# DEVELOPMENT OF CODE FOR SIMULATION OF ACCELERATION OF IONS FROM INTERNAL SOURCE TO END OF EXTRACTION SYSTEM IN CYCLOTRONS AND PRELIMINARY DESIGN STUDY OF 8-MeV CYCLOTRON FOR PRODUCTION OF RADIOISOTOPES 

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## Abstract

From the users' point of view modern cyclotrons must be compact, energy-saving, low-radiation and very reliable facilities. To provide all these characteristics, a very detailed design study of all systems of an accelerator under development is required.

Thus, particle tracking from the "beginning" to the "end" in modern cyclotrons with small gaps in the main acceleration region and with efficient extraction systems becomes a very important task for designers.
Codes for beam dynamics simulation at the center, main acceleration region and through the extraction system of the cyclotron have been developed. It is possible to monitor all main beam parameters at the different stages of acceleration, radial, axial and phase motion of the beam and the energy increase. During tracking particles through the extraction system it is possible to calculate rms envelopes of radial and vertical motion of the beam and beam losses at the aperture of the extraction system elements.
A preliminary design of a compact $8-\mathrm{MeV}$ proton cyclotron was studied using created codes. The accelerator is supposed to have a four sector compact magnet system with the pole 64 cm in diameter.

## DESCRIPTION OF THE CODE

The developed package of codes is intended for analysis of magnetic field maps (obtained after the main parameters of the cyclotron are chosen and the magnet system is created), calculation of particles dynamics in the main acceleration region, and tracking of particles through the extraction system of the cyclotron. These codes were written in the Mathematica 5.1 environment working under the Windows platform.

## Equations

Full differential equations describing the particle motion inside the 6D phase space volume ( $\mathrm{r}, \mathrm{r}^{\prime}, \mathrm{z}, \mathrm{z}^{\prime}, \mathrm{E}$, $\psi$ ), where ( $\mathrm{r}, \mathrm{z}$ ) are the transverse particle coordinates, E is the energy, $\psi$ is the RF phase, are integrated with the azimuth angle as an independent variable by the means of Runge-Cutter method of the 4-th order:

$$
\begin{aligned}
& r^{\prime \prime}-\frac{2 r^{\prime 2}}{r}-r=\frac{q c}{\sqrt{E^{2}-E_{0}^{2}}}\left(1+\frac{r^{2}}{r^{2}}+\frac{z^{\prime}}{r^{2}}\right)^{1 / 2}\left[\left(r^{2}+r^{2}\right) B_{z}-r^{\prime} z^{\prime} B_{r}-r z^{\prime} B_{\varphi}\right]- \\
& -\frac{q E}{E^{2}-E_{0}^{2}}\left(1+\frac{r^{\prime 2}}{r^{2}}+\frac{z^{\prime 2}}{r^{2}}\right)\left(r r^{\prime} \varepsilon_{\varphi}-r^{2} \varepsilon_{r}\right)
\end{aligned}
$$

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$z^{\prime \prime}-\frac{2 r^{\prime} z^{\prime}}{r}=\frac{q c}{\sqrt{E^{2}-E_{0}^{2}}}\left(1+\frac{r^{\prime 2}}{r^{2}}+\frac{z^{\prime 2}}{r^{2}}\right)^{1 / 2}\left[\left(r^{2}+z^{\prime 2}\right) B_{r}-r^{\prime} z^{\prime} B_{z}-r^{\prime} r B_{\varphi}\right]-$
$-\frac{q E}{E^{2}-E_{0}^{2}}\left(1+\frac{r^{\prime 2}}{r^{2}}+\frac{z^{\prime 2}}{r^{2}}\right)\left[\left(r z^{\prime} \varepsilon_{\varphi}-r^{2} \varepsilon_{z}\right)\right.$
$t^{\prime}=\frac{\left(r^{2}+r^{\prime 2}+z^{\prime 2}\right)^{1 / 2}}{c\left(1-E_{0}^{2} / E^{2}\right)^{1 / 2}}$
$E^{\prime}=q\left(r E_{\varphi}+r^{\prime} E_{r}+z^{\prime} E_{z}\right)$
where $\left(B_{z}, B_{r}, B_{\varphi}\right)$, and $\left(\varepsilon_{z}, \varepsilon_{r}, \varepsilon_{\varphi}\right)$ are components of the magnetic and electric fields, $t$ is the time, $q$ is the charge of the particle, $E$ is the total energy, $E_{0}$ is the rest energy of the particle, $c$ is the velocity of light, ' means the differentiation with respect to the azimuth angle.

