A NEW METHOD FOR EMITTANCE RECONSTRUCTION USING A SCRAPER IN A DISPERSIVE REGION OF A LOW ENERGY STORAGE RING *

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Abstract

Beam scraping is a standard method for beam emittance measurements at low energies and will be applied at the Extra Low ENergy Antimatter (ELENA) ring. However, in ELENA, as in many other low energy storage rings, the scraper is located in a position of finite dispersion which poses a unique challenge when reconstructing the emittance from beam intensity data. A new algorithm for ELENA and other machines that use a scraper in a dispersive region has been developed for the purpose of determining the rms emittance of the beam. It combines data obtained by scraping the beam from opposite sides with information on the storage ring lattice. In this contribution, the new algorithm is presented, tested using simulations and compared with alternative methods for emittance reconstruction.

INTRODUCTION

Emittance measurement is an essential diagnostic in controlling and supplying high quality beams from most particle accelerators. The use of a scraping device is often employed to perform these measurements, being a practical device that provides information of the beam profile, has the capability to probe low intensity beams and their halos, and also having the ability to collimate the beam, which can be very useful during the commissioning of a new machine.

A scraper system usually comprises one or more scraper blades which move slowly into the beam along a certain axis and destroy the beam in doing so. As the beam is destroyed its intensity is recorded as a function of the scraper blade's position along the axis of interest, e.g. by monitoring showers of secondary particles as in the Antiproton Decelerator (AD) [1] or by a standard Beam Current Transformer (BCT) as in the PS booster at CERN. The data taken from these two instruments provides information on the beam profile, and combined with known parameters at the scraper's position, the emittance of the beam can be ascertained.

ELENA is a low energy storage ring designed to accept antiprotons with a kinetic energy of 5.3 MeV and decelerate them down to 100 keV with the help of an electron cooler to keep the beam from blowing up [2].

The ELENA ring will employ the use of a scraper (Fig. 1) for emittance diagnostics as these devices are well suited to low energy machines. However, the lack of a location with zero dispersion in the ELENA optics complicates the calculation of the emittance from the beam density measurement performed by scraping.

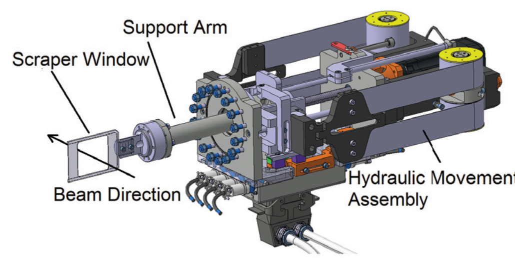

Figure 1: CAD illustration displaying the design of the scraper in ELENA.

Steps have been taken to develop and test, through simulations, an algorithm for reconstructing the emittance of an arbitrary shaped beam in a region of non-zero dispersion. These simulations have been done in the context of the ELENA ring, however the methods and algorithm presented here may be used in other storage rings where similar challenges present themselves.

Previously, an algorithm for calculating the emittance of a Gaussian beam with longitudinal momentum spread in a dispersive region, was tested [3] and has since been shown to work with a high level of accuracy in ELENA ring simulations.

Here, we present a new method to reconstruct the rms emittance of a a beam with an arbitrary transverse profile, which relies on two separate scraping measurements from opposite sides at a position with dispersion.

ALGORITHM

When the beam is scraped from positive and negative x, we may describe the remaining fraction of the beam as:

$$F_{\pm}(x_s) = \frac{N_{\pm}(x_s)}{N_0}. \tag{1}$$

where $N_{\pm}(x_s)$ is the number of particles remaining when the scraper is at x_s and N_0 is the total number of particles. This quantity is the Cumulative Distribution Function (CDF) of the beam and can be fit to the scraper data. Differentiating this quantity with respect to x_s, we obtain the corresponding Probability Density Function (PDF) of the beam, $f_{\pm}(x_s) = \frac{\mathrm{d}F_{\pm}(x_s)}{\mathrm{d}x_s}$. An example of these quantities taken from simulation data can be seen in Fig. 2.

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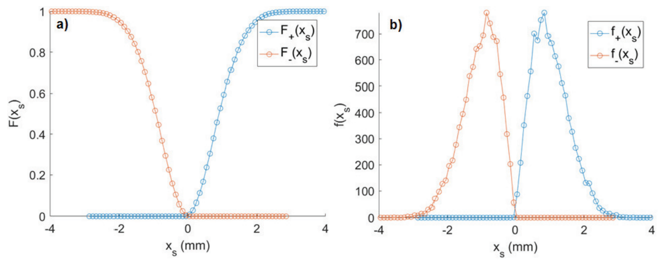

Figure 2: An example of the a) PDFs and b) CDFs given when beams are scraped from positive and negative x in the simulations presented here.

