2D WAKE FIELD CALCULATIONS OF TAPERED STRUCTURES WITH DIFFERENT FDTD DISCRETIZATION SCHEMES

C. Schmidt, H.W. Glock, U. van Rienen
Rostock University, Germany

Abstract

Recent particle accelerator designs demand for advanced prediction of parasitic effects like wake fields, even in structures of comparatively weak influence like tapers. In the case of smoothly tapered components even well established codes like MAFIA [1] demonstrate strong discretization dependency of the results or solver instabilities, making alternative approaches necessary for such applications. Grid dispersion is assumed to cause these problems. In Ref. [2] an alternative discretization scheme is described, using a homogeneous rotated mesh intended to eliminate such grid dispersion effects. In order to study the dependence on the discretization applied, we use this scheme to calculate wake fields in prototype taper structures of rotational symmetry. Furthermore a comparison is provided with results of a non-rotated mesh, MAFIA runs and - so far applicable - analytical approaches.

INTRODUCTION

In particle accelerators, tapered structures are used as a smooth transition between different beampipe-radii to reduce wake-field effects. For high performance accelerators even those weak effects are not negligible. Unfortunately the simulation of scattered fields with computer codes like MAFIA© [1] yields strong discretization dependency, which is shown in Figure 5. The directional dependence of wave phase velocity referring to the grid orientation, commonly denoted as grid dispersion, is assumed to cause this failures. To overcome this drawback we use an alternative discretization scheme introduced in [2]. This scheme is based on the well known grid dispersion minimum for wave propagation along the grid diagonals in 3D-calculations [3]. The restriction on cylindrical symmetric problems allows a treatment in 2D. Therefore two discretization schemes, the ordinary (r, z) and the new $45°$rotated one are implemented using Mathematica® [4]. To test the predicted lower dispersion error, we performed a time integration of a resonant TM_0 mode in an entirely closed cylindrical resonator. The frequency deviation of the simulated oscillation to the analytically calculated eigenfrequency is a direct measure of the phase velocity deviation. Finally wake-field calculations of tapered structures with both schemes were performed. Therefore the recursion algorithm was extended by an expression for the exciting fields of the bunch. To observe longitudinal wake field effects we restricted the calculation to an ultrarelativistic bunch movement on the structures axis.

IMPLEMENTATION OF THE ROTATED MESH

To perform a calculation on a rotated mesh we transform the (r, z)-coordinate system into a new (u, v)-system rotated by $45°$. In Figure 1, the allocation of the field components is shown. The length of a mesh cell, given by the step size h, is equidistant for u- and v-direction. For a good boundary approximation we allow for half filled mesh cells at the non-tapered surface.

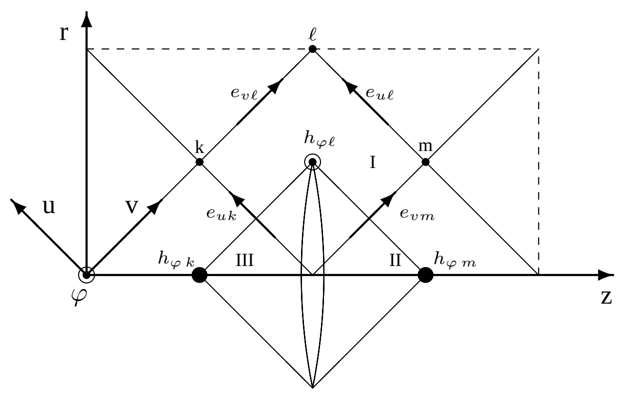

Figure 1: Field allocation and integration surfaces

Starting from the well known integral form of Maxwell's equations, we find the discretized form at a single mesh point for the three field components, performing the integration on the quadratic surface I (Eq.(1)) and the two conical surfaces II and III that lead to Eqs. (2) and (3), resp.:

$$\partial_t\, h_l = -\frac{1}{\mu}\frac{1}{h}\left(e_{vm} + e_{u\ell} - (e_{v\ell} + e_{uk})\right) \qquad (1)$$

$$\partial_t\, e_{uk} = \frac{1}{\varepsilon}\frac{1}{h}\frac{1}{\left(r_k - \frac{h}{2\sqrt{2}}\right)}\left[h_k\left(r_k - \frac{h}{\sqrt{2}}\right) - h_\ell\, r_k\right] \qquad (2)$$

$$\partial_t\, e_{vm} = \frac{1}{\varepsilon}\frac{1}{h}\frac{1}{\left(r_m - \frac{h}{2\sqrt{2}}\right)}\left[h_\ell\, r_m - h_m\left(r_m - \frac{h}{\sqrt{2}}\right)\right] \qquad (3)$$

