}$, as $n, n' \to \infty$, is called the Kolmogorov distribution and is a consequence of a bivariate Donsker's theorem cf. [4]. The practical application of this is that if n and n' are sufficiently large, the null-hypothesis can be rejected for values of the test statistic that are larger than the upper quantile in the limit distribution. For finite n, n' one may instead use the rule to reject the null hypothesis if

$$D_{n,n'} > c(\alpha)\sqrt{\frac{nn'}{n+n'}}, \tag{3}$$

for a level α test, where c(α) is a tabled value. Visually, the distance of interest is the distance illustrated in Figure 2.

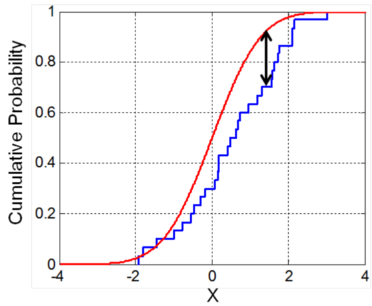

Figure 2: The $D_{n,n'}$ distance represented visually.

Normalised Distributions

Many factors effect the protons through the linac, and this obscures the effects from the non-linearities that is known to be a major factor in the increase in beam halo and emittance. A method of monitoring these non-linear effects on the particles is to normalise the distributions. A simple variable change removes the effect of correlation [1]

$$\begin{cases} x_i = x_i \\ p_i = \frac{\alpha_i}{\beta_i} x_i + \sqrt{\beta_i} x_i' \end{cases} \quad (4)$$

Extreme Value Theory

In EVT one is interested in modelling of the estimation for the extremal part of a distribution. In order to do inference in this setting, such as e.g. perform tests for unknown parameters, the extremes of a data set are examined. At ESS a quantitative study has been conducted where 20 000 simulations on a linac with imposed stochastic errors in the QPs (gradient and alignment erros), accelerating cavities (errors on the accelerating field, phase and their alignment) and input beam. With these simulations, different observations of the losses are presented and form a solid foundation for statistical data analysis. In the Block-Maxima approach, consider the value of the observed losses (in W) in a certain section to be:

$$\bar{x}_k = (x_1, x_2, ..., x_k) \quad (5)$$

that are observations of the random vector

$$\overline{X_k} = (X_1, X_2, ..., X_k) \quad (6)$$

and consider k different independent, identically distributed realizations $\overline{X}_{k,1}, ..., \overline{X}_{k,n}$ of the random vector $\overline{X_k}$. Then a standard result from EVT states that the block maxima

$$M_n = \max_{1 \le j \le k} (X_{j,n}) \quad (7)$$

properly normalised and scaled, converges in distribution to a generalised extreme value distribution

$$GEV(x; \mu, \sigma, \xi) = exp\{-[1 + \xi(\frac{x-\mu}{\sigma})]^{-\frac{1}{\xi}}\} \quad (8)$$

as $n \to \infty$, where $-\infty \le \mu \le \infty$, $\sigma > 0$ and $-\infty \le \xi \le \infty$ is the location, scale and shape parameter respectively. One important parameter of estimation is the return-value, x_p, which is the value when

$$P(X \le x_p) = 1 - p = 1 - \frac{1}{m} \quad (9)$$

with

$$\hat{x_p} = \hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}}(1 - (-log(p))^{-\hat{\xi}}) \quad (10)$$

as the inverse of the GEV function with a predefined p-value describing the probability that a big observation x_p will occur within m observations (p = $\frac{1}{m}$). $\hat{\mu}, \hat{\sigma}$ and $\hat{\xi}$ denotes the Maximum Likelihood estimate (MLE) of μ, σ and ξ.

Monte Carlo Methods - The Empirical Bootstrap Method

Monte Carlo (MC) methods are useful when generating data for computational purposes. In this report the empirical (or non-parametric) bootsrap method is used to generate distributions from which one can calculate the return value, $\hat{x_p}$. The method is based on a re-sampling routine where the distribution of maximum losses are regenerated to calculate the return value for each iteration [5]. This forms a distribution of return values from which a 100(1-α)% confidence interval (CI) of the same can be constructed.

Profile Likelihood Method

Another approach to obtain the CI for the return value is to express the likelihood function in terms of the return value, $\hat{x_p}$. Then one can maximize likelihood function to obtain the CI by two-sided likelihood ratio test [6].

Data Generation

To be able to perform comprehensive studies on the maximum losses in the ESS linac, simulation is the main tool for pre-operational studies. The code TraceWin [7] is used for particle tracking and error simulations. Errors that can occur in the actual construction of the linac are represented with stochastic properties because of the uncertainties in the accuracy in the construction of the designated lattice. Misalignments of elements (QPs and cavities in this study), degradation of the fields in the cavities and quadrupoles and errors in the input beam distribution have been imposed on the system trying to take a realistic approach.

RESULTS

The results from the distribution analysis and the EVT are presented below.

