STUDY OF FAST ION INSTABILITY AT KEKB ELECTRON RING

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Abstract

We have studied fast ion instability (FII) in the electron ring (HER) at KEKB. We introduce an application of Singular Value Decomposition (SVD) to study multi-bunch beam oscillations. The SVD analysis[1] has been performed for a matrix consists of vertical bunch positions in a turn-by-turn and bunch-by-bunch to extract wave lengths which characterized the behavior of multi-bunch beam instabilities. Multi-bunch beam instabilities were also observed by changing the beam size with adjusting the dispersion function. We found that the multi-bunch beam instabilities were stronger as the beam size decreased. We compared the wave length and growth time with the linear approximate calculation of the FII. Observed multi-bunch beam instability is consistent with the FII.

1 INTRODUCTION

For multi-bunch beam oscillations, it can be assumed that

$$y_n^\mu(t) = A_\mu e^{\frac{t}{\tau_\mu}} \cos\left(\frac{2\pi\mu n}{M} - \omega_\beta t\right), \tag{1}$$

where n is the number of a bunch, M is the total number of bunches, μ is the number of a mode, τ_μ is the growth (damping) time corresponding to the mode, and ω_β is the betatron frequency. Equation (1) implies a periodicity of $y_0^\mu(t) = y_M^\mu(t)$. Because a single monitor utilized to measure the position of the bunch, the arrival time of the n-th bunch to the point of observation after k-th revolution can be written as follows:

$$T_{k,n} = (M-1-n)\frac{l}{c} + M\frac{l}{c}k, \tag{2}$$

where l is the distance between neighboring bunches along the orbit and c is the light velocity. Using $T_{k,n}$ instead of t and taking sums of mode μ, the bunch position:

$$\begin{aligned} y_n(k) = \sum_\mu A_\mu \Bigg[& \left\{ e^{\frac{Ml}{c\tau_\mu}k} \cos\left(\omega_\beta M \frac{l}{c} k\right)\right\} \times \\ & \left\{ e^{\frac{(M-1-n)l}{c\tau_\mu}} \cos\left(\frac{2\pi\mu n}{M} + \omega_\beta (M-1-n)\frac{l}{c}\right)\right\} \\ & - \left\{ e^{\frac{Ml}{c\tau_\mu}k} \sin\left(\omega_\beta M \frac{l}{c} k\right)\right\} \times \\ & \left\{ e^{\frac{(M-1-n)l}{c\tau_\mu}} \sin\left(\frac{2\pi\mu n}{M} + \omega_\beta (M-1-n)\frac{l}{c}\right)\right\} \Bigg]. \end{aligned} \tag{3}$$

The data from the beam position monitor can be stored in a matrix Y of K rows by M columns, where K is the total number of turns recorded. The matrix Y can be decomposed by singular value decomposition (SVD),

$$Y = U^T W V, \tag{4}$$

where U and V are orthogonal matrices. The eigen vectors which are row vectors U represent time patterns and V represent spatial patterns, respectively. The singular values corresponding to the eigen vectors appear in the diagonal matrix W. The number of significant singular values shows the number of the physical bunch oscillation. The form of Eq.(4) invokes Eq.(3). Two significant singular values above noise level should be found for each physical mode from Eq.(3) .

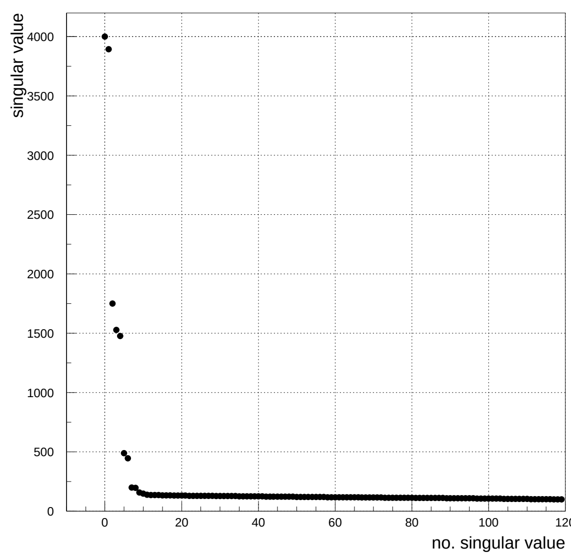

Figure 1: Typical singular values.

2 OBSERVATION OF MULTI-BUNCH BEAM OSCILLATION

Multi-bunch beam oscillations have been observed in the electron ring (HER) at KEKB. Instabilities of bunches should be suppressed by the feedback system. The data was taken by bunch oscillation recorder (BOR)[2] after turning off the feedback system. The positions of all bunches passing through the BOR were stored in the memory turn by turn up to 4096 turns which corresponds to 40 msec. The filling pattern was 8 bunch trains. Each bunch train contained 120 bunches which were equally spaced at intervals

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of 8 nsec (4 bunch spacing). The gaps between neighboring bunch trains were 160 nsec, and 1 μsec for the gap between the last and the first bunch train. The beam current was 240 mA and the average pressure with beam was 1.4×10^{-7} Pa for HER during the experiment.