We may begin by expressing the second moment of the quantity $(x_s - x_r)$ in terms of the density functions $f_\pm(x_s)$ which describe the beam when scraped from positive and negative x:

$$\begin{aligned} \langle (x_s - x_r)^2 \rangle_\pm &= \int_{-\infty}^{+\infty} \mathrm{d}x_s (x_s - x_r)^2 f_\pm(x_s) \\ &= \bar{x}_\pm^2 + \sigma_\pm^2 - 2\bar{x}_\pm x_r + x_r^2, \end{aligned} \tag{2}$$

where $\langle \cdots \rangle$ denotes the expectation value, x_s is the scraper position, x_r is an estimate of the closed orbit of the beam, $\bar{x}_\pm$ is the mean value of the measured distribution (PDF) and $\sigma_\pm$ is the rms measured beam size.

Considering $x_{s\pm} = x_0 + \delta D \pm \sqrt{\beta} A$, we may also express this quantity in terms of the rms emittance, ϵ_{rms}:

$$\begin{aligned} \langle (x_s - x_r)^2 \rangle_\pm &= (x_0 - x_r)^2 + D^2(\bar{\delta}^2 + \sigma_\delta{}^2) + 2\beta\epsilon_{rms} \\ &\quad + 2(x_0 - x_r)D\bar{\delta} \pm 2(x_0 - x_r)\sqrt{\beta}\bar{A} \\ &\quad \pm 2D\sqrt{\beta}\langle \delta A \rangle, \end{aligned} \tag{3}$$

where D is the dispersion at the position of the scraper, β is the beta function at the position of the scraper, x_0 is the closed orbit of the beam, A is the phase space amplitude, δ is the particle momentum offset, $\bar{\delta} = \langle \delta \rangle$, $\bar{A} = \langle A \rangle$ and $\sigma_\delta^2 = \langle (\delta - \bar{\delta})^2 \rangle$ is the relative particle momentum offset.

Equating Eqs. (2) and (3), taking the difference for scraping from the two sides and rearranging, we obtain an expression for the rms emittance of the beam which relies only on values that can either be calculated from the measurement data or accurately estimated a priori:

$$\epsilon_{rms} = \frac{1}{4\beta}\left[\sigma_+^2 + \sigma_-^2 + \frac{(\bar{x}_+ - \bar{x}_-)^2}{2}\right] - \frac{D^2\sigma_\delta^2}{2\beta}. \tag{4}$$

Equation (4) forms the basis of the algorithm and explicitly shows the subtraction of the term which accounts for the momentum spread.

SIMULATIONS

To test the algorithm, simulations using the PTC tracking module of MAD-X were carried out [4]. 10,000 macro particles were used to represent the beam of nominal intensity (2.5×10^7 antiprotons) and tracked. For convenience, the smooth regular movement of the scraper blade has been replaced by a stepwise movement. The position is kept constant for 360 turns and then changed by 0.1 mm. At the lowest energy of 100 keV foreseen for ELENA, this corresponds on average to a nominal speed of the blade of 40 mm/s. A Python script to automate and control this process was written [5].

For the scraper blade coming from positive (negative) values of the x-axis, all particles with positions x larger (smaller) than the scraper edge x_s are removed. The phase space data saved at each scraper step was used to graphically represent the scraping process, an example of which can be seen in Fig. 3. For this simulation values of $\beta_x = 0.6877$ m and $D_x = 1.292$ m were used at the scraper.

To fully test the algorithm, beams of various input emittances, longitudinal momentum spreads and transverse profiles were run through the simulation. Bi-gaussian beams based on results from [6] were generated, with a range of emittances, $0.4 - 1.2$ mm mrad and a range of longitudinal momentum spreads $0 - 1 \times 10^{-3}$ (nominal value $\approx 3 \times 10^{-4}$).

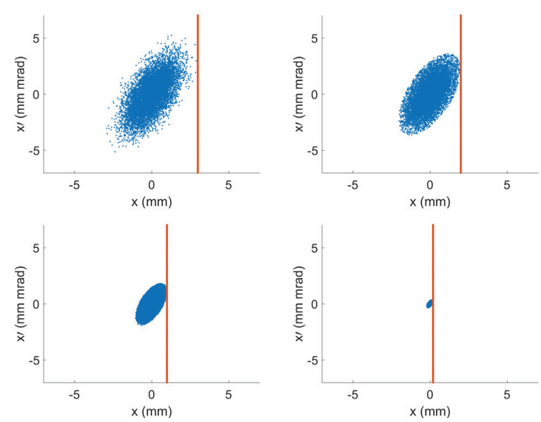

Figure 3: Transverse horizontal phase space of the antiproton beam during various stages of the simulated scraping process. The scraper blade is represented by the vertical red line.

