

# HIGHER ORDER MULTIPOLE ANALYSIS FOR 952.6 MHz SUPERCONDUCTING CRABBING CAVITIES FOR JEFFERSON LAB ELECTRON-ION COLLIDER\*

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## Abstract

The proposed electron ion collider at Jefferson Lab requires a crabbing cavity system to increase the luminosity in the colliding beams. Currently several superconducting crabbing cavity designs are being reviewed as the design option for the crabbing cavity. Knowledge of higher order mode multipole field effects is important for accurate beam dynamics study for the crabbing system, in selecting the design that meets the design specifications. The multipole components can be accurately determined numerically using the electromagnetic field data in the rf structure. This paper discusses the detailed analysis of higher order multipole components for the operating crabbing mode and design modifications in reducing those components.

## INTRODUCTION

The proposed electron-ion collider for Jefferson Lab (JLEIC) consists of two figure-8 rings; one for electrons and the other for protons [1]. The electrons are accelerated using the existing CEBAF machine and the ions are accelerated at the linac booster ring as shown in Fig. 1.

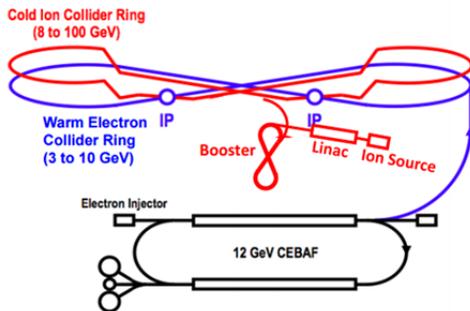


Figure 1: JLEIC electron and ion collider rings.

The luminosity goal of the JLEIC is in the range of low-to-mid  $10^{33} \text{ cm}^{-2}\text{sec}^{-1}$  per interaction point. Bunch crabbing concept will be implemented in increasing the luminosity at the interaction points (IP). A set of local crabbing cavities will be installed before to enforce the head-on collision of incoming bunches at each interaction point and after in order to cancel the crabbing effect on the beam [2].

The proton beam parameters and the operating frequency of 952.6 MHz set tight requirements on the crabbing cavities. The beam parameters for the crabbing system are listed in Table 1. The large crossing angle of 50 mrad and the small betatron function at crab cavity for the proton beam corresponds to a high transverse kick of 20.82 MV required per IP per side.

Table 1: JLEIC Crab Crossing Design Parameters

Parameter	Electron	Proton	Units
Beam energy	10	100	GeV
Bunch frequency	952.6		MHz
Crab crossing angle	50		mrad
Betatron function at IP	10		cm
Betatron function at crab cavity	200	363.44	m
Integrated transverse voltage	2.8	20.82	MV

## CRABBING CAVITY OPTIONS

Several superconducting crabbing cavity geometries are studied in order to achieve a compact crabbing cavity system for JLEIC. Any dimensional constraints and design requirements related to impedance threshold, higher multipole components etc. that may apply to the crabbing cavities are under study. The crabbing cavities are also required to have large beam aperture to accommodate the proton beam; yet be a compact design operating at the high frequency of 952.6 MHz. The designs considered are the single cell and multi-cell rf-dipole design and squashed elliptical design as shown in Fig. 2 [3, 4]. Currently all these design options are analysed with varying beam aperture radii. The rf properties of the three design options with a beam aperture of 70 mm are listed in Table 2.

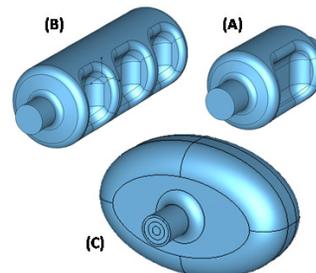


Figure 2: Crabbing cavity design options: (A) single cell rf-dipole cavity, (B) multi-cell rf-dipole cavity, and (C) squashed elliptical cavity with beam aperture of 70 mm.

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The transverse kick of each cavity design is determined with a peak magnetic field of  $\sim 100$  mT per cavity. The single cell cavity requires the highest number of cavities at each IP. However, multi-cell cavity has two similar order modes and the squashed elliptical cavity has one lower order mode.