There is no periodicity for multi-bunch oscillations in the ring because of the existence of large gaps. Therefore, we introduce the wave length, λ, instead of the mode. The SVD analysis can be applied to multi-bunch trains when the mode number, μ, is just replaced with Ml/λ in Eq.(1). We present the vertical multi-bunch oscillations in this analysis. The SVD analysis is performed for the matrix consists of the bunch positions within a bunch train. Typical singular values obtained from a matrix of vertical bunch oscillations are shown in Fig. 1. A few significant singular values can be identified as a couple which represents the physical multi-bunch oscillations as described in the previous section. The largest two singular values show the dominant multi-bunch oscillation. Figures 2(a) and (d), (b) and (e), (c) and (f) show the time and spatial patterns corresponding to the largest, third, and fourth singular values. The second and fifth time and spatial patterns are similar to those of the first and fourth except for phases. Figure 2(b) shows a drifting of the closed orbit because the spatial pattern represents that common drift for all bunches from Fig. 2(e). The SVD analysis can distinguish physical sources from noise effects. The spatial patterns represent the behavior of each bunch oscillation involved in a bunch train. The bunch oscillations in the head of the bunch train is very small in comparison with the tail as shown in Fig. 2(d). This behavior may show ions are created by each passing bunch,

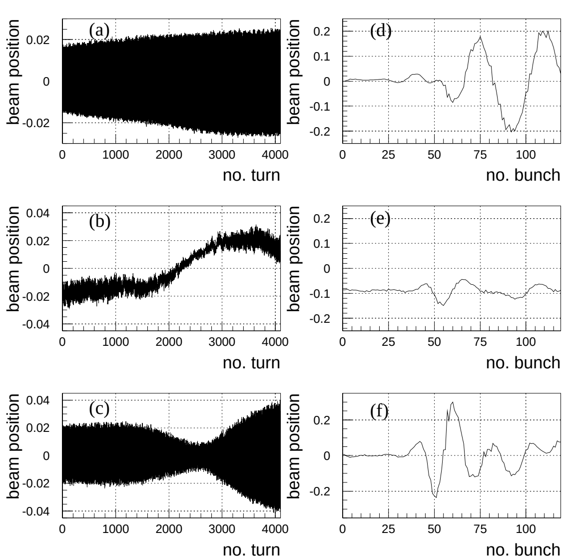

Figure 2: Typical time and spatial patterns correspond to the first(a)(d), third(b)(e), and fourth(c)(f) singular values in Fig. 1. The beam positions are shown in the arbitrary unit.

which leads to a linear increase of the ion density along the bunch train. The created ions should be lost in the gap and not perturb the motion of the head of the next bunch train.

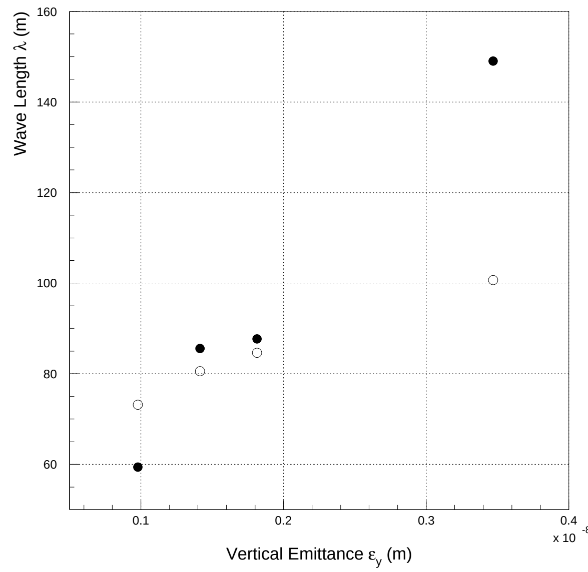

Figure 3: Wave length as a function of the vertical emittance. The closed circle shows measured wave length and the open circle shows calculations with the analytic formula.

A candidate is *Fast Ion Instability (FII)* which is a unique phenomenon observed in multi-bunch electron beams[3, 4]. The position of the n-th bunch after traveling a time t is approximately given by

$$y_n(t) \propto \cos\left(\frac{2\pi}{\lambda} ln - \omega_\beta t\right). \tag{5}$$

The wave length which is due to the ion oscillations under the influence of the electron beam can be written by

$$\lambda = 2\pi\sqrt{\frac{M_{ion} l \Sigma_y(\Sigma_x + \Sigma_y)}{2zNm_e r_e}}, \tag{6}$$

where z is ion charge in unit of e, M_{ion} is ion mass, N is total number of electrons in a bunch, m_e and r_e are electron mass and classical electron radius, Σ_x and Σ_y are the sum in quadruture of the r.m.s. size of the electron beam and the ion cloud in the x and y coordinate, respectively.

