ISBN: 978-3-95450-210-3 doi:10.18429/JACoW-FEL2019-TUP072

ORBITAL ANGULAR MOMENTUM FROM SASE

J. Morgan[1,2], B.W.J. M[c]Neil[1,2], B.D. Muratori[2,3], P. Williams[2,3], A. Wolski[2,4]
[1]SUPA, Department of Physics, University of Strathclyde, Glasgow
[2]Cockcroft Institute, Warrington, UK
[3]ASTeC, STFC Daresbury Laboratory, Warrington, UK
[4]University of Liverpool, Liverpool, UK

Abstract

To reach very short wavelengths and high intensities of light, free-electron lasers, FELs, are used which produce radiation from amplified noise in an electron beam. In this SASE regime, mode competition dictates that the dominant transverse mode of the radiation will be Gaussian. A method is proposed to suppress the Gaussian mode via phase shifts which allows higher order Laguerre-Gaussian modes to be amplified. These modes are of interest as they carry orbital angular momentum, OAM. Techniques for generating OAM radiation with a FEL have been proposed previously, however, this is the first look at altering mode competition in order to get a dominant OAM mode starting from the initial shot noise in the electron beam.

INTRODUCTION

Recently, much attention has been paid to light which carries OAM. This light has helical phase-fronts characterized by $e^{il\phi}$, where ϕ is the azimuthal coordinate and l is an integer number named the topical charge. The magnitude of l gives the number of intertwined helices in the phase front and the sign of l gives the handedness of these helices. Conventional methods for generating OAM light require downstream optics which convert the radiation from a standard laser [1]. This has its limitations. The optical elements' damage threshold limits the brightness and wavelength of light which is transmitted and constraints arise from the lasers themselves. In contrast, in the FEL, the phase structure of light can be controlled through the manipulation of the electrons themselves and offers the benefit of having a wide range of wavelengths accessible.

Previous work has shown that OAM can be produced in a FEL through a variety of methods. Recently, OAM radiation has been produced at FERMI through harmonic lasing schemes involving helical undulators as well as using a spiral zone plate to convert the radiation downstream from normal FEL output [2]. Another method from Hemsing and colleges creates OAM radiation by first bunching electrons into a helix through second harmonic interaction with a helical undulator. [3].

The current methods for producing OAM radiation in a FEL have their limitations. The intensity of light from harmonic lasing schemes is less than that at the fundamental frequency [2]. Other methods rely on seeding the FEL either with an OAM seed laser for amplification or with a prebunched electron beam. When a FEL is seeded in this way, the output is restricted by the quality of seeds available. This causes difficulty at very short wavelengths as a seed may not be available at the required wavelength and the intensity of the seed must be large to overcome the initial shot noise in the beam. It would be useful, instead, for the initial seed for amplification to come from the shot noise in the electron beam itself. This work looks at the feasibility of just this, generating OAM through suppression of the Gaussian mode.

THEORY

Electrons enter the undulator with random phases due to shot noise in the electron gun. In the self-amplified-spontaneous emission (SASE) mode of operation, the initial amplitude due to the noise acts as a seed for the FEL interaction and is amplified. The incoherence of the electrons is mimicked by the radiation they produce. This radiation can be described by a superposition of the orthogonal Laguerre-Gaussian beams, $LG_{pl}(\phi, \hat{r})$,

$$E(\phi, \hat{r}) = \sum_{l=-\infty}^{\infty} \sum_{p=0}^{\infty} a_{pl} LG_{pl}(\phi, \hat{r}), \tag{1}$$

where a_{pl} is the initial mode amplitude. The Laguerre-Gaussian, LG, modes are chosen as they provide a convenient mode basis and are often used in the study of OAM beams. These modes are written in terms of their OAM index l and the radial mode index p. The fundamental Gaussian mode is found when $p = l = 0$.

All of the modes will have a contribution from the initial electron density. However, due to their transverse profile, the higher order modes have longer gain lengths [4] with the shortest gain length belonging to the Gaussian mode. The Gaussian mode, therefore, dominates FEL interaction, suppressing the higher order modes, leading to the Gaussian mode of operation typical of a FEL. Here we demonstrate that suppressing the Gaussian mode will lead to the amplification of the higher order modes.

Suppression of the Gaussian Mode

It is possible to disrupt the interaction between electrons and radiation through a relative phase shift. A longitudinal delay of the electrons which shifts the electron phase relative to the fundamental wavelength by $\Delta\theta$ will shift the electrons relative to the nth harmonic by $n\Delta\theta$. It has been demonstrated that the exponential gain of the fundamental wavelength can be suppressed when $\Delta\theta$ is a non-integer multiple of 2π and can increase the power in higher harmonics [5]. Instead of considering the higher harmonics of

ISBN: 978-3-95450-210-3 doi:10.18429/JACoW-FEL2019-TUP072

the radiation, this work looks at the higher order transverse modes at the fundamental wavelength.