## Electric and Magnetic Fields

Particle energy gain inside the accelerating gaps is calculated in accordance with the particle phase, amplitude of RF voltage and gap geometry. The electric field is represented [1] as:

$$
E_{y}=\frac{U}{t \sqrt{2 \pi}} \exp \left(-0.5(y / t)^{2}\right) ; E_{z}=\frac{y z}{t^{2}} E_{y}\left\{1+\frac{1}{6}\left(\frac{z}{t}\right)^{2}\left[\left(\frac{y}{t}\right)^{2}-3\right]\right\}
$$

where $E_{y}, E_{z}$ are the horizontal and axial components of the electric field,

$$
t=0.2 H+0.4 W
$$

(see fig. 1), $U$ is the dee voltage.


Figure 1: Geometry of the dee system assumed in the code.
Considering the particle transit time, the resulting expression describing the electric field components is $\varepsilon_{y, z}(r, \varphi, z, t)=E_{y, z} \cos \left[h\left(2 \pi f t(i)+\psi(i)+\phi_{k}\right)\right]$,
where $f$ is the resonant orbital frequency, $h$ is the harmonic number of acceleration, $t$ is time, $\psi$ is the particle phase with respect to RF voltage at the starting position, $\phi_{k}$ is the initial RF phase of acceleration gap number $k$. The magnetic field map in the median plane of
the cyclotron $\mathrm{B}_{\mathrm{z}}(\mathrm{r}, \varphi, 0)$ is used in the code. Components of the magnetic field outside the median plane are calculated using Maxwell's equations $\operatorname{div} \vec{B}=0$, and $\operatorname{rot} \vec{B}=0: B_{r}=z \frac{d B_{z}}{d r}, B_{\varphi}=\frac{z}{r} \frac{d B_{z}}{d \varphi}$, $B_{z}=B_{z}(z=0)-\frac{z^{2}}{2}\left(\frac{d^{2} B_{z}}{d r^{2}}+\frac{1}{r} \frac{d B_{z}}{d r}+\frac{1}{r^{2}} \frac{d^{2} B_{z}}{d \varphi^{2}}\right)$.

## STUDY OF THE 8-MEV PROTON CYCLOTRON

A compact $8-\mathrm{MeV}$ proton cyclotron for production of radioisotopes is under development in the Department of New Accelerators of JINR. After choosing the main parameters (see Table 1) of the machine and creating a model of the magnetic system, codes for dynamics calculation were used for assessing the cyclotron design from point of view of beam dynamics. As in any other cases of the design process, the initial crude model was iteratively refined.

Table 1: Main Parameters of $8-\mathrm{MeV}$ cyclotron

| Number of sectors | 4 |
| :--- | :--- |
| Sector angular width, deg | $15-21$ |
| Hill gap, mm | 20 |
| Valley gap, mm | 70 |
| Average magnetic field, T | $\sim 1.45$ |
| Orbital frequency, MHz | 22.1834 |
| Acceleration harmonic number | 4 |
| Dee voltage, kV | 50 |

## Magnetic field map analysis

Closed orbit analysis [2] was used to define the focusing properties of the magnetic system. Frequencies of free betatron oscillations are seen in Fig.1. There is focusing in both vertical and radial directions in the magnetic field map used.


Figure 1: Frequencies of betatron oscillations.

## Acceleration in the main region

An "equilibrium-accelerated" particle (which has the amplitude of free radial oscillations 1 mm or less) was found by choosing the optimal starting position in the (r, $\mathrm{p}_{\mathrm{r}}$ ) phase plane, initial phase relative to RF voltage, value
of the gap between the ion source and the puller, and the angle between the ion source and the axis of symmetry of the cyclotron. The results of simulation of the acceleration of this particle are presented in Fig. 2-3.


