DETERMINATION OF LONGITUDINAL BUNCH PROFILE USING SPECTRAL FLUCTUATIONS OF INCOHERENT RADIATION*

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Abstract

Single-shot spectrum measurements of the radiation emitted by an electron bunch provide a novel way to characterize the bunch shape. Shot noise fluctuations in the longitudinal beam density result in radiation with a spectrum that consists of spikes with width inversely proportional to the bunch length. The variance of the Fourier transform of the spectrum is proportional to the convolution function of the beam current averaged over many bunches. After the convolution function is found, the phase retrieval technique can be applied to recover the bunch shape. This technique has been used to analyse the shape of the 4-ps-long bunches at the Low-Energy Undulator Test Line at the Advanced Photon Source.

1 INTRODUCTION

Measurement of the longitudinal profile of a beam is an important diagnostic tool for accelerators. For bunch lengths in the range of picoseconds, such measurements can be done by a streak camera. Shorter bunches usually require some kind of special techniques. It has been recently proposed that longitudinal properties of an electron bunch can be obtained through measurement of the fluctuations of undulator radiation from the bunch [1]. First measurements of the single-shot spectra of the undulator spontaneous emission at the Accelerator Test Facility at Brookhaven National Laboratory (ATF/BNL) with resolution required to demonstrate 100% fluctuations of intensity were recently reported [2].

This report presents results of single-shot spectrum measurements obtained at the Low-Energy Undulator Test Line at the APS. The measurements repeat those made at the ATF/BNL; however, data processing in the experiment described here is expanded. The spectrum intensity fluctuations are used not only for the bunch length extraction but for reconstructing the detailed longitudinal bunch profile.

This paper begins with a brief theoretical introduction. Then it describes the bunch length extraction following the ATF experiment. The rest of the paper is devoted to the experimental recovering of the bunch profile.

Let us consider the microscopic picture of the electron beam current at the entrance into the undulator. The electron beam current consists of electrons arriving at the entrance of the undulator at some particular time t_k:

$$I(t) = -e \cdot \sum_{k=1}^{N} \delta(t - t_k),$$

where N is the number of electrons in a bunch. The Fourier transform of the current and the electric field emitted in the undulator can be written as

$$\bar{I}(\omega) = \int_{-\infty}^{\infty} e^{i\omega t} I(t)dt = -e \cdot \sum_{k=1}^{N} e^{i\omega t_k},$$

$$E(\omega) = e(\omega) \cdot \sum_{k} e^{i\omega t_k}, \qquad (1)$$

where $e(\omega)$ is the Fourier transform of an individual particle travelling through the undulator and is proportional to $\sin(\omega-\omega_0)T/(\omega-\omega_0)T$. The summation of large numbers of exponentials with different arguments results in an appearance of sharp spikes in the $E(\omega)$ dependence. This leads to bunch-to-bunch fluctuations in intensity of the incoherent radiation spectrum.

To extract the bunch length information from the spectrum, one can calculate the second order correlation of the Fourier harmonics $\bar{I}(\omega)$:

$$\left\langle \left|\bar{I}(\omega)\right|^2 \left|\bar{I}(\omega')\right|^2 \right\rangle = e^4 \sum_{n,m,p,q} \exp\left[i\omega(t_n - t_m) + i\omega'(t_p - t_q)\right].$$

The second order is necessary because measurements provide the spectrum of intensity. The beam current averaged over many bunches can be written in the form

$$\langle I(t) \rangle = -eNF(t),$$

where $F(t)$ is the electron bunch form factor. For the case $N\left|\bar{F}(\omega)\right|^2 \ll 1$ (which is true for our situation) the expression for the correlation above can be simplified:

$$\left\langle \left|\bar{I}(\omega)\right|^2 \left|\bar{I}(\omega')\right|^2 \right\rangle = e^4 N^2 \left(1 + \left|\bar{F}(\omega - \omega')\right|^2\right). \qquad (2)$$

So the spectrum correlation is related to the square of the Fourier transform of the bunch form factor. This formula can be used to calculate the bunch length using the correlation of the radiation spectrum.

2 SPECTRUM MEASUREMENTS AND BUNCH LENGTH EXTRACTION

Radiation spectra were measured using a high-resolution spectrometer [3] with a cooled CCD imager, which provides a resolution of 0.4 Å per pixel. Measurement of the single-shot spectrum is shown in Figure 1. The spectra are composed of spikes of random

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amplitude and frequency that have a characteristic width $\Delta\omega \sim 1/\tau_b$ (where τ_b defines the bunch length) and intensity fluctuation of almost 100%. The shape of an individual spectrum changes randomly from shot to shot, but the average of many shots approaches the familiar wiggler spectrum.