Permittivity ε and permeability μ are defined as either ∞ at the boundaries or ε_0, μ_0 (i.e.vacuum) inside the computational domain. Time discretization is performed explicitly with the so called leap-frog algorithm [5]. Next these recursions are applied for all grid points and written in matrix form with the system-matrix $\mathbf{A}$ (for details refer to [6]). Thus the whole recursion reduces to a matrix-vector multiplication in the form of (4), where n describes the time step index.

$$\begin{pmatrix} h_\varphi^{n+1} \\ e_u^{n+0,5} \\ e_v^{n+0,5} \end{pmatrix} = \mathbf{A} \cdot \begin{pmatrix} h_\varphi^{n} \\ e_u^{n-0,5} \\ e_v^{n-0,5} \end{pmatrix} \tag{4}$$

Wake field excitation

For the calculation of wake fields, we need to introduce the fields (index e) excited by the bunch [7].

$$e_u^e(r) = e_v^e(r) = \frac{1}{\sqrt{2}} \frac{-q_i}{4\pi\,\varepsilon_0} \frac{1}{r^2}\,\gamma, \tag{5}$$

$$h_\varphi^e(r) = \frac{-q_i}{4\pi\,\mu_0} \frac{Z_0}{r^2}\,\gamma, \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{6}$$

γ denotes the relativistic factor and q_i the charge of the bunch. The excitation only takes place at the material boundaries, which leads to (comp. [6]):

$$\tilde{h}_\varphi^e|^n = \left(1 - \frac{\mu_0}{\mu}\right)\left(h_\varphi^e|^{n+1} - h_\varphi^e|^n\right) \tag{7}$$

$$\tilde{e}_u^e|^{n-0,5} = \left(1 - \frac{\varepsilon_0}{\varepsilon}\right)\left(e_u^e|^{n+0.5} - e_u^e|^{n-0.5}\right) \tag{8}$$

$$\tilde{e}_v^e|^{n-0,5} = \left(1 - \frac{\varepsilon_0}{\varepsilon}\right)\left(e_v^e|^{n+0.5} - e_v^e|^{n-0.5}\right) \tag{9}$$

Electrical bunch fields being tangential to ideal conducting surfaces require the appearance of opposing scattered (index s) fields in order to fulfil the Dirichlet boundary conditions.

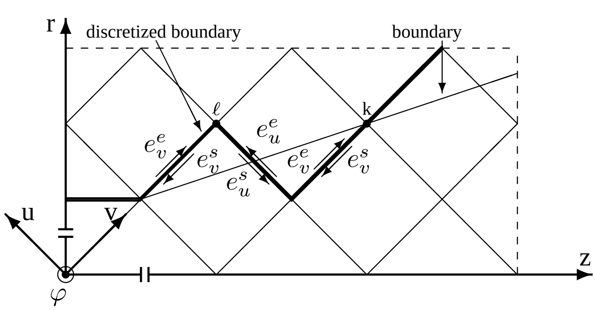

Figure 2: E-field boundary treatment and approximation

Referring to Figure 2, no scattered field will be generated in mesh cell ℓ because of $e_u^e = e_v^e$. But in k the absence of the electric field u-component (no boundary) causes an excitation of a scattered field.
To implement this inhomogeneity in the existing recursion formula we first merge the field components in a vector and add it to (4).

