OPTIMIZATION OF DYNAMIC APERTURE WITH CONSTRAINTS ON LINEAR CHROMATICITY

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Abstract

This paper presents numerical technique to optimize dynamic aperture with constraints on linear chromaticity of optical functions. By solving a set of linear equations at each iteration step of dynamic aperture optimization, the linear chromaticity is kept unchanged. The variable range of tuning knobs is taken into account in order to make the technique applicable to practical use. Numerical simulations assuming the SuperKEKB design lattice [1] are performed, and it is demonstrated that the dynamic aperture obtained with the presented scheme is almost comparable to that without constraints. Luminosity simulations assuming weak-strong model show that the constraints lead to improvements of luminosity performance.

INTRODUCTION

Optimization of dynamic aperture (DA) is a common and important topic in existing and future accelerators. There is unfortunately no theoretical strategy for the optimization since it is almost impossible to analytically predict the stability of six-dimensional $(x, p_x, y, p_y, z, \delta)$ motion under complicated nonlinear forces. A traditional way to optimize DA is correction of the chromaticity evaluated with some fixed off-momentum $(\delta \neq 0)$ particles. The correction of the chromaticity, however does not necessarily results improvement of DA. One of the reasons for this is that such fixed-momentum picture is broken down due to the synchrotron motion of the particles. More brute-force approaches using numerical optimization technique are therefore applied to maximize DA, and the resultant chromaticity is in general far from that of the chromaticity correction. Meanwhile the chromaticity control plays a key role in machine tuning. For instance, in collider machines the chromaticity at the interaction point has great impact on the luminosity performance. This paper presents a scheme that optimizes DA with constraints on linear chromaticity.

OPTIMIZATION SCHEME

Our approach is based on the correction of linear chromaticity at each iteration step of the DA optimization. The optimization parameters are modified such that the linear chromaticity is unchanged when the parameters are changed by a DA optimizer.

The horizontal (x) and vertical (y) tune chromaticity caused by thin sextupole magnets with integrated field strength k_2^i $(i = 1, 2, \cdots)$ are given by

$$\frac{\partial \nu_{x,y}}{\partial \delta} = \pm \sum_i \frac{1}{4\pi} \eta_x^i \beta_{x,y}^i k_2^i, \tag{1}$$

where δ, η and β are momentum deviation respect to the reference momentum, dispersion function and betatron function, respectively. The superscript i denotes the optical function is taken at i-th sextupole magnet. The sextupole fields also induce chromaticity of Twiss parameters β and α as

$$\begin{aligned} \frac{1}{\beta_{x,y}} \frac{\partial \beta_{x,y}}{\partial \delta} &= \mp \sum_i \frac{\beta_{x,y}^i \eta_x^i}{2 \sin 2\pi\nu_{x,y}} C_{x,y} k_2^i, \\ \frac{\partial \alpha_{x,y}}{\partial \delta} &= \mp \sum_i \frac{\beta_{x,y}^i \eta_x^i}{2 \sin 2\pi\nu_{x,y}} \left(S_{x,y} + \alpha_{x,y} C_{x,y} \right) k_2^i, \end{aligned} \tag{2}$$

where the parameters $C_{x,y}$ and $S_{x,y}$ are introduced with betatron phase $\phi_{x,y}$ by

$$C_{x,y} \equiv \cos\left(2\left|\phi_{x,y} - \phi_{x,y}^i\right| - 2\pi\nu_{x,y}\right), \tag{3}$$

$$S_{x,y} \equiv \sin\left(2\left|\phi_{x,y} - \phi_{x,y}^i\right| - 2\pi\nu_{x,y}\right). \tag{4}$$

Equations (1) and (2) are written in a matrix form $\boldsymbol{A}\boldsymbol{k}_2 = \boldsymbol{C}$, where the matrix $\boldsymbol{A}$ is a response matrix from the sextupole-field $\boldsymbol{k}_2$ to the vector $\boldsymbol{C}$ which consists of the linear chromaticity shown in the left-hand side of Eqs. (1) and (2).

