APPLICATION OF MODIFIED KV-DISTRIBUTIONS TO STUDY THE PHASE PORTRAIT TRANSFORMATION OF INTENSE BUNCHES IN MAGNETIC FIELDS

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Abstract

Modified KV-distribution functions are applied to study the intense bunch behaviour in transverse magnetic fields. The function used allow to consider both the emittance-dominated and charge-dominated bunches in 2D and 3D approximations. Bunch phase portrait transformation caused by the coupled particle oscillations in magnetic field is investigated. Particular case is proved to exist characterized by the absence of the emittance transfer.

INTRODUCTION

Application of KV-distribution is convinient way to study the charged particle beam behavior in accelerator structures [1]. KV-distribution describes the quasistationary continuous beam. To provide the bunch dynamics study or the study of the most general case of nonstationary beam the modification of KV-distribution is needed. First, such modification in 2D approximation was developed in [2] for the description of nonstationary ring of the particles. Further, various 3D models are developed (for example, [3-6]), which give the possibility to investigate the ellipsoidal bunch dynamics with most physical generality. The common approximation of all these models is the requirement of the linearity of the self-consistent field acting on the particles. This approximation allows to obtain the beam envelope equations analytically. In present paper the analogous model is used to study the phase portrait transformation in nonuniform magnetic fields.

MODEL DESCRIPTION

It is well known that magnetic field acting the charged particle beam results in the coupling of the particle oscillations in the plane transverse to the magnetic field lines. The phase ellipses of the bunch corresponding to the coordinate space and the velocity space rotate with different angular velocities that results in the effect of emittance transfer in laboratory coordinate system. This effect becomes more complicated in nonuniform magnetic field, for instance, in multipole lenses, magnetic mirrors and gorns, as well as in optics fringe fields. It seems to be important to study this effect for the non-relativistic beams with significant own space charge forces. For this goal the next model may be applied.

For the simplicity let us consider the case when the beam particle motion along the magnetic field line may be neglected, and the character of the field nonuniformity is that the field is weakly changed across the beam profile. So the motion equations of the particles may be written as:

$$\begin{aligned} \ddot{x} &= \frac{eB\dot{y}}{mc} + \frac{e}{m}\frac{\partial\Phi}{\partial x_1} \\ \ddot{y} &= -\frac{eB\dot{x}}{mc} + \frac{e}{m}\frac{\partial\Phi}{\partial y_1} \end{aligned} \qquad (1)$$

Here (x,y) are laboratory coordinates, (x_1, y_1) are the coordinates in the coordinate system, connected with the beam profile center of mass, Φ - the beam own potential, B - the magnetic field strength value, m and e - the mass and the charge of the beam particle respectively, c - the speed of light.

The most affect of the field nonuniformity is expected to be from the first term of the field spatial coordinate series. Let us suppose that the field gradient is a constant value, so one can write:

$$\omega_B(x) = \omega' x \sigma(x), \qquad (2)$$

where ω_B is the cyclotron frequency, ω' is the gradient of the cyclotron frequency, σ - Heavyside function.

During the beam motion in the field with constant gradient the beam profile central particle moves in transverse plane according the equation:

$$\frac{dx_0}{dt} = \pm\sqrt{(v^2 - \frac{\omega'^2 x_0^4}{4})}\,, \qquad (3)$$

where coordinate x_0 corresponds to the beam profile central particle, v - the beam drift velocity.

In general case the dynamics of the beam profile should be described by the system of the ten ordinary differential equations of 1st order in usual derivatives ([7]). But if we consider the case, when the angular velocity of the elliptical cross-section of the beam is connected with the cyclotron frequency value by means of relation

$$\dot{\theta} = \frac{\omega_B}{2}, \qquad (4)$$

and the suppose that the relation is satisfactory:

$$\dot{x}_0 \cos 2\theta = \dot{y}_0 \sin 2\theta\,, \qquad (5)$$

One can obtain the separation of the variables in the equations of the motion, corresponding to the coordinate system, connected with the main axes of the beam cross-section ellipse (x_2,y_2):

$$\ddot{x}_2+\frac{\omega_B^2}{4}x_2=\omega'\dot{x}_0\cos^2\theta\frac{x_2}{\cos 2\theta}+Q\frac{x_2}{(R_x+R_y)R_x}$$
$$\ddot{y}_2+\frac{\omega_B^2}{4}y_2=-\omega'\dot{x}_0\sin^2\theta\frac{y_2}{\sin 2\theta}+Q\frac{y_2}{(R_x+R_y)R_y} \quad (6)$$