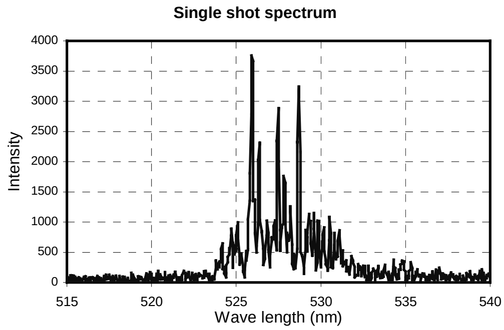

Figure 1. Typical single-shot spectrum.

To extract the bunch length from the spectral data, Eq. (2) for the second-order spectrum correlation is used. The normalized autocorrelation of the spectral intensity averaged over many shots is calculated from the spectrum measurements:

$$C_n = \left\langle \sum_i P(\omega_i) P(\omega_{i+n}) \right\rangle / \left\langle \sum_i P(\omega_i)^2 \right\rangle ,$$

where $P(\omega_i)$ is the signal in the i-th CCD channel and n is the shift in channels. The correlation averaged over 100 shots is plotted in Figure 2.

For the ideal case of a zero emittance beam and diagnostics with sufficient spectral resolution, 100% fluctuation of the spectral intensity will occur. However, when the beam size is large or the detector spectral resolution is poor, the spectrum will be similar to a spectrum emitted by several independent sources. The fluctuation level will be reduced, and a pedestal will appear in the spectrum.

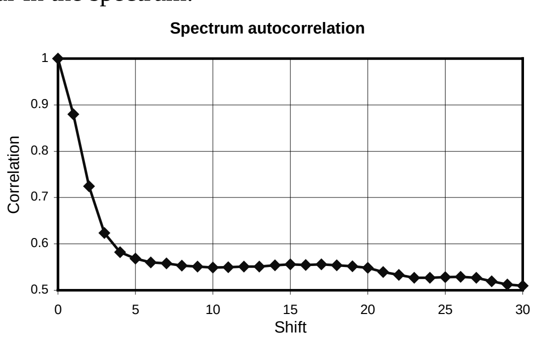

Figure 2. Spectrum autocorrelation.

The asymptotic level of the correlation curves at large n shows how large the pedestal was; for 100% intensity fluctuations this level should be equal to 0.5 (see Eq. (2)). The value of this level can be used to characterize how many independent modes contributed to the radiation measured in one channel.

The spike width at half maximum according to Figure 2 is about two pixels. The frequency step corresponding to one pixel is $\delta\omega = 2.4 \cdot 10^{11}$ rad/s. Therefore, assuming the beam to be Gaussian, the sigma of the Gaussian distribution is

$$\tau_b \approx \frac{1}{n \cdot \delta\omega} \approx 2\,ps\,.$$

This gives the FWHM length of the bunch to be equal to 4.5 ps.

Accuracy in determining the spike width is not so great due to the size only being a few pixels. This means the resolution of the spectrometer is not enough for our bunch length. However, shorter bunches result in wider spikes and improved accuracy.

3 PROFILE MEASUREMENTS

Measurement of spectral intensity fluctuations can be used not only to determine the bunch length but also to recover a longitudinal bunch profile. It has been shown [1] that the variance of the Fourier transform of the spectrum is proportional to the convolution function of the beam current. After the convolution function is found, a phase retrieval technique can be used to recover the shape of the pulse in many practical cases.

These two steps are described below. For each step both simulation and experimental results are presented.

3.1 Calculation of the Convolution Function of the Bunch Shape

Let us denote the detector signal in channel m for measurement n as $P_{m,n}$. The Fourier transform of the spectrum is

$$\Gamma_{k,n} = \sum_{m=1}^{N_{ch}} P_{m,n} e^{2\pi imk / N_{ch}} ,$$

where N_{ch} is the number of channels in the detector (CCD). After accumulation of N_p number of pulses large enough for statistical analysis, the following quantity is computed:

$$d_k = \sum_{m=1}^{N_p} \left| \Gamma_{k,m} - \frac{1}{N_p} \sum_{n=1}^{N_p} \Gamma_{k,n} \right|^2 .$$

It can be described as the average deviation of the signal in the k-th channel from its average. The quantity d_k gives the convolution function of the particle density in the bunch averaged over N_p bunches (the theoretical proof of this can be found in Ref. [1]).

Figure 3 represents the convolution function of the longitudinal bunch distribution extracted from spectral measurements described above. The number of spectra used for averaging is 100 with a slit size of 50 μm. This plot can also be used to determine the bunch length. If we assume that the bunch has a Gaussian shape, then its convolution is also a Gaussian with $\tau = \sqrt{2}\tau_b$. A fitted Gaussian function is also shown in Figure 3 as a solid line.