The chromaticity $\boldsymbol{C}$ will be changed from $\boldsymbol{C}$ to $\boldsymbol{C} + \Delta\boldsymbol{C}$ when one sets a different set of sextupole strengths to survey DA. In the presented scheme we adjust sextupole strength in order to keep the chromaticity unchanged during the DA optimization. The adjustment $\Delta\boldsymbol{k}_2$ can be obtained by solving a linear equation, $\boldsymbol{A}\Delta\boldsymbol{k}_2 = -\Delta\boldsymbol{C}$. The variable range of the sextupole should be taken into account since the field strength has upper and lower limits in a practical case. Finally, the problem to be solved becomes

$$\begin{cases} \boldsymbol{A}\Delta\boldsymbol{k}_2 = -\Delta\boldsymbol{C} \\ \Delta k_{\text{min}}^i \leq \Delta k_2^i \leq \Delta k_{\text{max}}^i, \quad i = 1, 2, \cdots \end{cases} \tag{5}$$

where the parameters Δk_{min}^i and Δk_{max}^i are lower and upper limits of the adjustment, respectively. Equation (5) turns to a quadratic program, and can be solved in some numerical algorithm. We employ the active set method [2], which iteratively applies the Lagrange's method of undetermined multiplier.

In collider machines, chromaticity of x-y coupling is also essential parameters to maximize the luminosity performance. There are several parametrization techniques for betatron coupling in accelerators. Four optical functions r_{1-4} used in this paper are introduced in order to transform the transverse motions to two eigen betatron motions as

$$\begin{pmatrix} u \\ p_u \\ v \\ p_v \end{pmatrix} = \begin{pmatrix} \mu & 0 & -r_4 & r_2 \\ 0 & \mu & r_3 & -r_1 \\ r_1 & r_2 & \mu & 0 \\ r_3 & r_4 & 0 & \mu \end{pmatrix} \begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix}, \tag{6}$$

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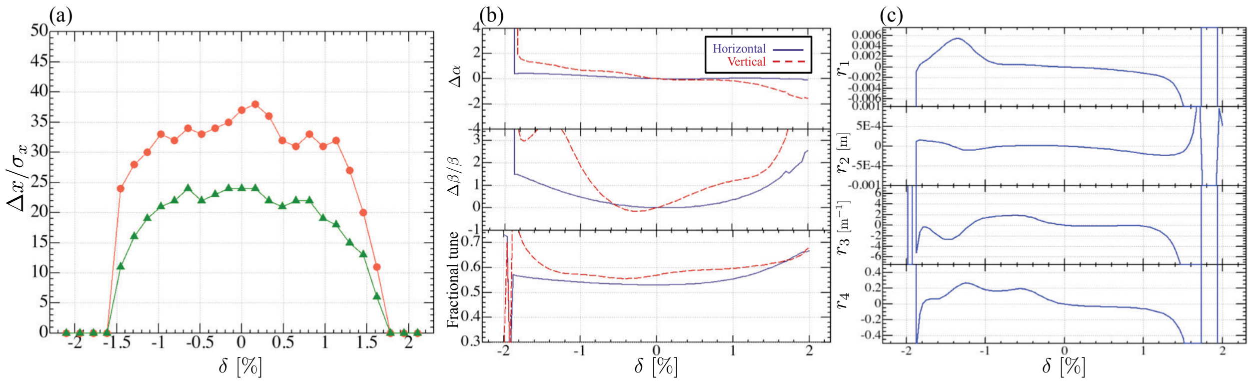

Figure 1: Optimization results without any constraints on chromaticity, where (a) DA, (b) chromaticity of Twiss parameters, and (c) chromatic x-y coupling at the interaction point are shown. The dots and triangles in (a) represent DA evaluated with initial betatron phases $(\phi_{x0}, \phi_{y0}) = (0, 0)$ and $(\pi/2, \pi/2)$, respectively.

