# PBO LAB TOOLS FOR COMPARING MARYLIE LIE MAPS AND TRANSPORT TRANSFER MATRICES 

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#### Abstract

Particle optics codes frequently utilize either a Lie algebraic formulation or a matrix formulation of the equations of motion. Examples of codes utilizing the Lie algebra approach include MARYLIE and MAD, whereas TRANSPORT and TURTLE use the matrix formulation. Both types of codes have common application to many particle optics problems. However, it is often a very tedious exercise to compare results from the two types of codes in any great detail. As described in a companion paper in these proceedings, differences in the choice of phase space variables, as well as the inherent differences between the Lie algebraic and matrix formulations, make for unwieldy and complex relations between results from the two types of codes. Computational capability has been added to the PBO Lab software that automates the calculation of TRANSPORT transfer matrices from MARLYIE Lie maps through third order. Graphical tools have been added so that quick and easy visual comparisons of transfer maps and matrices can be made. Quantitative comparison tools have also been developed to calculate the differences between transfer matrices computed from TRANSPORT directly and those computed from the MARYLIE maps. The tools have been applied to a representative set of magnetic optics elements and a quadrupole example is illustrated here.


## INTRODUCTION

The Particle Beam Optics Laboratory (PBO Lab ${ }^{\text {TM }}$ ) is a set of software application modules developed to support beamline design [1], personnel training [2], and accelerator operations [3]. PBO Lab provides a common graphic user interface (GUI) for constructing and editing iconic object-based computer models of accelerators and beamlines. The broad range of applications that may be addressed, together with the ease-of-use provided by the intuitive GUI, has made PBO Lab a popular package and it is now used by more than ninety laboratories in over twenty countries [4].

PBO Lab has a number of optics code modules, each of which incorporates a specific particle optics modeling or simulation program. In particular, the MARYLIE [5] and TRANSPORT [6] programs have been available as PBO Lab Modules for sometime. The common graphic interface greatly simplifies the task of carrying out calculations for a given beamline using these and other codes working within the PBO Lab framework. PBO Lab
assures that the input files for the optics codes represent the same beamline: each input file is derived from the same beamline object model [7]. However, comparing detailed output results from MARYLIE and TRANSPORT, such as the $2^{\text {nd }}$ and $3^{\text {rd }}$ order transfer maps, has still involved considerable work. To assist with such comparisons, several new tools have been added to PBO Lab aimed at providing both qualitative and quantitative capabilities for comparing transfer maps computed by different optics codes. In this paper, we apply these tools to the MARYLIE and TRANSPORT computation of transfer maps for a representative magnetic quadrupole.

## THE MAP AND MATRIX TOOLS

The new tools developed to support the comparison of MARYLIE maps and TRANSPORT matrices fall into three basic categories. (1.) Algorithms have been written to compute transfer matrices, in terms of the phase space variables used by TRANSPORT, from the coefficients of the Lie map polynomials of the phase space variables used by MARYLIE. (2.) Graphical displays have been implemented that plot the "spectra" of the $2^{\text {nd }}$ and $3^{\text {rd }}$ order TRANSPORT matrices. (3.) A transfer matrix difference tool has been developed that creates a new transfer matrix, through $3^{\text {rd }}$ order, that is the element by element difference between two transfer matrices. Each of these tools has been implemented with easy to use GUI features that characterize the PBO Lab software.