Table 2: RF Properties of 952.6 MHz Crabbing Cavities Shown in Fig. 2 with A Beam Aperture of 70 mm

Parameter	(A)	(B)	(C)	Units
LOMs	None	770, 869	692	MHz
Nearest HOM	1406	1367	1041	MHz
Deflecting voltage ( $V_t^*$ )		0.315		MV
Peak electric field ( $E_p^*$ )	5.32	5.55	2.18	MV/m
Peak magnetic field ( $B_p^*$ )	13.53	11.42	7.72	mT
Geometrical factor ( $G$ )	166.0	178.4	339.8	$\Omega$
$[R/Q]_t$	50.4	219.0	49.7	$\Omega$
$R_t R_s$	$8.4 \times 10^3$	$3.9 \times 10^4$	$1.7 \times 10^4$	$\Omega^2$
At $E_t^* = 1$ MV/m				
$V_t$ per cavity	2.1	4.2	2.1	MV
$E_p$	41	50	29	MV/m
$B_p$	104	102	103	mT
Total no. of cavities (e/p)	12 / 72	4 / 20	4 / 28	

### MULTIPOLES IN A RF CAVITY

In deflecting/crabbing cavities the rf field of the fundamental operating mode varies across the beam aperture leading to field non-uniformity. This field non-uniformity corresponds to higher order multipole components that may lead to perturbations in beam dynamics such as linear tune shift, chromaticity shift, and chromatic coupling [5]. The higher order multipole components also affect the long term stability in colliders, storage rings and synchrotron machines especially with proton or heavy ion beams [6]. The higher order multipole components are calculated as follows using the Panofsky-Wenzel Theorem.

$$a_n + ib_n = \frac{in}{\pi\omega} \int_0^\infty \int_0^{2\pi} \frac{1}{r^n} E_z(r, \phi, z) e^{in\phi} e^{i\omega z/c} d\phi dz \quad (1)$$

where  $a_n$  and  $b_n$  are skew and normal multipole components.  $E_z$  in the longitudinal electric field at an offset of  $r$ . This paper evaluates the higher order multipole components for the crabbing cavity options.

### MULTIPOLE FIELD ANALYSIS

The higher order multipole components are calculated using Eq. 1 for beam aperture radii of 60 mm and 70 mm for the three design options shown in Fig. 3. The  $b_1$  corresponds to the transverse kick in the crabbing cavity.

### RF-Dipole Cavity Design

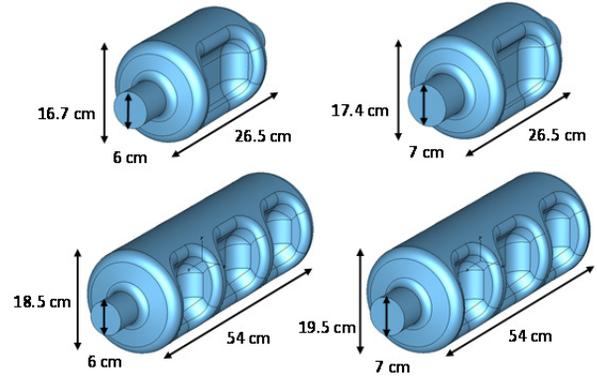


Figure 3: 952.6 MHz single cell (top) and multi-cell (bottom) rf-dipole cavities with beam apertures of 60 mm and 70 mm

In the rf-dipole geometry the transverse electric field gives the major contribution to transverse kick where the transverse magnetic field reduces it slightly. Table 3 and Table 4 list the higher order multipole components up to  $b_7$  for single cell and multi-cell rf-dipole cavity designs with flat trapezoidal-shaped poles for the transverse kick of 1 MV.

Table 3: Higher Order Multipole Components for Single-Cell Rf-Dipole Cavity at  $V_t = 1$  MV

Component	60 mm aperture	70 mm aperture	Units
$b_1$	3.33	3.33	mT m
$b_2$	$-5.0 \times 10^{-4}$	$-5.5 \times 10^{-4}$	mT
$b_3$	853.2	797.5	mT/m
$b_4$	1.55	1.18	mT/m <sup>2</sup>
$b_5$	$-1.2 \times 10^5$	$-4.4 \times 10^4$	mT/m <sup>3</sup>
$b_6$	-3544.4	-1866.0	mT/m <sup>4</sup>
$b_7$	$-4.2 \times 10^7$	$-1.5 \times 10^7$	mT/m <sup>5</sup>

Table 4: Higher Order Multipole Components for Multi-Cell Rf-Dipole Cavity at  $V_t = 1$  MV.

Component	60 mm aperture	70 mm aperture	Units
$b_1$	3.33	3.33	mT m
$b_2$	$-3.0 \times 10^{-4}$	$-3.0 \times 10^{-4}$	mT
$b_3$	697.1	610.0	mT/m
$b_4$	0.92	0.63	mT/m <sup>2</sup>
$b_5$	$-1.2 \times 10^5$	$-5.4 \times 10^4$	mT/m <sup>3</sup>
$b_6$	-2092.5	-1002.7	mT/m <sup>4</sup>
$b_7$	$-4.1 \times 10^7$	$-1.4 \times 10^7$	mT/m <sup>5</sup>

The multipole components for the four cavity designs with single cell and multi-cell designs with varying beam aperture have similar values where larger aperture gives slightly lower values. Due to cylindrical symmetry the even  $b_n$  components are negligible in the rf-dipole cavity

design. The multipole components can be reduced by curving the poles as shown in Fig. 4.