In order to make sure that the FII is present, we have measured the wave length of the vertical multi-bunch oscillations with changing the vertical beam size. The vertical beam size was adjusted by the vertical dispersion. The vertical dispersion was controlled with a local bump (sin-like) made at a pair of sextupoles which are connected via -I transformation. Thus, dispersion can be generated without changing the condition of the xy coupling. The wave length was measured from the spatial patterns as well as the conventional harmonic analysis. Figure 3 shows the wave length in a bunch train as a function of the vertical emittance. The wave lengths calculated by the analytic formula(Eq.(6)) are also plotted when CO^+ is assumed as ions and the size of the ion cloud is the same as the electron beam. The electron beam size was measured by a synchrotron radiation (SR) interferometer in the both x and y direction[5]. It is found that the behavior of the measured wave length is similar to the analytic calculation.

We also measured the growth time of the multi-bunch oscillations as shown in Fig. 4. The growth time predicted from the FII can be written by

$$\tau_{FII} = \frac{\gamma\lambda\Sigma_x\Sigma_y}{8\pi c n^2 l z n_{ion} r_e \overline{\beta}}, \tag{7}$$

where γ is the Lorentz factor, $\overline{\beta}$ is the average vertical beta function, and n_{ion} is the number of ions created by an electron bunch per unit orbit length. In practice, the multi-bunch instabilities are suppressed by a large positive chromaticity. The head-tail damping was estimated by $\tau_{HT} = 6.4$ msec from tune shifts depend on the beam current[6]. If we assumed that the damping effect was only the head-tail effect, the growth time of the FII can be extracted by

$$y_n(t) \propto \exp\left(\sqrt{\frac{t}{\tau_{FII}}} - \frac{t}{\tau_{HT}}\right). \tag{8}$$

In the case of the smallest vertical emittance (0.98×10^{-9}m), we found that the growth time was 0.15 msec at the bunch number 100 out of 120 in the first train. If the growth is exponential instead of quasi-exponential, we obtain the growth time of 4.6 msec. Analytically, the growth time is calculated to be 0.48 msec in the above condition with Eq.(7). The number of created ions, n_{ion} is estimated from the relative pressure of the CO^+ measured by a residual gas analyzer.

Figure 5 shows maximum growth-amplitudes of each vertical bunch oscillation for several vertical emittances. We found that the multi-bunch beam instability was stronger as the beam size decreased. This behavior also implies that the observed multi-bunch instability depends on the beam size.

All evidence is consistent with the observed multi-bunch instability as being the Fast Ion Instability.

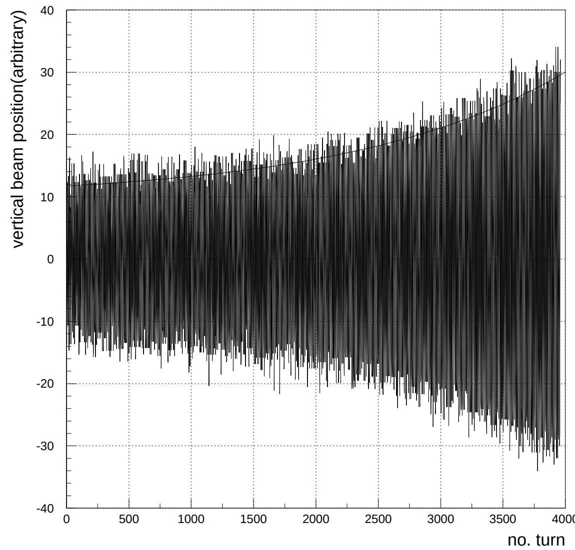

Figure 4: Growth of the bunch oscillation in the case of the smallest emittance(0.98×10^{-9}m). The bunch number is 100 out of 120 in the first train. The solid line shows fitted function for the envelope using Eq.(8).

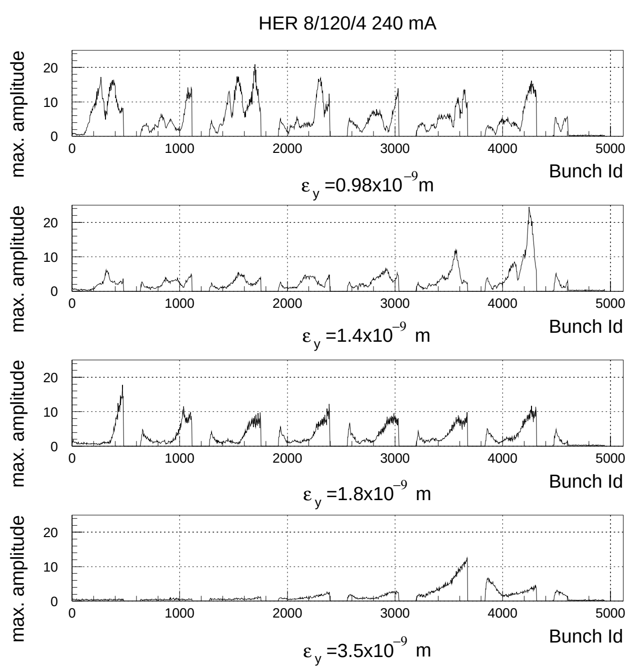

Figure 5: Maximum growth-amplitude (arbitrary) of vertical multi-bunch oscillations for the vertical emittance.

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