## TRANSPORT matrices from MARYLIE maps

The basic theoretical framework for computing TRANSPORT transfer matrices from MARLYE Lie maps is described in a companion paper [8]. Two different procedures for implementing that framework have been developed. Different GUI elements are used for the two procedures. These GUI elements are referred to here as the MARYLIE command "Make TRANSPORT Matrix Piece" and the "Map->Matrix" button, following the names used in PBO Lab. The Make TRANSPORT Matrix Piece command utilizes the MARYLIE type code "ptm" to compute and write to a file the $R, T$ and $U$ matrices in MARYLIE coordinates. The file is then read, the formulae described in [8] are applied to convert the matrices to TRANSPORT coordinates, and then the transfer matrices are loaded into a Matrix Piece - an object in PBO Lab that stores the data and has useful methods and tools available for manipulating the data. The Map-> Matrix button appears on a Map Piece -
another PBO Lab object similar to the Matrix Piece, but developed to support specific attributes [9] of Lie maps. [The Matrix and Map Pieces have been parts of the PBO Lab GUI since inception, but we have added to their functionality as part of this work.] Once a map has been computed and loaded into a Map Piece (e.g. using MARYLIE), the Map ->Matrix button can be used to create a Matrix Piece that contains the $R, T$ and $U$ matrices in TRANSPORT coordinates. The Map->Matrix approach does not use MARYLIE - the map polynomial coefficient and coordinate transformations are both done by PBO Lab. Since separate code is used for the two procedures they provide independent checks on the algorithms for computing transfer matrices from maps.

## Graphic displays of matrix and map spectra

Since the initial commercial release of the MARYLIE Module for PBO Lab [10], the Map Piece has included two very useful graphics tools, Display F3 and Display F4. These display tools are activated with buttons (of the same name) and provide histogram type plots of the coefficients of the $f_{3}$ and $f_{4}$ map polynomials. As part of the work described here, similar graphics tools have been added to the Matrix Piece: Display T-Matrix and Display U-Matrix. Figure 1 illustrates a Matrix Piece together with a display of the $2^{\text {nd }}$ order (T-matrix) elements, plotted as a histogram or "spectrum" of the elements. The $\mathrm{T}_{\mathrm{ijk}}$ indexing scheme follows that of TRANSPORT, so that only the 126 independent elements are included. Submatrices of $\mathrm{T}_{\mathrm{ijk}}$ and $\mathrm{U}_{\mathrm{ijk}}$ can be viewed using the radio button controls that appear in the Matrix Piece window.

The matrix data shown in Figure 1 is for a 10 cm long quadrupole with a field gradient of $2.3473 \mathrm{~T} / \mathrm{m}$. The beam particle (reference trajectory) is a 234.568077 MeV proton (mass 938.272310 MeV ), which corresponds to $\beta=0.6$. The quadrupole coefficient $\mathrm{K} 1=1.0$. The results were obtained using the PBO Lab MARYLIE command: Make TRANSPORT Matrix Piece. Note that, in addition to the transfer matrix data, the Matrix Piece created with this command includes data for the length, energy gain, and initial (input) energy for the selected beamline. These parameters are important when the Matrix Piece itself is used for calculations with other PBO Lab Modules (e.g. TRANSPORT or TURTLE).

The data displayed for this example quadrupole show that the (non-trivial) R-matrix elements are of order 0.1 to 1.0 , and the T -matrix elements are all of order 0.1 or smaller. A similar spectrum plot for $3^{\text {rd }}$ order, not illustrated here due to space limitations, shows that the Umatrix elements are also all of order 0.1 or smaller.

The same transfer matrices may be computed directly by TRANSPORT, using the PBO Lab TRANSPORT command: Make a Matrix Piece. The T-matrix spectrum for that calculation is virtually identical to that shown in Fig, 1; there is no discernable difference. The Matrix Tool provides more quantitative comparisons.

