FUZZY MARKOV MODELING IN AUTOMATIC CONTROL OF COMPLEX DYNAMIC SYSTEMS

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Abstract

A novel modeling technique for automatic control purposes is discussed. A fuzzy Markov system is proposed to describe both determined and random behavior of complex dynamic plants. The main advantage is its high computational speed. Another benefit of this method is its flexibility and applicability to both linear and nonlinear systems.

A controlled Markov chain represents a fuzzy system with a rectangular membership function. Its output is the probability distribution, not a variable value. This approach represents an attempt to overcome the primary difference between non-randomness of fuzzy sets and Markov chain theory, which deals with random phenomena.

1 INTRODUCTION

Most fuzzy logic applications are intended for control and analysis purposes [1, 2]. Another group of applications is system state prediction [3]. Conventional fuzzy systems cannot operate with random phenomena.

Control processes in real-life plants consist of determined and random elements. Stochastic processes can be described using a Markov modeling approach [4], which provides high computational speed because it utilizes only operations of move and comparison. However, this approach allows simulation of a limited number of system states depending on state quantisation. Furthermore, the transition probability matrix must have large size to achieve high accuracy of modeling. This disadvantage can be avoided using a combination of Markov modeling with fuzzy logic.

In order to extend the application area of both techniques, a fuzzy Markov modeling approach was proposed [5]. Fuzzy systems are often referred to as "universal approximators" [6]. Therefore, fuzzy Markov systems could be used for smooth nonlinear approximation of a multidimensional probability density function. In this case, a Markov model represents a fuzzy inference system with the transition probability matrix stored within the rule base.

Recently, Adaptive-Network-Based Fuzzy Inference Systems (ANFIS) were used for chaotic time series prediction [3]. Similarly, stochastic time series simulation can be carried out using fuzzy inference combined with Markov modeling.

2 MARKOV MODELING

Generalization of the Markov approach to fuzzy systems is based on the following statements. Consider a dynamic model described by the nonlinear difference equation

$$x(t+1) = f\{x(t), u(t), e(t)\}, \qquad (1)$$

where $x(t)$ is the state vector, $u(t)$ is the input vector, $f(x)$ is a nonlinear function and $e(t)$ is an independent Gaussian random vector. This is also a Markov process [7]. The order of this Markov process depends on the order of Eq.(1). A controlled Markov chain can be obtained via state quantisation.

Stochastic processes within a dynamic system can often be assumed to be stationary and ergodic. In this case, the Markov chain is homogeneous and its dynamics are described by the transition probability matrix P. In a high order case, the matrix represents an appropriately dimensioned hypercube. A first order model represents a three-dimensioned matrix

$$P = \{Pijk\}, \qquad (2)$$

$$Pijk = P\{x(t+1)=Xj \mid x(t)=Xi\ , u(t)=Uk\}. \qquad (3)$$

This is the probability of transition from the state Xi to the state Xj under control Uk. The state probability is described as

$$\rho_i = \int_{X_i-\Delta/2}^{X_i+\Delta/2} p(x)dx = P\{x \in [X_i - \Delta/2; X_i + \Delta/2]\}, \qquad (4)$$

where $X = (X1, \dots , Xn)$ is the vector of interval centers; ρ_i is the probability of x being in the i-th interval and Δ is the interval length.

Markov simulation represents realization of a random value with desired distribution. At every time $(t+1)$ the new value $x(t+1)$ is determined as having probability density

$$p(x(t+1)) = f(x(t),u(t)), \qquad (5)$$

where $p(x(t+1))$ is obtained as the appropriate row of the transition probability matrix P.

A Monte Carlo procedure is used to simulate such a system. Generation of a random value with desired distribution is performed using a uniform distribution generator and a functional transform. At the first stage, the current input $u(t)$ and output $x(t)$ are measured. Having been compared with the interval centers, the input and output are transformed to the Markov state $\{Xj,Uk\}$ via the following formulas

Xj: $j = \arg\min |x(t)-Xj|$, (6)

Uk: $k = \arg\min |u(t)-Uk|$. (7)

At the second stage, the vector p is extracted from the transition probability matrix P using the indices j and k. The elements of the vector p represent the corresponding state probabilities $pi=P\{Xi\}$ for the time instant t+1.