Examination of the transverse phase profile of the Laguerre-Gaussian modes indicates that a rotational shift, $\Delta\phi_r$, of the electron beam about the longitudinal axis results in a relative phase shift between the electrons and the transverse modes of $l\Delta\phi_r$. The total phase change, $\Delta\Psi_l$, between the electrons and the different l modes from the combination of longitudinal and rotational shifts is, therefore,

$$\Delta\Psi_l = \Delta\theta + l\Delta\phi_r \tag{2}$$

Eq. 2 describes how, through careful selection of $\Delta\theta$ and $\Delta\phi_r$, different relative phase changes between the electrons and the OAM modes can be achieved. If successive repetition of the shifts causes the exponential gain of the Gaussian mode to be disrupted - such that the gain length of the Gaussian mode is longer than of the higher order modes - then a dominant OAM mode will self-select for amplification.

RESULTS

Initial Results

The FEL is modeled using the FEL simulation code Puffin [6]. Presented first, is the result of rotating the electrons along the longitudinal axis according to the rotation matrix,

$$R(\phi_r) = \begin{bmatrix} \cos\phi_r & 0 & -\sin\phi_r & 0 \\ 0 & \cos\phi_r & 0 & -\sin\phi_r \\ \sin\phi_r & 0 & \cos\phi_r & 0 \\ 0 & \sin\phi_r & 0 & \cos\phi_r \end{bmatrix} \tag{3}$$

which acts on the phase space vector constructed from the vaiables (x, p_x, y, p_y), where x and y are the transverse coordinates, and p_x and p_y are the conjugate of momenta. The rotation is applied - along with a longitudinal shift - between undulator modules each around a gain length long. In order to alter mode competition to select for a LG_{01} mode, the shift pairs are chosen so that $\Delta\Psi_1 = 2\pi$. The set-up utilizes three alternating pairs of shifts, the longitudinal shifts, $\Delta\theta = \pi/2, \pi$ and $3\pi/2$ and the corresponding rotational shifts $\Delta\phi_r = 3\pi/2, \pi$ and $\pi/2$ respectively. These sections are repeated until the end of the undulator lattice.

The results of this set-up are displayed in Fig.1. Decomposition of the power into the different Laguerre-Gaussian modes demonstrates that suppressing the competing transverse modes means the LG_{01} mode dominates the interaction. This causes the LG_{01} mode to grow over an order of magnitude above the Gaussian mode. Also included in the figure is the bunching factor of the different helical modes calculated using [7],

$$b_l = \langle \exp(i\theta_j - il\phi) \rangle, \tag{4}$$

where the brackets indicate the ensemble average over the whole electron beam. The b_1 factor has exponential growth as the electrons propagate through the undulator while the bunching factors for the competing modes b_0 and b_{-1} grow

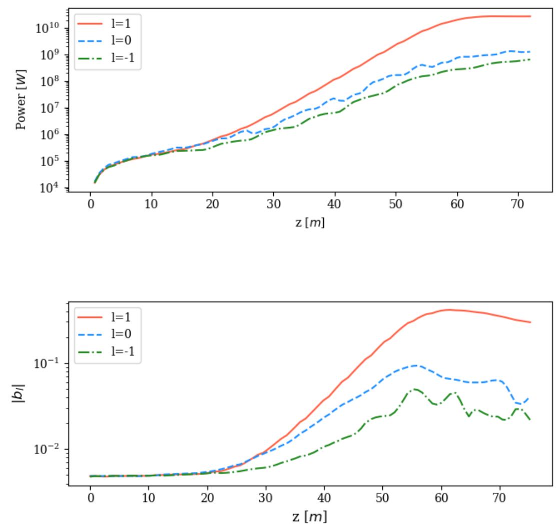

Figure 1: Time-averaged power decomposition of the fundamental frequency into Laguerre-Gaussian modes (top) and mean helical microbunching factor (bottom) when the phase shifts $\Delta\theta = \pi$, $\Delta\phi_r = \pi$; $\Delta\theta = 3\pi/2$, $\Delta\phi_r = \pi/2$; and $\Delta\theta = \pi/2$, $\Delta\phi_r = 3\pi/2$ are applied between undulator modules. This results in the most power being contained in the LG_{01} mode.