$$\begin{pmatrix} h_\varphi^s|^{n+1} \\ e_u^s|^{n+0,5} \\ e_v^s|^{n+0,5} \end{pmatrix} = \mathbf{A} \cdot \begin{pmatrix} h_\varphi^s|^{n} \\ e_u^s|^{n-0,5} \\ e_v^s|^{n-0,5} \end{pmatrix} - \begin{pmatrix} \tilde{h}_\varphi^e|^{n} \\ \tilde{e}_u^e|^{n-0,5} \\ \tilde{e}_v^e|^{n-0,5} \end{pmatrix} \tag{10}$$

RESULTS

Validation

To compare the different discretization schemes, we use a cylindrical resonator of the length 0.5m and a radius of 0.1m. As a starting point for the time integration, the field components of the resonant fundamental mode are calculated analytically. We observe the magnetic field component at a fixed random grid point and compare the time differences of zero crossings between our calculation, MAFIA and the analytical result after roughly 10000ns (115 oscillations).

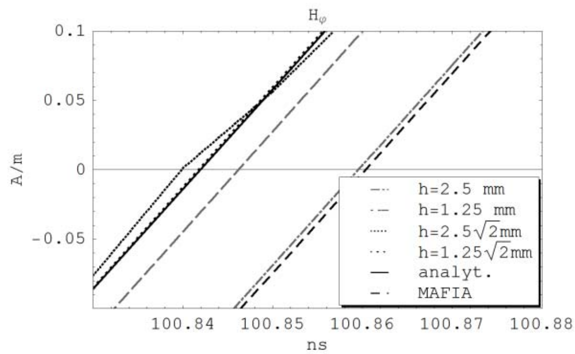

Figure 3: Comparison of zero crossings (h-values having a $\sqrt{2}$ factor indicate a rotated mesh.)

Figure 3 demonstrates that our result on a non-rotated mesh is very close to the MAFIA result (Δt = 1ps, h = 2.5mm). Furthermore, the results of the rotated mesh are much closer to the analytic one, even for a coarser discretization. The interrelation between time deviation and discretization is shown in Table 1 for both schemes.

Table 1: Convergence behaviour for (r, z)-grid (left column) and (u,v)-grid (right column)

h/mm	$\Delta t_{(h)}/\Delta t_{(h/2)}$	h/mm	$\Delta t_{(h)}/\Delta t_{(h/2)}$
10	3.985	$10\sqrt{2}$	2.166
5	4.002	$5\sqrt{2}$	4.554
2.5	4.024	$2.5\sqrt{2}$	11.09
1.25	4.113	$1.25\sqrt{2}$	3.083
0.625		$0.625\sqrt{2}$	

Evaluating the expression [3, Eq.2.70a] for the numerical phase velocity in the Yee algorithm in the case of small h and with θ being the wave propagation angle in the grid leads to $\Delta t \propto h^2(3 + \cos(4\theta))$. This h^2-dependency is resembled very well by the (r,z)-grid as Table 1 illustrates, whereas the (u,v)-grid shows a more complicated behaviour.

Wake field calculations

We compare three codes (MAFIA, the non-rotated scheme and the scheme with the rotated mesh) regarding their convergence behavior in the simulation of tapered structures. Therefore, we double stepwise the number of mesh lines. Figures (5), (6), (7) show the longitudinal wake potential for the three codes.

We assume a Gaussian bunch with a FWHM of $\sigma = 1$cm and perform a time step $\Delta t = h/(10\, c_0)$ where c_0 represents the speed of light in vacuum. Using ideal electric conductors for all boundaries demands increasing of the waveguide lengths in order to suppress reflection errors from the upper and lower z-boundaries.

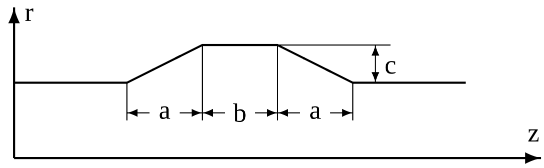

Figure 4: 2D cross section of a tapered structure

Figure 4 shows a sketch of the simulated tapers. In the following, we present for the longitudinal wake potential computed by the three different codes. It is plotted on the structure's axis, with s as the distance behind the head of the bunch.