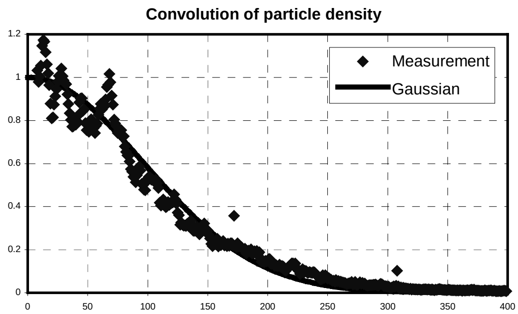

Figure 3. Convolution of longitudinal particle density calculated using measured spectra. Solid line is a fitted Gaussian function.

Knowledge of the convolution function does not allow a unique restoration of *I(t)*. However, as shown in Ref. [4], the use of a phase retrieval technique allows one to recover the beam profile in many practical cases.

3.2 Reconstruction of the Bunch Shape from the Convolution Function

It is possible to extract both the amplitude and the phase information of the radiation source by applying a Kramers-Kronig relation to the convolution function. This technique of phase extraction was well developed in the optics of solids for the problem of reflectivity. We will briefly describe the technique following Ref. [4].

We denote the longitudinal particle density as *S(z)* and its Fourier transform as *S(ω)*, and write it in the following form

$$S(\omega) = \rho(\omega) e^{i\psi(\omega)} . \qquad (3)$$

Here $\rho(\omega)$ corresponds to the Fourier transform of our convolution function. Phase ψ can be extracted using the expression

$$\psi_m(\omega) + \psi_{Blaschke}(\omega) = -\frac{2\omega}{\pi} P \int_0^{\infty} dx \frac{\ln \rho(x)}{x^2 - \omega^2} + \sum \arg\left(\frac{\omega - \omega_j}{\omega - \omega_j^*} \right) ,$$

where ψ_m is the minimal phase and the ω_j's are the zeros of *S(ω)* in the upper half of the complex frequency plane. If *S(ω)* has no zeros, the contribution from $\psi_{Blaschke}(\omega)$ equals zero, and the expression above gives the minimal phase. This minimal phase is a good approximation to the actual phase in cases where the bunch density has no nearby zeros in the upper half of the complex frequency plane. The final expression for calculating the minimal phase is

$$\psi_m(\omega) = -\frac{2\omega}{\pi} P \int_0^{\infty} dx \frac{\ln[\rho(x)/\rho(\omega)]}{x^2 - \omega^2} .$$

The density distribution function can now be obtained from the inverse Fourier transform of Eq. (3):

$$S(z) = \frac{1}{\pi c} \int_0^{\infty} d\omega \cdot \rho(\omega) \cdot \cos\left(\psi_m(\omega) - \frac{\omega z}{c} \right) .$$

One can see that the plot of the convolution function is very noisy despite averaging over 100 single shots. This noise produces high frequency content in the spectrum; therefore, before taking the Fourier transform of the convolution function, it has been smoothed by filtering the high frequencies. The result of calculating the bunch shape using the smoothed convolution is presented in Figure 4. A FWHM length of the bunch according to the plot is 4 ps, which corresponds to the estimates made above.

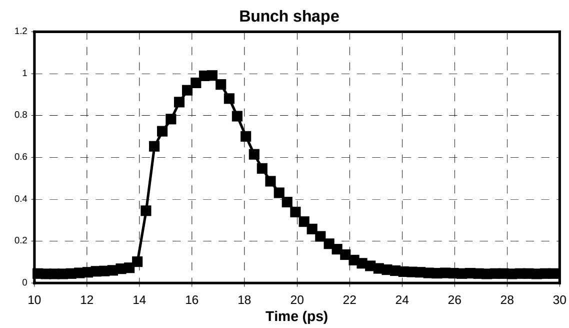

Figure 4. Longitudinal bunch shape recovered from spectrum measurements.

4 CONCLUSIONS

A new technique for recovering a longitudinal bunch shape from single-shot spectrum measurements has been implemented, and a bunch profile has been measured. An important feature of this method is that it can be used for bunches with lengths less than a millimeter – the shorter the bunch, the less requirements for the spectrometer.

To reconstruct the bunch shape from the convolution function the technique suggested in Ref. [4] has been utilized, but a different approach has been used to build the convolution. By using spectrum fluctuations to construct the convolution one can avoid measuring the spectrum of coherent far infrared radiation of the bunch and making any assumptions about the asymptotic behavior of this spectrum.

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