PROGRESS TOWARDS CRAB CAVITY SOLUTIONS FOR THE ILC

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Abstract

In order to achieve acceptable luminosity for ILC crossing angles greater than ~2 mrad, RF deflection cavities must be used to rotate electron and position bunches leading up to the IP. A bunch that passes through a deflection cavity at a phase where the deflection averages to zero, receives a crab kick leading to a finite rotation at the IP. For a beam energy of 500 GeV and a crossing angle of 20 mrad the required crab kick is about 11.4 MV at 1.3 GHz and 3.8 MV at 3.9 GHz. Cavities are needed on both beams and are likely to be positioned about 12 m before the IP. Any RF phase error between the bunch and the cavity leads to a deflection of the bunch in addition to a rotation of the bunch. Any differential phase error between the cavities leads to differing deflections and consequential loss in luminosity.

An updated analysis of system requirements and phase tolerances with respect to original calculations [1] is given. Issues on cavity and frequency choice are discussed.

CRAB KICK REQUIREMENT

The action of a crab cavity is most simply understood with reference to a pillpox cavity without beam-pipes excited in a TM110 dipole mode. This mode has no electric field on axis and constant transverse magnetic flux density B on axis. The transverse momentum kick for a relativistic particle passing through a single cell crab cavity or a deflection cavity is therefore given as

$$p(t_o) = \int_{t_o}^{t_1} e\,c\,B\cos(\omega t)\,dt = -\frac{e\,c\,B}{\omega}\left\{\sin(\omega t_1) - \sin(\omega t_o)\right\}$$

where, ω is the angular frequency, t_o is the entry time of the particle and t_1 is the exit time of the particle. The maximum deflection is obtained when $\omega t_o = -\pi/2$ and $\omega t_1 = \pi/2$ hence there is an optimum cavity cell length $d = \pi c/\omega$. For a cell of optimal length, but for an arbitrary entry time, the transverse kick expressed as a voltage V_{kick} is given as

$$eV_{kick} = c\,p(t_o) = \frac{2ec^2B}{\omega}\sin(\omega t_o) = eV_{cav}\sin(\omega t_o)$$

where V_{cav} is the value of V_{kick} for $\omega t_o = -\pi/2$.

A crab cavity is a displacement cavity operated with a 90^o shift on the beam timing so that the particles enter at times around $t_o = 0$ when B is maximum and pass the centre of the cavity when B is zero. The angular kick x' on a particle that enters the cavity at $t_o = \Delta t$ is given as

$$x'_c(\Delta t) = \frac{c\,p(\Delta t) - c\,p(0)}{mc^2} = \frac{V_{cav}}{E_o}\sin(\omega\Delta t) \qquad (1)$$

Referencing this to a bunch of length $2\sigma_z = 2c\Delta t$ whose centre is at the centre of the cavity when $t_o = 0$ gives

$$x'_c(\sigma_z) = \frac{V_{cav}}{E_o}\sin\left(\frac{\omega\sigma_z}{c}\right) \qquad (2)$$

The angle that bunch ultimately takes at the IP depends on the optics as well as the momentum kick to its ends. Assuming the crab cavity is positioned at distance L_c back from the horizontal optical centre of the final focusing quadrupole doublet, then at distance L after the optical centre, the displacement of a particle that starts on axis at the crab cavity is given as

$$x = \left\{(1+\delta_1)\left(L + L_c - \frac{L\,L_c}{f}\right) + (\delta_2 + L\delta_3)\right\}x'_c$$

$$\approx \left\{L\left(1 - \frac{L_c}{f}\right) + L_c\right\}x'_c \qquad \approx \left\{L - \left(\frac{L}{f} - 1\right)L_c\right\}x'_c$$

where f is the effective horizontal focal length of the final focusing doublet and δ_1, δ_2 and δ_3 are small coefficients that depend on the focusing element. The first approximate form shows that when the crab cavity is closer to the final doublet than its effective focal length then the displacement arising as a consequence of the crab kick grows monotonically with L towards and beyond the IP. If the crab cavity is set back from the final focusing doublet by a distance equal to the effective focal length the displacement associated with the crab kick is constant towards and beyond the IP. Applying the second approximate form near the IP then as L is greater than f at the IP, then as L_c increases the displacement arising from the crab kick decreases. The current ILC optics deck for the 20 mrad crossing angle [2] places the crab cavity right next to the final focusing quadrupole. Figure 1 traces particle paths for this location.