Distribution Analysis

The comparison of the distributions can be done by comparing the input distribution with the distribution at the n:th element in the accelerator to see how the distributions change through the linac. However, in this report the change in distribution between the n:th and the $n+1$:th element

[1] Where x and x' is are the coordinates in the phase-space notation. α and β are the Courant-Snyder parameters

ISBN 978-3-95450-182-3

is covered [8] to try to capture the positions in the linac where the big changes of distribution occur. By applying the two-sample KS test one can measure the supremum distance (2) between the n:th and the $n+1$:th cumulative distribution. To further visualise this quantity, the supremum distance is plotted at each location in the linac, as Figure 3 shows.

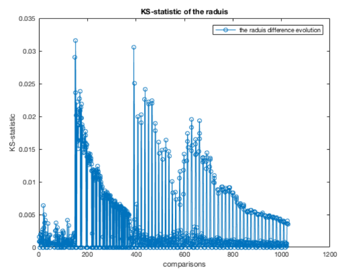

Figure 3: The figure represents the supremum distance between the n and n+1 distribution in the linac for the x-x' plane.

As seen in Figure 3, the major peaks are present between element 179-181, 399-401 and around element 600 which are the intersections between the MEBT-DTL, DTL-Spokes and Spokes-MBL. In the first two cases there is a change from a low phase advance structure to a high phase advance one and the opposite happens at the end of the DTL, while at the Spokes-MBL the main cause is the frequency jump. From a statistical point of view, the change of the locations of the particles must be statistical significant in order to be able to say that a distribution is different from another. This test is done using the KS-test which return a 0 if the distributions can't be seen at different, at a statistical significance level α, and 1 if they can.

EVT

The section of interest for the EVT approach in this report is the MEBT because of its consistency in the amount of losses throughout the simulations. For each of the 20 000 simulations, a maximum loss is registered in the MEBT section and an empirical maximum loss distribution can be formed, as Figure 4 shows.

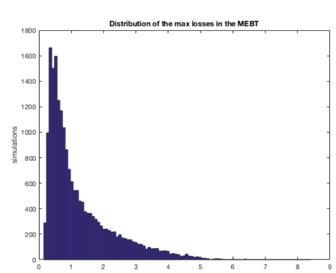

Figure 4: The figure represents the distribution of maxumum losses in the MEBT.

Using (9) the return value is obtained and the confidence interval is constructed by performing the empirical bootstrap method on the maximum loss distribution and calculating a return-value for each iteration (Figure 5). The profile likelihood method can also be used for the same purpose.

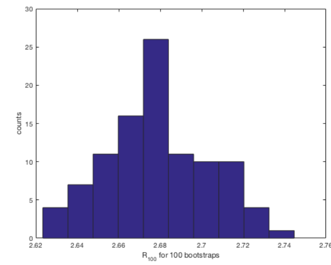

Figure 5: The figure shows the distribution of the return values using the empirical bootstrap method.

The two methods gave similar results and the maximum loss one would expect if 100 different machines were to be tested, with the design values used in this study, would be between [2.6396 2,7229] W using the Bootstrap method and between [2,6191 2,7359] W using the profile likelihood method, with a 95 % certainty.

DISCUSSION

This report highlights a statistical method to monitor the beam distribution evolution through the ESS linac. By using statistical tests one can determine where the big changes occur thus presenting the critical points in the machine. Big changes in the distribution may cause particles to deviate from its intended envelope. Knowledge of such changes might be useful before and during the operation of the machine, pointing to the usefulness of these methods.

The EVT approach aims to predict extreme events of losses in the linac. Because much time is spent in the preparatory stages of the accelerator projects one could use the approach to design the lattice to handle extreme events. The strenght of EVT in combination with MC methods is to make powerful predictions and give estimates of the maximum loss one might expect at a certain probability level.

REFERENCES

[1] S. Peggs *et al.*, "ESS Technical Design Report", ESS, Lund, Sweden, pp. 268–269, 2013.

[2] G. Blom *et al.*, "Sannolikhetsteori och statistikteori med tillämpningar", *Studentlitteratur AB*, pp. 320–324, 2005.

[3] Frank J. Massey, Jr., "The Kolmogorov-Smirnov Test for Goodness of Fit", *Journal of the American Statistical Association*, vol. 46 no. 253, pp. 68-78, 1951.

[4] A. Van der Vaart and J.A. Wellner, *Weak Convergence of Empirical Processes*, Springer-Verlag, New York, 1996.

[5] G.H Givens *et al.*, *Computational Statistics*, John Wiley & Sons inc., 2013.

[6] S. Coles, *An Introduction to Statistical Modeling of Extreme Value*, Springer, 2001.

[7] D. Uriot and N. Pichoff, “Status of the TraceWin Code”, in *IPAC’15*, Richmond, VA, USA, May 2015, pp. 92–94.

[8] L.P. Alexander, “Beam losses along the ESS LINAC due to nonlinear effects - a statistical review”, Lund University, 2016.

ISBN 978-3-95450-182-3