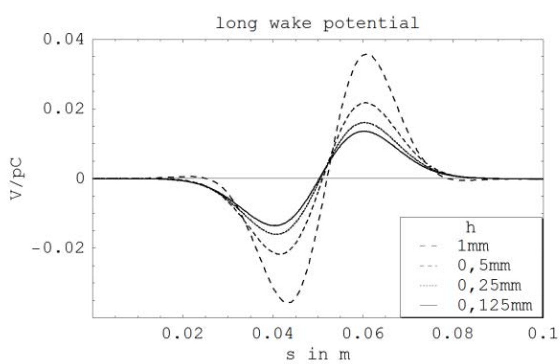

Figure 5: MAFIA results for a = 100 mm, b = 500 mm, c = 4 mm

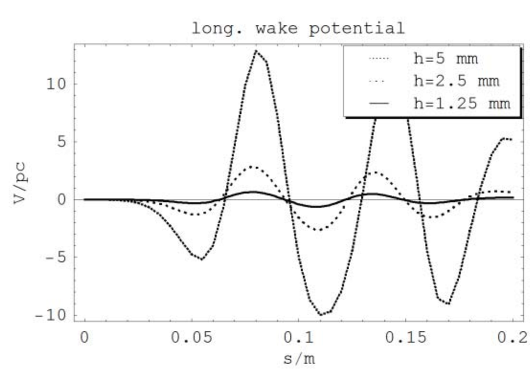

Figure 6: Non-rotated mesh with a = 100 mm, b = 100 mm, c = 50 mm

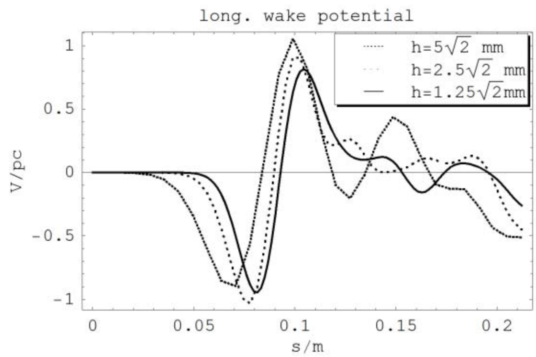

Figure 7: Rotated mesh with a = 100 mm, b = 100 mm, c = 50 mm

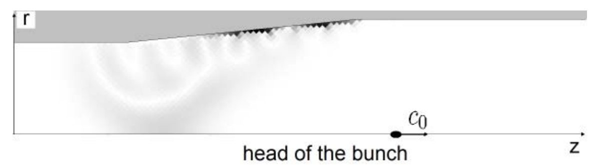

Figure 8: Local snapshot of the scattered h-field in a tapered structure, calculated with the rotated mesh

CONCLUSIONS

The numerical calculation of wake potentials in tapered structures shows a strong dependence on discretization, even for well established codes. By implementing both a conventional 2D discretization and - following a proposal from literature - a mesh rotated by $45°$ reduced phase velocity deviation of the latter one was demonstrated using a resonator oscillation as well defined test system. Nevertheless only the discretization dependence of the conventional grid follows theoretical considerations. Wake potential calculations yield significantly different results, those of the rotated mesh being less depending on mesh size.

REFERENCES

[1] MAFIA V 4.107: CST GmbH, Bad Neuheimer Str. 19, D-64289 Darmstadt, Germany

[2] R.Hampel, I.Zagorodnov, T.Weiland. New discretization scheme for wake field computation in cylindrically symmetric structure. Proceedings of EPAC, 2004, pp 2559

[3] A.Taflove, The Finite-Difference Time-Domain Method, Artech House, 1998

[4] Mathematica®5.0: Wolfram Research, Champaign, IL, USA

[5] K.S. Yee, Numerical Solution of Initial Boundary Value Problems Involving Maxwells Equations in isotropic Media, IEEE-AP, 1966, p.302-307

[6] C. Schmidt, diploma thesis, Inst. f. Allgemeine Elektrotechnik, Universitaet Rostock, 2006

[7] G.L.Palumbo, V.G.Vaccaro, M.Zobov, Wake Fields and Impedance, CAS Fifth Advanced Accelerator Physics Course, 1995