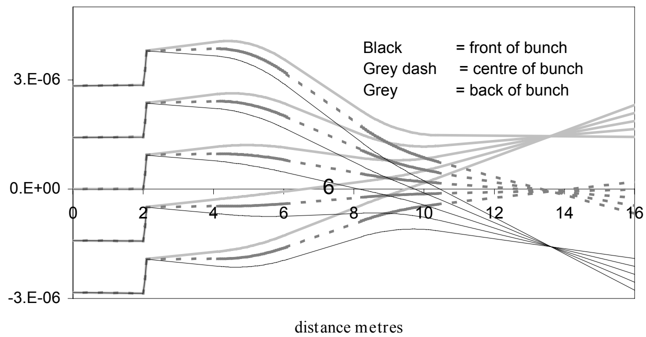

Figure 1: Particle paths from crab cavity to IP

The figure traces from five lateral locations near the bunch centre. A single cell of optimum length at position 2 m provides the crab kick. The focusing quadrupole extends from 4 m to 6 m and the defocusing quadrupole from 8.1 m to 10.2 m. The bunch length is 0.3 mm hence the angle of rotation at any point is computed as the lateral distance between the front and back of the bunch, i.e. the vertical distance between the black solid line and the grey solid line divided by the bunch length. At the focus at position 13.8 m the angle is 10 mrad as required. Note that particles approach the crab cavity with a small divergence. As they pass through the cavity they get a small displacement of the order of 100 nm. As the crab cavity is very close to the focusing quadrupole there is very little change in x' as it passes through. The divergence increases substantially as the bunch passes through the de-focusing quadrupole. Divergence at the IP is of course relevant to the extraction optics. The ratio of the divergence created by the crab cavity x'_c to the displacement at the IP, x_{ip} is referred to here as R_{12} i.e.

$$x_{ip} = R_{12}\, x'_c\,. \tag{3}$$

For the current 20mrad crossing angle optics deck R_{12}=16.3 m (*note in fig.1 R_{12}=13.8*). Using (2) and (3) the crab cavity kick is therefore calculated as

$$\theta_r = \frac{x_{ip}}{\sigma_z} = R_{12}\frac{V_{cav}}{E_o}\frac{1}{\sigma_z}\sin\left(\frac{\omega\sigma_z}{c}\right) \approx R_{12}\frac{V_{cav}}{E_o}\frac{\omega}{c} \tag{4}$$

where θ_r is half the crossing angle, i.e. 10 mrad. This equation shows that the cavity voltage decreases with frequency hence if voltage is limited by the magnetic flux density, a higher frequency gives a shorter cavity system. Specifically for E_o=500 GeV and $\nu = 3.9$ GHz then $V_{cavity} = 3.75$ MV (*note estimates in [1] used a smaller R_{12}*).

CAVITY TIMING ERRORS

As a crab cavity is a displacement cavity operated with a 90° shift on the beam timing, any error in this timing will displace the centre of the bunch. From (3), (1) and (4) a cavity timing error Δt (i.e. a phase error) causes a bunch centre displacement

$$\Delta x_{ip} = R_{12} x'_c(\Delta t) = c\,\theta_r \frac{\sin(\omega\Delta t)}{\omega} \approx c\,\theta_r\,\Delta t \tag{5}$$

A horizontal displacement of an electron bunch with respect to a positron bunch will lead to luminosity loss. The luminosity budget for crab cavity timing errors set by the GDE is currently at 2%. If at the IP the positron bunch has a horizontal displacement of $0.5\Delta x$ and the electron bunch has a displacement of $-0.5\Delta x$ and both bunches have Gaussian profiles then the integral that determines the geometric luminosity contains the term

$$f(x) = \frac{1}{2\pi\sigma_x^2}\ \exp\left[-\frac{(x+0.5\,\Delta x)^2}{2\sigma_x{}^2}\right]\ \exp\left[-\frac{(x-0.5\,\Delta x)^2}{2\sigma_x{}^2}\right]$$

The luminosity reduction factor is therefore given as

$$S = \exp\left(-\frac{\Delta x^2}{4\sigma_x{}^2}\right) \tag{6}$$

From (6) using the nominal ILC parameter set [2] then for a horizontal beam size at the ip of $\sigma_x = 655$ nm, then a luminosity reduction of 2% ($S = 0.98$) occurs for a displacement of 186 nm, which for 20 mrad crossing gives a timing tolerance of 0.062 ps, corresponding to a phase tolerance at 1.3 GHz of 0.029° or 0.087° at 3.9 GHz. In the next section it is seen that this timing tolerance only relates to the relative phase of the electron and positron cavities. (Note that Low Q ILC parameters give Δx = 141 nm hence the timing tolerance = 0.047 ps)

BUNCH TIMING ERRORS

The anticipated jitter on the arrival time of bunches at the crab cavities from the linacs will be 0.4 ps or more. If one bunch arrives on time and another is late (or early) then at the IP, the bunch that arrives on time can be regarded as a small segment of a long virtual bunch that crosses the beam-line at half the crossing angle and the late (or early) bunch can be regarded as a small segment of this virtual bunch that lies off the beam-line, see figure 2. The virtual bunch rotates about its intersection on the beam line hence segments along it will stay in line up to the point where the virtual bunch starts to bend. The curvature comes from the sine dependency in (2). If the cavities are synchronised then virtual bunches on the positron and electron beams are in line.