ANALYSIS

The phase space and intensity data were analysed using MATLAB [7]. A small test was performed to determine whether a polymorphic spline fit should be used to analyse the data against the accuracy of using simple numerical methods (Fig 4). It was found that both the splining method and simple numerical methods returned negligible errors, so simple numerical methods were chosen for their simplicity and speed. In this analysis, however, all results were checked against the spline method.

Emittance Reconstruction

Initially intensity data, $F_\pm(x_s)$, was taken from an ideal Gaussian beam with zero longitudinal momentum spread to test the algorithm was working properly. The algorithm

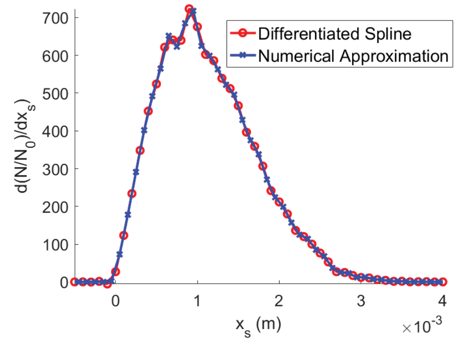

Figure 4: Comparison of a probability distribution obtained using a polymorphic spline and simple numerical method.

consistently reconstructed the beam's input emittance with an accuracy of within 1%.

Table 1: Reconstructed rms emittance values for a bi-Gaussian beam for different momentum spreads with input emittance, $\epsilon_{x,in} = 1\mu$m.

$\sigma_\delta(\times 10^{-4})$	$\epsilon_{x,out}$ (μm)	ϵ_x Error (%)
1	1.0120	1.20
3	1.0127	1.27
5	1.0143	1.43
10	1.0132	1.32

Further tests, running the simulation for a bi-Gaussian beam of varying input momentum spreads resulted in very accurate estimations for the input emittance by the algorithm, as seen in Table 1. The desired accuracy of the scraping device is to estimate the rms emittance to within 10% of its original value.

Errors

There are various sources of systematic error that could compromise the accuracy of the system if not understood and prevented. These include incorrect estimations of the longitudinal rms momentum spread, or of lattice properties such as the beta function (β_x) at the scraper, both of which are currently under investigation.

Because the algorithm relies on two separate measurements of the beam, there is a chance that the closed orbit may not be consistent for each scan. As the algorithm relies on the relative average position of the particles during a positive and negative scan, a closed orbit offset between scans would affect the accuracy of the algorithm.

To investigate the magnitude of this effect, the beam was held with a closed orbit $x_0 = 0$ mm for the first scan, and then moved by varying degrees during the second scraper scan (from the opposite direction). The results are displayed in Fig. 5.

It can be seen that beams with a larger emittance are less susceptible to the negative effects of the closed orbit offset, outside of the range of 0.1 mm the reconstructed emittance value will exceed 10% for a smaller beam. It should be noted that the method of simulating a closed orbit offset is analogous to a discrepancy in the relative position of the scraper blades used for scraping from the positive and negative directions. We advise that this value should be known to within 10% of this precision (0.01 mm).

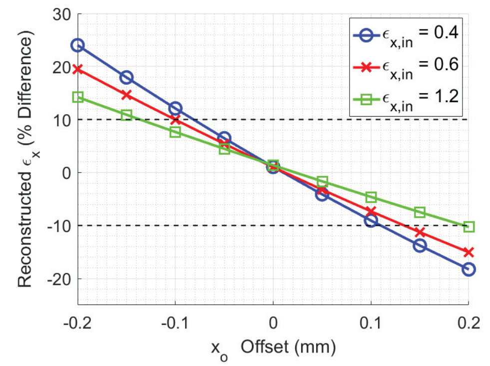

Figure 5: Effects of a closed orbit offset between positive and negative scraper scans on the reconstructed value of the horizontal rms emittance.

CONCLUSION AND PROSPECTS

We have developed and tested, using simulations, an algorithm for determining the rms emittance of a beam travelling in a region of non-zero dispersion using a scraping system. The system works for beams of an arbitrary transverse beam profile, and we have shown that it is compatible with varying magnitudes of rms emittance and rms momentum spreads much greater than the design value.

This algorithm was designed with the purpose of providing a solution to the challenges posed by the ELENA ring at CERN, however the methods presented here could be used in any machine, especially similar low energy storage rings, where similar challenges present themselves. A more detailed description of the simulations and results in this contribution can be found in [8].

The methods of testing the algorithm will soon be benchmarked against real data taken from the ELENA ring once commissioning is fully under way.

Additional studies into systematic errors are being carried out to ensure the least possible sources of error during operation. Finally, further investigations such as how heating processes like intra-beam scattering will affect the beam during scraping, or the impact of antiproton transmission through the scraper blade on the reconstructed value, are planned.

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