Finally, the obtained distribution is utilized for generating the random output $x(t+1)$. A transformation method [8] is used for generating a random number with a known distribution

$$F(x) = \int_{-\infty}^{x} p(x)dx \ . \qquad (8)$$

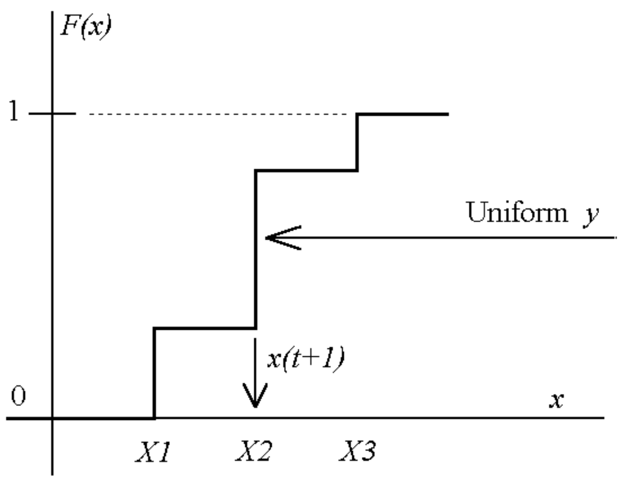

Figure 1: Generation of desired distribution.

A uniform random number y is chosen between 0 and 1, as shown in Fig.1, and the following transform is applied:

$x(t+1)=F^{-1}(y)$. (9)

A Markov chain can be considered as a fuzzy system with a rectangular membership function with no overlap. The terms Xi, Xj, Uk can be substituted with conventional linguistic variables like PB, ZO, NB, etc. This paper focuses on general features of fuzzy Markov modeling. The results are then to be generalized on numerous fuzzification and defuzzification techniques.

3 FUZZY MARKOV SIMULATION

The procedure of fuzzy Markov simulation includes four main stages: fuzzification, inference, defuzzification and randomization.

Before simulation, general parameters of the fuzzy system are chosen and the rule base is created from experimental data. This is usually performed using neural networks or evolutionary computation methods. Possible criteria for optimization of a fuzzy Markov system can be mean square errors of simulated spectral and distribution properties compared with desired characteristics. In order to fulfil the criteria, the following degrees of freedom can be used: the order of the Markov model, the model of fuzzy inference, the type, number and position of membership functions.

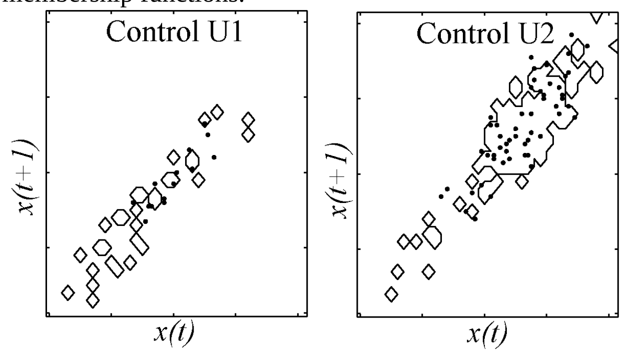

Figure 2: Contour plots of transition probability matrix.

Consider simulation of a first order SISO (single input and single output) Markov model. An example of a transition probability matrix is demonstrated in Fig.1. Contour plots show two typical sections of the matrix. Note that the maximum probability density is situated around a diagonal line representing deterministic dynamic properties of the system.

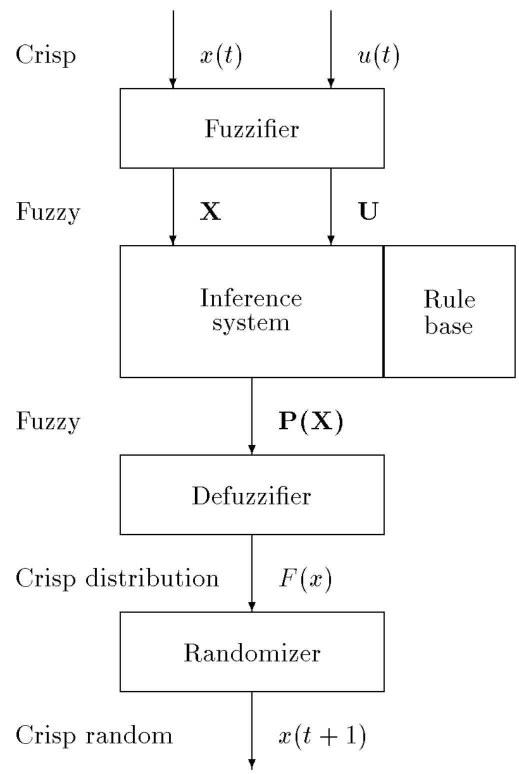

Figure 3: Fuzzy Markov system.

