

# USE OF MOVABLE BEAM POSITION MONITORS FOR BEAM SIZE MEASUREMENTS

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## Abstract

The use of beam position monitors (BPMs) as non-intercepting emittance monitors has been proposed in 1983 by Miller et al. The emittance measurement relies on the beam size dependency of the BPM signals. It is shown that the original proposal can be improved by using movable BPMs. Changing the BPM position with a stepping motor allows accurately calibrating the beam size measurement. The absolute scale on the beam size measurement is given by the scale of the stepping motor and can be determined in the laboratory and measured in situ. Uncontrolled changes of the beam position can be monitored through the use of a BPM triplet.

## 1 INTRODUCTION

It has been proposed in [1] that the quadrupole term in the signal of standard four button beam position monitors (BPM's) can be used for a non-intercepting measurements of beam size. Recent publications have shown that the method can indeed be used successfully for measuring beam size [2,3]. Here, we propose an improvement of the method. It is shown that the use of BPM step movers greatly simplifies the method. It is explained how movers can be used for a precise calibration of the absolute scale of beam size.

## 2 THEORY

We consider BPMs that consist of four pickup buttons with an angle of  $\pi/2$  between neighbouring buttons. The pickup buttons shall be located at upper left, top, bottom, left, and right position, as illustrated in Figure 1. The distance of a button to the BPM centre is given by  $a$  and its azimuthal angle by  $\theta$ . The image current in a four-polar beam position monitor has been calculated by Miller et al [1] in 1983. We consider an infinitely long line current  $I(r, \phi)$  at radial location  $r$  and azimuthal angle  $\phi$ . The image current density  $J_{image}(r, \phi, a, \theta)$  on a conducting cylinder of radius  $a$  at azimuthal angle  $\theta$  is then:

$$J_{image}(r, \phi, a, \theta) = \frac{I(r, \phi)}{2\pi a} \frac{a^2 - r^2}{a^2 + r^2 - 2ar \cos(\theta - \phi)} \quad (1)$$

A multipole expansion in powers of  $(r/a)$  is performed and the image current is integrated over all  $r$  and  $\phi$ . For this a Gaussian beam distribution in  $r$  and  $\phi$  is assumed, with  $x_b$  and  $y_b$  being the horizontal and vertical centres of gravity and  $\sigma_x$  and  $\sigma_y$  being the corresponding standard deviations.

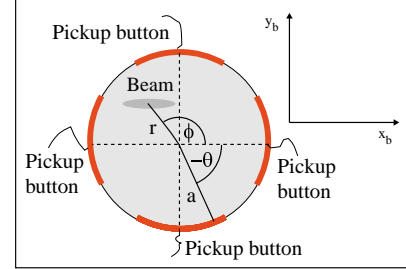


Figure 1: The pickup buttons are at a distance  $a$  from the centre. The beam offsets with respect to the centre are denoted as  $x_b$  and  $y_b$  for horizontal and vertical direction.

For a total beam current  $I_b$  the induced image current  $J(a, \theta)$  at radius  $a$  and azimuthal angle  $\theta$  is calculated:

$$J(a, \theta) \cong \frac{I_b}{2\pi a} \left\{ \underbrace{1}_{\text{MONOPOLE}} + 2 \underbrace{\left[ \frac{x_b \cos \theta + y_b \sin \theta}{a} \right]}_{\text{DIPOLE}} + 2 \underbrace{\left[ \left( \frac{\sigma_x^2 - \sigma_y^2}{a^2} + \frac{x_b^2 - y_b^2}{a^2} \right) \cos(2\theta) + 2 \frac{x_b y_b}{a^2} \sin(2\theta) \right]}_{\text{QUADRUPOLE}} + 2 \left[ 3 \left( \frac{\sigma_x^2 - \sigma_y^2}{a^2} \right) + \frac{x_b^2 - 3y_b^2}{a^2} \right] \frac{x_b}{a} \cos(3\theta) + 2 \left[ 3 \left( \frac{\sigma_x^2 - \sigma_y^2}{a^2} \right) + \frac{3x_b^2 - y_b^2}{a^2} \right] \frac{y_b}{a} \sin(3\theta) \right\} \quad (2)$$