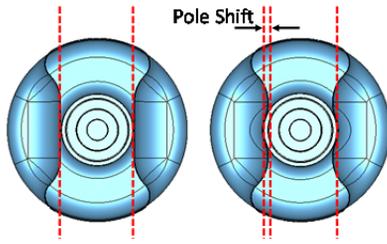


Figure 4: RF-dipole design with flat poles (left) and curved poles (right).

The cavity can be designed to eliminate the effects due to  $b_3$  with adequate curving of the poles as shown in Fig. 5, however this increases the other higher order components such as  $b_5$  and  $b_7$ . Table 5 shows the modified multipole components for the multi-cell rf-dipole design.

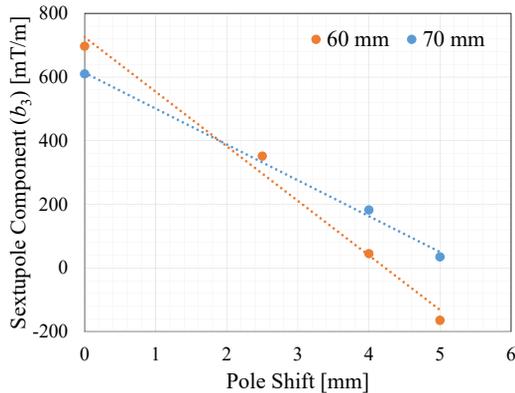


Figure 5: 952.6 MHz single cell (top) and multi-cell (bottom) rf-dipole cavities with beam apertures of 60 mm and 70 mm.

Table 5: Higher Order Multipole Components for Multi-Cell Rf-Dipole Cavity with Curved Poles at  $V_t = 1$  MV

Component	60 mm aperture	70 mm aperture	Units
$b_1$	3.33	3.33	mT m
$b_2$	$-3.7 \times 10^{-10}$	$-7.7 \times 10^{-9}$	mT
$b_3$	45.3	35.1	mT/m
$b_4$	1.16	$-1.6 \times 10^{-5}$	mT/m <sup>2</sup>
$b_5$	$-9.1 \times 10^5$	$-5.7 \times 10^5$	mT/m <sup>3</sup>
$b_6$	-2644.7	-0.024	mT/m <sup>4</sup>
$b_7$	$-4.4 \times 10^8$	$-1.9 \times 10^8$	mT/m <sup>5</sup>

In the multi-cell rf dipole both  $b_5$  and  $b_7$  increase by about an order with the curved poles compared to that with flat poles.

### Squashed Elliptical Cavity Design

The squashed elliptical cavity is considered as a design option in JLEIC due to high operating frequency. These types of geometries are favourable in obtaining a compact

design at higher frequencies since the cavity transverse dimensions are inversely proportional to the frequency.

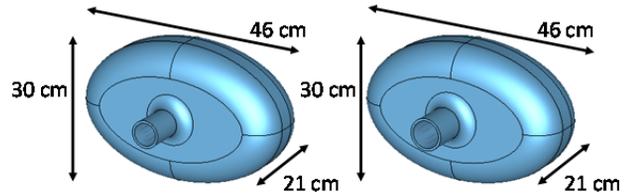


Figure 3: 952.6 MHz squashed elliptical cavity design with beam aperture radii of 60 mm (left) and 70 mm (right).

Table 6: Higher order multipole components for squashed elliptical cavity at  $V_t = 1$  MV

Component	60 mm aperture	70 mm aperture	Units
$b_1$	3.33	3.33	mT m
$b_2$	$-1.7 \times 10^{-9}$	$-1.2 \times 10^{-3}$	mT
$b_3$	77.9	76.4	mT/m
$b_4$	$-5.0 \times 10^{-6}$	2.56	mT/m <sup>2</sup>
$b_5$	-391.9	-2898.2	mT/m <sup>3</sup>
$b_6$	-0.012	-4065.1	mT/m <sup>4</sup>
$b_7$	$-1.8 \times 10^6$	$-1.2 \times 10^6$	mT/m <sup>5</sup>

The higher order multipole components of the squashed elliptical geometry has lower  $b_3$ ,  $b_5$  and  $b_7$  components compared to the rf-dipole cavity designs with flat poles. However, the squashed elliptical design has fewer degrees of freedom present in the geometry in further suppressing the multipole components.

## CONCLUSION

Higher-order multipole components have been calculated numerically for different crabbing cavity design options for JLEIC. Design requirements for the crabbing system for both electron and proton beams are under study. Several design options with varying beam aperture radii are considered in determining the best design that would meet the design requirements for JLEIC. Preliminary analysis shows that multipole components of all the cavity designs do not impose any limitation in considering them for crabbing cavity system for JLEIC.

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