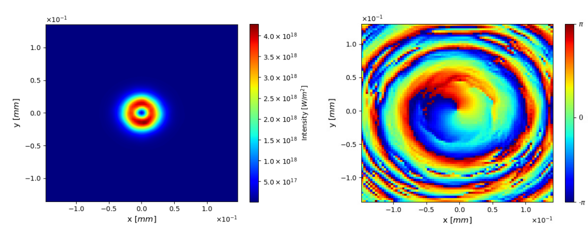

Figure 2: Intensity (left) and phase (right) at $z = 55.28\ m$ when the phase shifts $\Delta\theta = \pi$, $\Delta\phi_r = \pi$; $\Delta\theta = 3\pi/2$, $\Delta\phi_r = \pi/2$; and $\Delta\theta = \pi/2$, $\Delta\phi_r = 3\pi/2$ are applied between undulator modules.

at a slower rate. This set has not been optimized and further disruption of the Gaussian mode may be possible. Further evidence of the OAM mode is provided in Fig. 2 which shows a snapshot of the phase and intensity of radiation near saturation. The phase of the radiation has a transverse profile typical to the LG_{01} mode and the intensity is the expected doughnut structure of OAM modes.

How to Rotate an Electron Beam

The results presented above use a point transform to rotate the electron beam. Investigated here is a physical method to achieve such a transform. A beamline for rotating a beam through an arbitrary angle around the longitudinal axis can be constructed from a set of quadrupoles with appropriate tilt angles around that axis. The design that we present here is based on that of Talman [8], who used a similar system

ISBN: 978-3-95450-210-3 doi:10.18429/JACoW-FEL2019-TUP072

for achieving a "Möbius" transformation in a storage ring. The rotation matrix described by (3) can be created from,

$$\bar{M}\left(\frac{\phi_r}{2}+\frac{\pi}{4}\right)\bar{M}\left(-\frac{\pi}{4}\right) = R(\phi_r) \quad (5)$$

The matrices $\bar{M}$ are defined as,

$$\bar{M}(\theta) = R(\theta) M R^{-1}(\theta) \quad (6)$$

where:

$$M = \begin{bmatrix} \cos\mu & \beta\sin\mu & 0 & 0 \\ -\frac{1}{\beta}\sin\mu & \cos\mu & 0 & 0 \\ 0 & 0 & -\cos\mu & -\beta\sin\mu \\ 0 & 0 & \frac{1}{\beta}\sin\mu & -\cos\mu \end{bmatrix} \quad (7)$$

represents a phase advance through angles μ and $\mu + \pi$ in the transverse and horizontal spaces, respectively. The required transformation M can be achieved using a set of 5 quadrupoles arranged symetrically:

$$M = Q_1 D_A Q_2 D_B Q_3 D_B Q_2 D_A Q_1 \quad (8)$$

where Q_n is the transfer matrix for a quadrupole of focusing strength $k_1 L_n$ and $D_{A(B)}$ is the transfer matrix for a drift of length $L_{A(B)}$. A transformation $\bar{M}(\theta)$ can be constructed using the same set of quadrupole magnets, but with each set of quadrupoles tilted by an angle, θ, around the longitudinal axis.

Using the thin-lens approximation for the quadrupole magnets, the matrix M in (8) can be expressed in terms of the quadrupole strengths and drift lengths. Equation (7) then provides a set of constraints from which the quadrupole strengths and drift lengths can be found for a given μ and β. Not all values of μ and β admit physical solutions. However, a solution can be found for $\mu = \pi/2$, in which case the required focusing strengths are,

$$k_1 L_1 = \frac{L_B \xi}{L_A^2 - L_B^2} \qquad k_1 L_2 = -k_1 L_3 = \frac{\xi}{L_B} \quad (9)$$

and the drift length L_B is given by:

$$L_B = \frac{2}{3}\left(\eta + \frac{1}{2} + \frac{1}{\eta}\right) L_A \quad (10)$$

The quantities ξ and η are defined as:

$$\xi^2 = 1 + \frac{L_B}{L_A} \quad \eta^3 = \frac{27}{16}\frac{\beta^2}{L_A^2}\left(1 + \sqrt{1 - \frac{32}{27}\frac{L_A^2}{\beta^2}}\right) - 1 \quad (11)$$

A system to rotate a beam with the transfer matrix (3), can be constructed from two sets of five quadrupoles, with each set having the same drift length and quadrupole strength. In the first set, the quadrupoles are tilted by an angle $\phi_r/2+\pi/4$ around the longitudinal axis, where ϕ_r is the desired rotation angle in the beam; in the second set, the quadrupoles are tilted by an angle $-\pi/4$. To change the beam rotation angle requires changing the tilt angle of the first 5 quadrupoles: this may be done either mechanically or by constructing each magnet so as to resemble octupole magnets but with the current in the coils arranged to allow an arbitrary superposition of normal and skew quadrupole fields. A rotation of the field is then achieved by changing the ratio of normal to skew quadrupole field strengths.