This result is valid for beam offsets  $x_b$ ,  $y_b$  and beam sizes  $\sigma_x$ ,  $\sigma_y$  much smaller than the radius of the beam pipe. For practical applications these conditions will be true. Note that the result differs in the sextupole term from the result published by Miller et al. The relevant derivation is summarised in [5]. For the considered BPM geometry, the quadrupole signal  $q$  is constructed:

$$q = \frac{1}{T} \cdot \left[ J(a, 0) - J(a, \frac{\pi}{2}) + J(a, \pi) - J(a, \frac{3\pi}{2}) \right] \quad (3)$$

$T$  is the sum signal from all four buttons. The quadrupole signal  $q$  has been calculated for point-like and for wide buttons (covering 90 degrees around the pickup centre):

$$q = 2 \cdot \left( \frac{\sigma_x^2 - \sigma_y^2}{a^2} + \frac{x_b^2 - y_b^2}{a^2} \right) \quad (\text{point-like buttons}) \quad (4)$$

$$q = \frac{4}{\pi} \cdot \left( \frac{\sigma_x^2 - \sigma_y^2}{a^2} + \frac{x_b^2 - y_b^2}{a^2} \right) \quad (\text{wide buttons}) \quad (5)$$

The width of the buttons does not change the result, apart from an overall scaling factor. This result has also been

obtained in [3]. The quadrupole signal is sensitive to the difference in squared beam sizes and can therefore be used to determine the beam sizes.

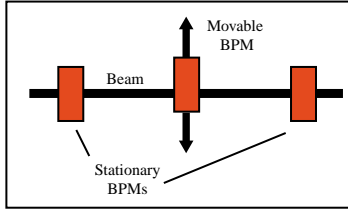


Figure 2: Two outer BPMs of a BPM triplet measure variations of the beam position in the middle BPM. The middle BPM is moved to measure the beam size term.

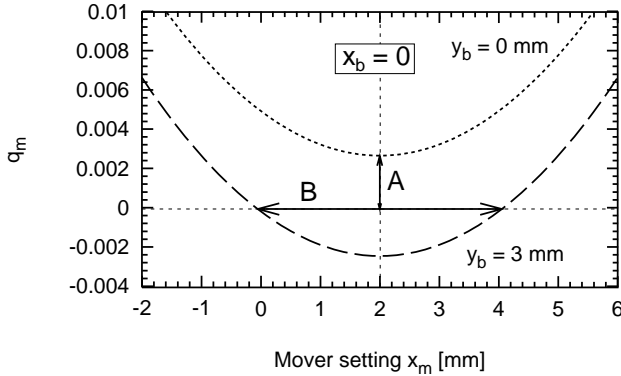


Figure 3:  $q_m$  as a function of the mover setting for example parameters. The minimum is at  $x_b = 0$ . The offset  $x_0$  was chosen to be 2 mm in this example. The two curves correspond to different values of  $y_b$ . We use LEP parameters (BPM at QL17, 94.5 GeV,  $a = 4.2$  cm,  $\sigma_x = 2.19$  mm,  $\sigma_y = 0.06$  mm).

### 3 BEAM SIZE MEASUREMENT

It is proposed to mount the BPMs on step movers that allow moving the BPM's independently in the horizontal and vertical directions. The range of such movers should be in the mm-range with step sizes of about  $0.5 \mu\text{m}$ . The absolute scale of movements can be measured in the laboratory and in situ. The settings of the mover shall be denoted by  $x_m$  and  $y_m$ . The true centre of gravities  $x_b$  and  $y_b$  can then be expressed as:

$$x_b = x_m - x_0 \quad (6)$$

$$y_b = y_m - y_0 \quad (7)$$

$x_0$  and  $y_0$  are arbitrary offsets. They can vary with time. To control changes in  $x_0$  and  $y_0$  two additional BPMs can be installed close to the movable BPM (see Figure 2). We introduce the observable  $q_m = C \cdot q$ , where the constant  $C$  depends on the width of the buttons. The observable  $q_m$  can then be written as:

$$q_m = \frac{\sigma_x^2 - \sigma_y^2}{a^2} + \frac{(x_m - x_0)^2 - (y_m - y_0)^2}{a^2} \quad (8)$$

It is evaluated by changing the mover settings  $x_m$  and  $y_m$ . The calculated response in  $q_m$  is shown in Figure 3.