Figure 2: Radial motion of the equilibrium particle.


Figure 3: Phase motion of the equilibrium particle.
One can see from Fig. 3 that the phase of the particle lies in $\pm 10^{\circ}$ interval relative to RF voltage, which means good agreement between the magnetic field used and the isochronous one.

Then a bunch of 1000 particles normally distributed in the $\left(r, p_{r}\right)$ and $\left(z, p_{z}\right)$ phase plane and uniformly distributed in the (w, RF-phase) phase plane was generated around the equilibrium particle and accelerated up to the final radii. The vertical and axial size of the bunch and starting angles of particles are specified by the geometry of the ion source hole (supposed to be $1 \times 10 \mathrm{~mm}$ in size), the RF-phase of the bunch equals $20^{\circ} \mathrm{RF}$ (at the next step of the design study simulation of action of the cutting slits is supposed). The results of this simulation are presented in Fig. 4-6 (200 particles were taken for more suitable presentation).


Figure 4: Axial motion of 200 particles during the first 3 turns.

There are considerable axial losses during the first turn ( $24 \%$ of particles) which caused by bad vertical focusing in the central region (the aperture of the dees equals 15 mm in this calculations).


Figure 5: Plan view of the cyclotron centre - motion of particles during the first 2 turns.


Figure 6: Radial amplitudes of 200 particles.

## Extraction

Beam extraction from this cyclotron is intended by means of the electrostatic deflector. Figure 7 shows the beam portrait ( z -r plane) at the deflector entrance.


Figure 7: Beam portrait at the deflector entrance.
Out of the initial number of particles $57 \%$ were observed at the deflector entrance, $17 \%$ fall to the top of the septum, and $2 \%$ fall to the top of the opposite electrode. Tracking of the bunch through the extraction system showed that $6 \%$ of particles were lost during the
extraction at the deflector inner surface. Figure 8 shows the plan view of the extracted beam.


Figure 8: Plan view of the extracted beam.
Thus, the analysis of beam dynamics in a compact 8MeV proton cyclotron was carried out. Acceleration of particles from the ion source to the exit of the extraction system was simulated. The main parameters of the machine subsystems were determined for this step of the cyclotron design:

- Ion source position and orientation in the centre plug.
- Dees geometry: angular span- $30^{\circ}$, aperture -15 mm .
- Beam losses during the main region acceleration 24\%.
- Deflector voltage -30 kV , aperture -3 mm , septum thickness -0.5 mm .
- Efficiency of the extraction system - $66 \%$.
- Total beam losses - $49 \%$.
- Beam parameters at the end of extraction: average energy - $8.6 \mathrm{Mev} \pm 0.8 \%$, emmitances: $\varepsilon_{\mathrm{r}}=8.0 \cdot \pi \cdot \mathrm{~mm} \cdot \mathrm{mpad}, \varepsilon_{z}=18.7 \cdot \pi \cdot \mathrm{~mm} \cdot \mathrm{mpad}$.


## CONCLUSIONS

The developed package of codes was used for design study of the $8-\mathrm{MeV}$ proton cyclotron for production of radioisotopes. At this stage the analysis of the magnetic system from the point of view of focusing properties and isochronism and simulation of the beam acceleration were carried out. The parameters of other cyclotron systems were chosen and will be used in further investigations.

In addition to the beam dynamic analysis, the possibility of studying the resonance crossing by monitoring of beam parameters can be used to set tolerances for the magnetic field and manufacturing errors, which is important for the cyclotron construction process.

## REFERENCES

[1] N.Hazewindus et al, "The Magnetic Analogue Method as Used in the Study of a Cyclotron Central Region", NIM, 118(1974), p. 125
[2] M. M. Gordon, "Computation of Closed Orbits and Basic Focusing Properties for Sector-focused Cyclotrons and the Design of "Cyclops", Particle Accelerators, 16(1984), pp. 39-62