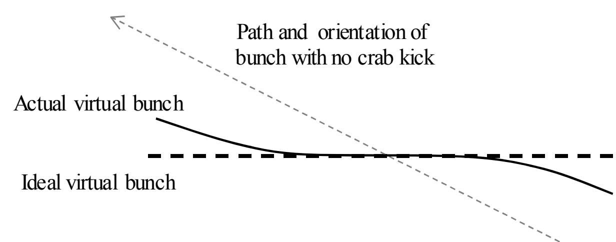

Figure 2:Shape of a very long bunch kicked by the crab cavity

In this way the crab cavities conveniently align miss timed bunches such that bunch timing errors can be as large as 8 ps before the associated displacement error gives rise to a luminosity reduction of 2%. This ability of the crab cavity to align bunches is not as useful as it seems as when bunches are late by as little as 0.6 ps, the collision point is shifted from the intended IP to the point where defocusing in the vertical plane causes luminosity loss to increase beyond 2%.

AMPLITUDE ERROR

From (4) one sees that the crab rotation angle of the bunch at the IP is linearly proportional to the cavity voltage, hence any variation in the voltage ΔV produces a proportional variation in the rotation of the bunch. The luminosity reduction factor S due to incorrect rotation is given by $S = (1-\sigma_z\Delta\theta_r/\sigma_x)^{-\frac{1}{2}}$

hence the acceptable amplitude variation is given as

$$\frac{\Delta V}{V_{max}} = \frac{1}{\theta_r}\frac{\sigma_x}{\sigma_z}\sqrt{\frac{1}{S^2}-1}$$

Taking S = 0.98, and using the ILC nominal parameter set [2] which gives $\sigma_x = 0.665\times10^{-6}$ m, $\sigma_z = 0.3\times10^{-3}$ m and $\theta_r = 0.01$ rads then $\Delta V/V_{cav} = 4.4\%$ which at a glance is a comfortably large. However this tolerance may actually be difficult to achieve if the bunches arriving at the crab cavity arrive off axis. It is estimated that bunches could arrive at the crab cavity with a horizontal offset as a much as 1.5 mm. Arriving off axis will result is a slightly differing transverse kick but more problematically the bunch will deposit or extract energy from the crab cavity. Successive bunches will alter the amplitude and indeed the phase of the field in the cavity for the next bunch. At first sight this is no different to the beam loading of an accelerator cavity. The difference is that the cavity to cavity phase tolerance here is far more stringent than the beam timing tolerance. Detailed calculations to determine whether the tolerances established can be maintained with off axis bunches are being undertaken. If the tolerances cannot be met, one solution might be to consider a larger cavity operating at a lower frequency.

BEAM –BEAM DISRUPTION EFFECTS

The estimated parameters here do not take account of beam disruption effects. Church [3] has undertaken detailed simulations of the crab cavity system and final focus optics using the simulation software MAD together with Guinea Pig to determine RMS tolerances. He predicts 2% luminosity loss for cavity timing errors of 0.042 ps, beam timing errors of 0.64 ps and amplitude errors of 4.4%.

CAVITY CHOICE

In order to synchronise the crab cavities with the linac they must operate at the same frequency, a harmonic or sub harmonic frequency of the main linac. The current BDS layout for 20 mrad crossing, places the crab cavities adjacent to the final focusing quadrupole doublets at a distance of 12 m from the IP. At this position the separation between the beams is about 240 mm. A 1.3 GHz dipole cavity barely fits at this location, higher frequency choices are 2.6 GHz, 3.9 GHz etc.

The most stringent tolerance for the crab cavity system is the phase difference between positron crab cavity and the electron crab cavity. Scaling results in [3] to the low Q, ILC parameter set, this tolerance could be as little as 0.03 ps. Such a tolerance is on the limit of what can currently be achieved with superconducting CW cavities [4].

The crab cavity power requirement is nominally given as $\frac{V_{cav}^2}{(R/Q)Q}$ which for a warm cavity with say Q = 8000, R/Q = 1400 Ω and V_{cav} = 3.8 MV gives 1.3 MW implying that a warm copper cavity system would need to be pulsed on only during the bunch train. Achieving the required cavity phase stability with a pulsed source operating at this power level and driving a low Q system represents a considerable challenge. On this basis CW superconducting cavities were favoured.

Choosing a higher frequency gives a shorter system because longitudinal size varies almost inversely with frequency, (note that maximum SCRF magnetic field drops slightly with frequency). Wakefield voltage however increases with frequency and hence may become a limiting factor [5]. Surface resistance also increases with frequency so power density in the couplers becomes more difficult to handle. An advantage of working at 1.3GHz is that IOTs with high phase stability can be used. The phase stability of a klystron is 10^o per 1% power supply variation, while for an IOT this is 1^o per 1% voltage variation.

In view of the huge amount of effort required to start developing a new superconducting cavity from scratch, the practical way forward is to build on existing technology. A clear choice was to develop a variant of the FNAL 3.9GHz CKM cavity [6] optimised for an ILC crab cavity solution. Collaborative work between the Cockcroft Institute and FNAL is now being undertaken to look at wakefields in the CKM cavity with respect to the ILC bunch structure. Current analysis favours a solution with four, nine cell cavities on each beam. It is anticipated that the cavities will be run CW and driven from a number of Klystrons, one per nine cell cavity.

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