The minimum of Equation 8 is obtained with  $x_m = x_0$ . Varying  $x_m$  and fitting the minimum of the observed  $q_m$  allows the determination of the absolute mover scale with respect to the physical centre of the BPM. With  $x_m = x_0$  the quadrupole signal  $q$  assumes the following value:

$$\begin{aligned} A = q \quad (x_b = x_m - x_0 = 0) &= \frac{q_m}{C} \\ &= C' \cdot \left( \frac{\sigma_x^2 - \sigma_y^2}{a^2} + \frac{(y_m - y_0)^2}{a^2} \right) \end{aligned} \quad (9)$$

The vertical mover setting  $y_m$  can be precisely measured in situ and the offset  $y_0$  can be determined just as described above for  $x_0$ . The minimum value  $A$  of  $q$  is then a simple function of the beam size difference. Miller et al. suggested using this measure to determine beam sizes. Unfortunately it contains a constant  $C'$  that depends on the BPM calibration and, as we have seen, on the BPM button width. Therefore we consider another observable, the zero-crossings of the  $q_m$  signal. They appear at:

$$x_m \quad (q_m = 0) - x_0 = \pm \frac{B}{2} = \pm \sqrt{(y_m - y_0)^2 + \sigma_y^2 - \sigma_x^2} \quad (10)$$

These solutions do only exist for large offsets  $y_b$ :

$$y_b = y_m - y_0 \geq \sqrt{\sigma_y^2 - \sigma_x^2} \quad (11)$$

As explained above, we can assume that  $y_m$  and  $y_0$  are known. From a measurement with appropriate vertical offset we can obtain the observable  $B$  (compare Figure 3). The beam size term  $\sigma_x^2 - \sigma_y^2$  is then obtained:

$$m = \sigma_x^2 - \sigma_y^2 = (y_m - y_0)^2 - \frac{B^2}{4} \quad (12)$$

The absolute measurement of the beam size term  $\sigma_x^2 - \sigma_y^2$  does only depend on the calibration of the mover scale. For realistic (LEP) parameters and a  $\mu\text{m}$  resolution BPM, it is estimated that the beam size term  $\sigma_x^2 - \sigma_y^2$  can be determined with an accuracy of at least 5%. This assumes that the image currents from the four buttons are sampled with equal efficiency. Large differences in the processing of the four image currents can introduce additional errors.

Turn-by-turn (or shot-by-shot) variations of the beam position have been a major problem for existing measurements of the quadrupole signals. Those measurements did average over many shots and the beam size measurement was found to be diluted by variations of the beam position. The usability of "quadrupole BPMs" was found to be significantly hampered [4]. The use of state of the art BPMs should allow to almost completely avoid this problem. If the data is analysed turn by turn then both the dipole and the quadrupole signals can be measured for every passage of the beam. The effect of changes in beam position for the quadrupole term can be measured, for example, with two close-by BPMs.

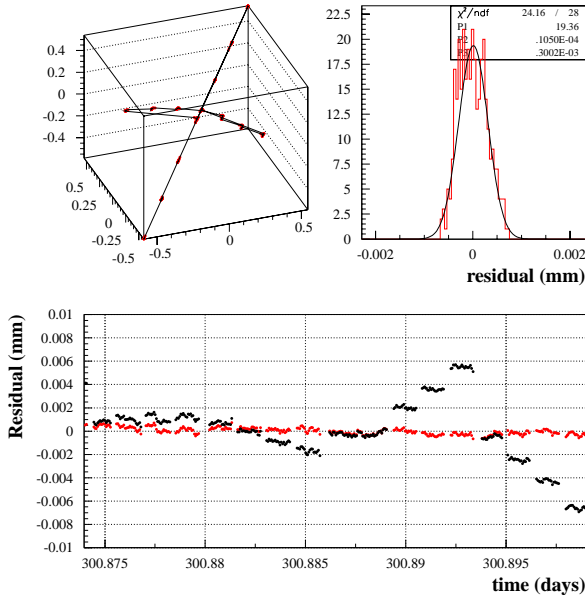


Figure 4: Results from triplet cross-calibration of BPMs.