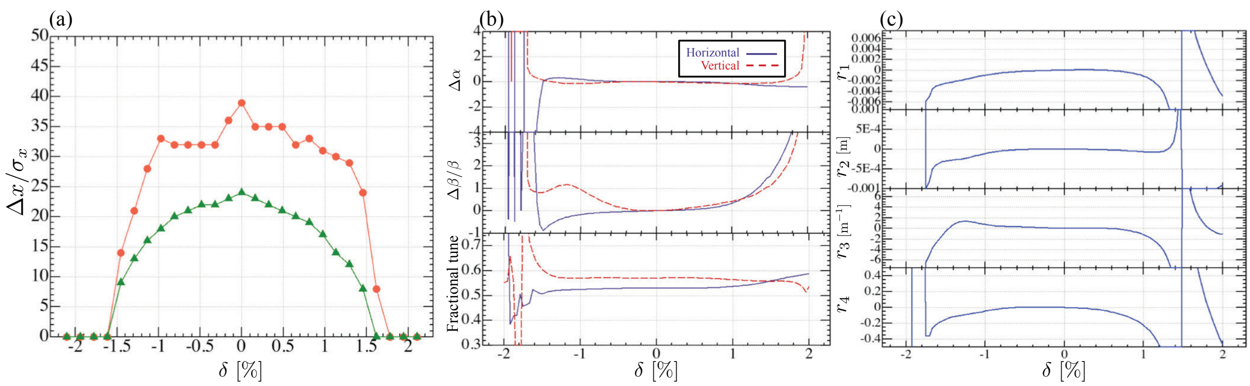

Figure 2: Optimization results with constraints on linear chromaticity of Twiss and x-y coupling parameters.

where $\mu^2 = 1-(r_1r_4-r_2r_3)$. The momentum dependency of r_{1-4} is called chromatic x-y coupling and considered as one of important parameters to be adjusted in collider machines. The linear chromaticity $\partial r_{1-4}/\partial\delta$ can be controlled by skew sextupole magnets. A response function of the chromatic x-y coupling up to first order of skew sextupole field strength is derived with a help of a perturbation theory, and the chromatic x-y coupling can be kept unchanged during the DA optimization as with chromaticity of Twiss parameters. It is note that not only chromaticity but also any distortion of optical function due to the DA optimization can be controlled by solving linear equations similar to Eq. (5) as long as the first order approximation is valid and a sufficient number of tuning knobs are available.

An other way to keep optical functions unchanged in the DA optimization is using multi-objective optimization technique that minimizes or maximizes multiple objective functions. The advantage of the presented method compared to multi-objective approach is that our approach is free from tuning of a weighting factor between DA and optical functions.

APPLICATION TO SUPERKEKB

Simulation study assuming the SuperKEKB design lattice is carried out to illustrate effectiveness of the presented scheme. SuperKEKB is a 7 GeV electron (HER) and 4 GeV positron (LER) collider based on the nano-beam scheme [3]. Although we performed study on both LER and HER, the results only for LER are presented in this paper.

The Nelder-Mead down-hill simplex algorithm [4] is chosen as the DA optimizer in this study. In the usual SuperKEKB lattice design, the down-hill simplex algorithm starts with chromaticity corrections. The DA optimizer uses Touschek lifetime evaluated by DA as a figure of merit rather than DA itself. Touschek lifetime is approximately evaluated by fitting DA by an ellipse and applying Bruck's formula [5]. The linear chromaticity of Twiss and x-y coupling parameters are corrected whenever the DA optimizer changes sextupole or skew sextupole strength. All simulation results presented here are obtained with the accelerator code SAD [6].

In the LER lattice, 54 families of normal sextupole magnets are available, and 12 of them are rotatable. Therefore we have totally 54 normal sextupole and 12 skew sextupole knobs in the optimization. The large number of tuning knobs is expected to allow us to optimize both DA and chromaticity by using the presented scheme. Although three octupole coils are installed into the final focus magnets, we concentrate correction of chromatic effects using these sextupole knobs in this paper.