An example of a rotation system has $L_A = 0.35\ m$, $\mu = \pi/2$ and $\beta = 1\ m$, the overall rotational beamline length is approximately 5 m and the maximum quadrupole strength is 2.53 m^{-1}. In practice, the length of the beamline is likely to increase when physical lengths are used for quadrupoles. Since there is no drift on either side of the set of five quadrupoles, the adjacent quadrupoles in the first and second sets may be combined into a single quadrupole.

DISCUSSION

Initial trials of the rotation system have been unsuccessful. The first issue concerns the total length of the rotation beamline. If the radiation diffracts too much between undulator modules, the interaction between the electrons and the radiation field is diminished and the Gaussian mode is not suppressed. This may not be a significant concern when the radiation wavelength is short and diffraction is low. There are also practical concerns due to the added length of the FEL, as the undulator line more than doubles in length due to the added shifts.

The second issue arising comes from the change in the longitudinal phase (z) position for different electrons. Variations in the transverse components of momentum change the z component of momentum p_z. Since the electrons will have different transverse momentum, depending on their distance from the beam radius, this causes a different longitudinal momentum variation for different electrons and leads to a debunching of the electron beam. Further work will examine if this debunching effect can be reduced.

CONCLUSION

The feasibility of generating light with OAM in a FEL from amplified shot noise in an electron beam is investigated. Trials in which a rotation of the electron beam is used to manipulate the relative phases between the electrons and the different OAM modes showed that suppressing a Gaussian mode will allow growth in the higher order $|l| = 1$ modes. However, although physical realisation of the transform matrix (3) has been demonstrated, the resulting transverse momentum changes debunched the electron beam. Further work is needed to design a system which could be implemented.

ACKNOWLEDGEMENTS

We gratefully acknowledge the support of STFC's ASTeC department for HPC access. Using the STFC Hartree Centre; The Science and Technology Facilities Council Agreement Number 4163192 Release #3; and the John von Neumann Institutefor Computing (NIC) on JUROPA at Jülich Supercomputing Centre (JSC), under project HHH20.

ISBN: 978-3-95450-210-3 doi:10.18429/JACoW-FEL2019-TUP072

REFERENCES

[1] A. M. Yao and M. J. Padgett, "Orbital angular momentum: origins, behavior and applications," *Adv. Opt. Photon.*, vol. 3, no. 2, pp. 161-204, 2011. doi:10.1364/AOP.3.000161

[2] P. R. Ribič *et al.*, "Extreme-ultraviolet vortices from a free-electron laser," *Phys. Rev. X.*, vol. 7, no. 3, p. 031036, 2017. doi:10.1103/PhysRevX.7.031036

[3] E. Hemsing *et al.*, "Helical electron-beam microbunching by harmonic coupling in a helical un-dulator," *Phys. Rev. Lett.*, vol. 102, no. 17, p. 174801, 2009. doi:10.1103/PhysRevLett.102.174801

[4] E. Hemsing, A. Gover, and J. Rosenzweig, "Virtual dielectric waveguide mode description of a high-gain free-electron laser.ii. modeling and numerical simulations," *Phys. Rev. A.*, vol. 77, no. 6, p. 063831, 2008. doi:10.1103/PhysRevA.77.063831

[5] B. W. J. McNeil, M. W. Poole, and G. R. M. Robb, "Inducing Strong Density Modulation with Small Energy Dispersion in Particle Beams and the Harmonic Amplifier Free Electron Laser", in *Proc. PAC'05*, Knoxville, TN, USA, May 2005, paper RPPT021, pp. 1718–1720.

[6] L. Campbell and B. McNeil, "Puffin: A three dimensional,unaveraged free electron laser simulation code," *Phys. Plasmas*, vol. 19, no. 9, p. 093119, 2012. doi:10.1063/1.4752743

[7] E. Hemsing, A. Marinelli, and J. Rosenzweig, "Generatingoptical orbital angular momentum in a high-gain free-electronlaser at the first harmonic," *Phys. Rev. Lett.*, vol. 106, no. 16, p. 164803, 2011. doi:10.1103/PhysRevLett.106.164803

[8] R. Talman, "A proposed möbius accelerator," *Phys. Rev. Lett.*, vol. 74, no. 9, p. 1590, 1995. doi:10.1103/PhysRevLett.74.1590