## 4 BEAM EMITTANCE

In the previous section it was shown how the beam size dependent term  $m$  is obtained. For a transport line it has been shown by Miller et al. how the emittances are obtained. Here we consider a storage ring. We assume that two measurements  $m_1$  and  $m_2$  are performed with BPMs located at two neighbouring focusing and defocusing quadrupoles. We further require that there is no source of emittance change between the two locations and that the beta functions are accurately known (there exist methods to measure the beta functions in a storage ring precisely). The values  $\beta_x(1)$ ,  $\beta_y(1)$ ,  $\beta_x(2)$  and  $\beta_y(2)$  shall be the beta functions at locations 1 and 2. The horizontal and vertical beam emittance is denoted by  $\epsilon_x$  and  $\epsilon_y$ . It can then be shown that the vertical beam emittance is obtained from  $m_1$  and  $m_2$  as:

$$\epsilon_y = \left( \frac{m_1 \cdot \beta_x^{(2)}}{\beta_x^{(1)} \cdot \beta_y^{(2)}} - \frac{m_2}{\beta_y^{(2)}} \right) / \left( 1 - \frac{\beta_y^{(1)} \cdot \beta_x^{(2)}}{\beta_x^{(1)} \cdot \beta_y^{(2)}} \right) \quad (13)$$

The horizontal emittance is then calculated via:

$$\epsilon_x = \frac{m_1 + \epsilon_y \cdot \beta_y^{(1)}}{\beta_x^{(1)}} \quad (14)$$

Note, that the described method for emittance determination requires that the following condition is fulfilled:

$$\beta_y^{(1)} \cdot \beta_x^{(2)} \neq \beta_x^{(1)} \cdot \beta_y^{(2)} \quad (15)$$

In other words, the phase advance in the two planes must be different. If more than two measurements are performed additional information about a possible betatron mismatch can be obtained.

## 5 STATUS OF TESTS AT LEP

In the context of the LEP spectrometer two BPM triplets have been installed in the LEP beamline. They are horizontally movable and it is planned to try some beam size measurements. Here we report some results on BPM resolution and stability. We denote the BPMs in a triplet as “1”, “2” and “3” with “2” as the middle position. The triplet residual is defined as:

$$R_x = x_2 - \frac{x_1 + x_3}{2} \quad (16)$$

For calibration purposes a series of beam rotations about the central BPM are done, followed by a series of parallel orbit bumps, up to  $\pm 600 \mu\text{m}$ . The aim of the procedure is to find the calibration of “1” and “3”, relative to “2”. In Figure 4 (top left) the reading of BPM2 is plotted against the readings of the other two, after subtraction of the mean BPM value. The two sets of beam movements result in two lines which together define a plane, the angles of which relative to the ideal plane  $z-(x+y)/2 = 0$  give the relative gains of the BPMs.

The triplet residual  $R_x$  would be zero for perfect BPMs, but in reality has a finite value due to noise and inaccuracies in the gain cross-calibration. Figure 4 (bottom) shows  $R_x$  against time before and after correction of the gains to that of BPM2. The histogram of  $R_x$  has a  $\sigma$  of 300 nm after the correction, over the time of 60 min taken to complete the sequence of beam movements. This implies that sub- $\mu\text{m}$  relative accuracy and stability has been achieved, the fluctuations from one BPM being around 200 nm.

## 6 CONCLUSION

The use of beam position monitors (BPMs) as non-intercepting emittance monitors was proposed in 1983 by Miller et al. The idea relies on the beam size dependency of the BPM signals. The original proposal can be improved by using movable BPMs. Changing the BPM position with a precise stepping motor allows accurately calibrating the beam size measurement. The absolute scale on the beam size measurement is given by the absolute scale of the stepping motor. Uncontrolled changes of the beam position can be corrected through the use of a BPM triplet. A BPM relative accuracy of 200 nm over 60 min was demonstrated. Tests at LEP are ongoing.

## REFERENCES

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