A block diagram of fuzzy Markov simulation is shown in Fig.3. The rule base contains information about the multidimensional transition probability matrix P.

Firstly, the control input $u(t)$ and output $x(t)$ are fuzzified using membership functions. The fuzzy values U and X with their membership degrees are used to infer the rules from the rule base. Having been extracted, the fuzzy rules are aggregated to the fuzzy probability density function $P(X)$ of the output for the time instant t+1.

After defuzzification, integration and normalization, the distribution function $F(x)$ is obtained. The normalization makes the function $F(x)$ be within the interval [0, 1].

$$F(x) = prob\{x(t+1) \le x\} = \frac{\int_{-\infty}^{x} p(x)}{\int_{-\infty}^{+\infty} p(x)}, \qquad (10)$$

where $p(x)$ is the result of defuzzification. Finally, a random value with the desired distribution $F(x)$ is generated using the transformation method from a uniformly distributed random number. The introduction of the modified defuzzification and randomization stages allows fuzzy simulation of a stochastic dynamic system.

4 EXAMPLE

The proposed fuzzy Markov modeling technique was applied in industrial system testing facilities [5]. The results of experiments with testing equipment enabled further investigation of the simulation technique.

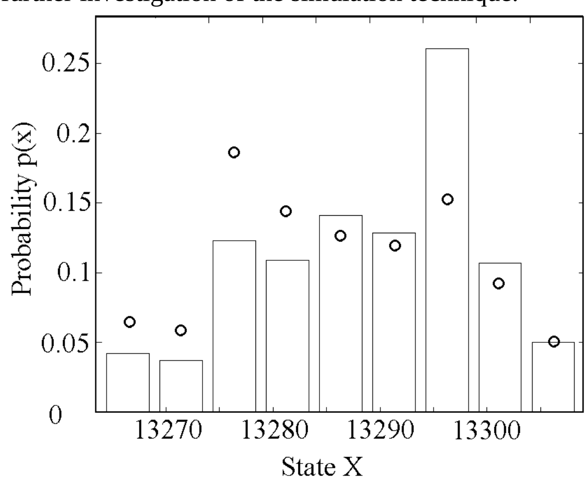

Figure 4: Simulated (solid) and demanded distributions (circles).

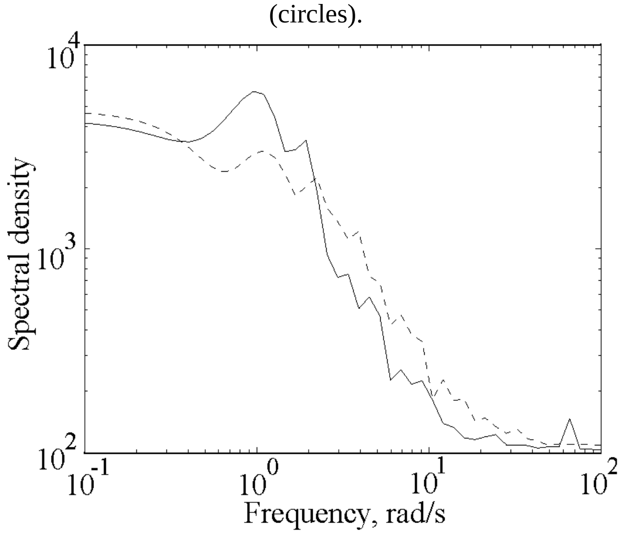

Figure 5: Simulated (solid) and demanded spectra (dashed).

Comparison of basic descriptive properties demonstrates viability of the proposed technique, as shown in Fig.4 and Fig.5. Slight differences between demanded and simulated descriptive properties exist because of variation in sample estimates. With unlimited observation time, the estimates converge to demanded characteristics.

The rule base contains all information about distribution and spectra in the transition probability matrix. This simple description becomes possible because the matrix simultaneously determines two types of system properties. It directly defines probabilities of transitions between states and indirectly determines final probabilities of states of a Markov chain. Conventional methods for stochastic modeling would require more complicated techniques to reflect both distribution and spectral properties.

5 CONCLUSIONS

A fuzzy Markov modeling methodology has been proposed to extend the application area of fuzzy systems. It easily describes any form of probability distribution. In addition to conventional stages of fuzzy modeling, Markov modeling also includes randomization. This transforms defuzzified “crisp” probability distribution into a signal. Application areas for fuzzy Markov modeling include simulation of complex stochastic systems, stationarity and stability analysis, systems identification and optimal control.

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