| 4) Matrix |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Energy Distance Display T-Matrix Display U-Matrix Matrix Tools |  |  |  |  |  |  |
| - R-Matrix CT-Matrix i: $\begin{aligned} & 1 \\ & \square\end{aligned}$ CU-Matrix $\quad \mathrm{i}: \sqrt{1+} \mathrm{j}: \sqrt{\square}$ |  |  |  |  |  |  |
| R-Matrix Elements $\mathrm{R}(\mathrm{i}, \mathrm{j})$ in meters and $1 /$ meters |  |  |  |  |  |  |
| i: $\mathrm{j}=1$ | 2 | 3 | 4 | 5 |  |  |
| $1 \longdiv { 0 . 9 9 5 0 0 4 }$ | 0.099833 | 0.000000 | 0.000000 | 0.000000 | 00.00 | 0000 |
| $2)-0.099833$ | 0.995004 | 0.000000 | 0.000000 | 0.000000 | 0.00 | 0000 |
| $3 \longdiv { 0 . 0 0 0 0 0 0 }$ | 0.000000 | 1.005000 | 0.100167 | 0.000000 | 0.0 .00 | 0000 |
| $4 \longdiv { 0 . 0 0 0 0 0 0 }$ | 0.000000 | 0.100167 | 1.005000 | 0.000000 | 0.00 | 0000 |
| $5 \longdiv { 0 . 0 0 0 0 0 0 }$ | 0.000000 | 0.000000 | 0.000000 | 1.000000 | 0.06 | 4000 |
| $6 \longdiv { 0 . 0 0 0 0 0 0 }$ | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 01.00 | 0000 |
| Parameters |  | Value | Unit |  | Guidanc | ce Limits |
| S Effective Length |  | 10.000000 | cm | $\checkmark$ | 0.0000 | 423.2364 |
| S Energy Gain |  | 0.000000 | MeV | $\checkmark$ | 0.0000 | $1.00 \mathrm{e}+010$ |
| S Input Energy |  | 234.568077 | MeV | $\checkmark$ | 0.0000 | $1.00 \mathrm{e}+010$ |
| Comments: |  |  |  |  |  |  |
| From MARYLIE - Transport Units |  |  |  |  |  |  |



Figure 1: Matrix Piece window (upper) and spectrum plot generated with Display T-Matrix button (lower).

## Matrix difference tool

The Matrix Tool button on the button bar at the top of the Matrix Piece opens the window shown in Figure 2.


Figure 2: Matrix Tool window used to create a Matrix Piece that contains the differences between Matrix Piece A and a selected Matrix Piece B.

The Matrix Tool window displays the Comment field of the Matrix Piece from which the tool was launched (Matrix A) together with a list of all other Matrix Pieces in the PBO Lab model. The user selects one item from this list (Matrix B) whose elements will be subtracted from the corresponding elements of Matrix A. The Create Difference Matrix button executes this subtraction and then loads the resulting data into a new Matrix Piece. The new Matrix Piece will have a Comment assigned that is the Matrix A Comment - Matrix B Comment, and will itself be a fully functional Matrix Piece with all of the supporting graphics and analysis tools available.

In the example shown in Figure 2, Matrix $A$ corresponds to the Matrix Piece window illustrated in Figure 1. Only one item appears in the list for Matrix B selection, corresponding to the matrix computed by TRANSPORT for the same quadrupole as used for Figure 1. Figure 3 illustrates the T-matrix spectrum that results from the difference Matrix Piece created from these.


Figure 3: T-Matrix spectrum for the difference between the MARYLIE and TRANSPORT matrices.

The data in Figure 3 shows, as expected, that the differences between the MARYLIE and TRANSPORT TMatrix elements are all quite small (note scale difference between Figure 3 and Figure 1), about 1 part in $10^{+6}$.

Similar comments apply to the R-matrix elements, and for all of the U-matrix elements $U_{i \mathrm{ijk}}$ except for $\mathrm{i}=5$. TRANSPORT does not compute 3 rd order path length terms ( $\mathrm{i}=5$ ), so the MARYLIE derived terms for $\mathrm{U}_{5 \mathrm{jkl}}$ appear directly in the difference matrix. The element $\mathrm{U}_{5 \text { III }}$ corresponds to the U-Matrix spectrum term 224. All components of the TRANSPORT U-Matrix spectrum for indices $>223$ will be zero. Another feature of the new matrix graphics tools can be used to focus only on the common elements. Using the Set Scale button on the UMatrix Histogram plot window to limit the indices to the range $1-223$ readily shows that the U-Matrices from MARYLIE and TRANSPORT are equal within this range.

## SUMMARY

Several new PBO Lab tools have been developed to aid in comparing the transfer maps of MARYLIE with the transfer matrices of TRANSPORT. Both graphical and analytical tools have been developed and are summarized. The tools have been applied to a representative set of magnetic optics elements to compare transfer matrices through $3^{\text {rd }}$ order, and one quadrupole example is described here. The new tools should prove valuable in comparing MARYLIE and TRANSPORT results, as well as results from other codes integrated into the PBO Lab framework.

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