Table 1: Optimization Results

Parameters	Figure 1	Figure 2
Touschek lifetime	549 sec	516 sec
$\partial\alpha_{x,y}/\partial\delta$	-11.6 / -62.6	0.1 / -9.7
$(\partial\beta_{x,y}/\partial\delta)/\beta_{x,y}$	-8.7 / 94.4	7.0 / 8.3
$\partial\nu_{x,y}/\partial\delta$	-0.3 / 2.5×10^{-2}	0.2 / 2.2×10^{-3}
$\partial r_1/\partial\delta$	-6×10^{-2}	3×10^{-2}
$\partial r_2/\partial\delta$	-8×10^{-3} m	-1×10^{-3} m
$\partial r_3/\partial\delta$	-213 m^{-1}	0.6 m^{-1}
$\partial r_4/\partial\delta$	-23.8	-2.5

Figure 1 shows DA together with the chromaticity of Twiss and x-y coupling parameters at the interaction point. Any constraints on the DA optimization are imposed in Fig. 1. As mentioned already, the relation between the chromaticity and DA is obscure from the viewpoint of traditional chromaticity corrections. An analogous plot obtained by using the presented scheme is shown in Fig. 2, where constraints on the linear chromaticity of Twiss and x-y coupling parameters at the interaction point are imposed. The presented scheme results a little bit smaller but almost comparable DA while excellent improvement of the chromaticity. Table 1 summarizes the estimated Touschek lifetime and the linear chromaticity of Twiss and x-y coupling parameters.

Numerical simulations on luminosity performance assuming weak-strong model are performed to check the advantage of the presented scheme. The simulations are performed with lattices shown in Figs. 1 and 2. Simulation results are summarized in Fig. 3, where specific luminosity is plotted as a function of the product of electron and positron bunch current $I_b^{e^-} I_b^{e^+}$. The luminosity performance is improved in the wide range of beam current by imposing the constraints in the DA optimization. The relatively steep luminosity reduction is observed at low beam current, $I_b^{e^-} I_b^{e^+} < 0.3$ mA^2. Although, the reduction is improved by imposing the constraints, it still exists. This behavior is already reported for example in Ref. [7]. Such steep reduction is not observed when the complicated final focus system including the detector solenoid is simplified with a quadrupole doublet. Therefore the lattice nonlinearity at the interaction region is probably responsible for this phenomenon. The presented study shows that the linear chromatic effects of Twiss and x-y coupling parameters explain some of the reduction. As for future works, geometrical effects will be added to the optimization algorithm in a same manner to mitigate the luminosity reduction further.

A separate study on the HER lattice is also carried out, and shows that consideration on chromatic effects in the DA optimization is essential to maximize luminosity performance.

SUMMARY

A numerical technique to adding constraints on linear chromaticity to the DA optimization is proposed. A set of linear equations is solved at each iteration step of the DA optimization in order to keep the chromaticity of Twiss and x-y coupling parameters unchanged. The upper and lower limits of optimization parameters are taken into account for practical use. Although only linear chromaticity is considered in this paper, our approach is applicable to any optical parameters as long as the first order approximation Eq. (5) is valid and a sufficient number of tuning knobs are available.

Simulation study assuming the SuperKEKB design lattice is carried out, and it is shown that the linear chromaticity almost kept unchanged during the DA optimization. The resultant DA is almost comparable to that without constraints. Numerical simulations on luminosity performance are performed to demonstrate usefulness of the presented method. It is confirmed that the constraints on linear chromaticity of Twiss and x-y coupling parameters improves luminosity performance in SuperKEKB.

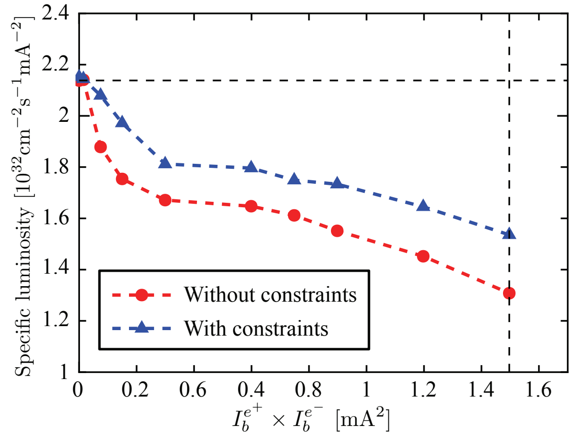

Figure 3: Specific luminosity as a function of the bunch current of the electron and positron beams. The horizontal and vertical broken lines represent the design specific luminosity and design bunch current, respectively.

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