Here Q is proportional to the linear density of the own beam charge, R_x and R_y are the semiaxes of the ellipse of the beam cross-section, corresponding to the plane of the beam rotation,

$$Q=\frac{4\pi e^2 n_0(t)R_xR_y}{m},$$

n_0 is the particle density, which is constant across the beam profile, but in general case is dependent on time.

The invariant I of the system (6) may be written as

$$I=\frac{(\dot{R}_x x_2-R_x\dot{x}_2)^2}{\varepsilon_1^2}+\frac{(\dot{R}_y y_2-R_y\dot{y}_2)^2}{\varepsilon_2^2}+$$
$$+\frac{x_2^2}{R_x^2}+\frac{y_2^2}{R_y^2} \quad (7)$$

Here ε_1 and ε_2 are the values proportional to the partial rms emittances of the beam.

If we take the distribution function f looking as

$$f=\kappa\delta(1-I), \quad (8)$$

where δ is Dirac-function, κ - the constant of normalization, one can obtain for the particle density:

$$n=\frac{\pi\kappa}{R_xR_y}\varepsilon_1\varepsilon_2\sigma(1-\frac{x_1^2}{R_x^2}-\frac{y_1^2}{R_y^2}). \quad (9)$$

The relation (9) proves that the model is self-consistent.

From the property of the invariant I one can obtain the equations for the semiaxes R_x and R_y:

$$\ddot{R}_x+\frac{\omega_B^2}{4}R_x=(\frac{\omega'\dot{x}_0\cos^2\theta}{\cos 2\theta}+\frac{Q}{(R_x+R_y)R_x})R_x+\frac{\varepsilon_1^2}{R_x^3}$$
$$\ddot{R}_y+\frac{\omega_B^2}{4}R_y=(-\frac{\omega'\dot{x}_0\sin^2\theta}{\sin 2\theta}+\frac{Q}{(R_x+R_y)R_y})R_y+\frac{\varepsilon_2^2}{R_y^3}$$

From relation (5) it is easy to find the conditions corresponding to the absence of the rms emittance transfer ([7]). If the bunch cross-section has the specific orientation of its main axes so the angle between the ellipse axes and the laboratory coordinate system axes is equal $\pi/4$, then it has the same angle of the value after the reflection in magnetic field (the 180° turn) and th emittance transfer is not observed. The bunch inlet should be the strong perpendicular to the magnetic field lines, and the magnet edge focusing is supposed to be absent too.

The arbitrary orientation of the beam cross-section with respect to the laboratory coordinate system connected with the magnet field geometry will result to the emittance transfer according the next relation:

$$V_n=\int dp_1dp_2...dp_n,$$

$$V_4=p_1p_2p_3p_4\frac{\Gamma(\frac{1}{2})^n}{\Gamma(\frac{n}{2}+1)}, \quad (10)$$

$$V_4=\frac{V_2^{(1)}V_2^{(2)}}{2}.$$

where V_n is the volume of phase space with dimension equal to n, and in our case $n=4$ or V_4 corresponds to the phase volume connected with the beam cross-section, $n=2$ or $V_2^{(1)}$ and $V_2^{(2)}$ correspond to phase volume in spaces (x,x') and (y,y').

In general case the effect of emittance transfer is affected by the initial mean angular momentum of the beam particle too ([4]).

To study the 3D task of phase portrait transformation in another model should be applied. In [7] the distribution function is used in which the third emittance corresponding to the coordinate direction perpendicular to the plane of the bunch rotation as a whole in magnetic field is equal to zero. To consider the more realistic case the model which involves the distribution function developed in [6] is supposed to be applied in future.

CONCLUSION

Analytical model is used to study phase portrait dynamics of the charged particle bunch in nonuniform magnetic field. The particular case is found characterized by the emittance transfer absence. The results may be applied while the magnetic optics is developed such as achromatic structures and magnetic mirrors.

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