STUDY OF HEAD-TAIL EFFECT CAUSED BY ELECTRON CLOUD

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Abstract

In positron or proton storage rings with many closely spaced bunches, an electron cloud can build up in the vacuum chamber due to photoemission or secondary emission. The electron cloud induces a transverse wake force during the beam passage. The medium range wake force (>ns) causes a multi-bunch instability, while the short range wake (<100ps) causes a head-tail effect. In this paper, we study the short range wake force and discuss the head-tail effect caused by the electron cloud.

1 INTRODUCTION

Two-stream instabilities caused by interaction of a positron beam with ionization electrons [1] or with photo-electrons and secondary electrons [2, 3] have been studied previously. In Ref.[3], we discussed that the two-stream instability appears as either strong or regular head-tail instability due to synchrotron oscillation and, possibly, chromaticity. In this report, we characterize the head-tail instability driven by the electron cloud using the concept of the wake force.

A positron beam emits a large number of photon by synchrotron radiation. The number of photons emitted by a positron per revolution is $N_\gamma = 5\pi\alpha\gamma/\sqrt{3}$. In storage rings with GeV energies, the number is several hundreds. The photons generate photoelectrons at the chamber wall, with a typical quantum efficiency of the order of 0.1. An electron cloud is build up in the vacuum chamber during the successive passage of closely spaced bunches. When the beam passes through the electron cloud the electron cloud induces an effective transverse wake force. This wake force causes a multi-bunch dipole mode instability [4], where a variation in the electron-cloud centroid position couples the motion of subsequent bunches. The wake force also has short range component at a frequency of several tens GHz, corresponding to the oscillation frequency of electrons within a bunch. The short range wake force gives rise to a single bunch phenomenon; i.e. a head-tail effect, which is discussed below. Although a single-bunch effect, the phenomenon will occur only in multi-bunch operation, since the electron cloud is built up from synchrotron radiation emitted by the preceding bunches. The head-tail mode of the single-bunch instability will be observed as a beam-size blow up.

As a concrete example, we study the single-bunch photo-electron instability for the Low Energy Ring of the KEK-B factory (KEKB-LER), with parameters as summarized in Table 1. At the beginning of the year 2000, the LER was operated with a beam current of 600 mA stored in 1000 bunches at 8 ns spacing. A blow up of the vertical beam size has been observed already early on during LER commissioning [6]. This blow up is not accompanied by any coherent beam motion, which is suppressed by transverse feedback and chromaticity, and the blow up is seen only in multi-bunch operation with a narrow bunch spacing. The single-bunch two-stream instability provides a plausible explanation of the observed beam blow up.

Table 1: Basic parameters of the KEKB LER

variable	symbol	value
particle type	—	e^+
circumference	L	3016 m
beam energy	E	3.5 GeV
bunch population	N_b	3.3×10^{10}
bunch spacing	t_{sep}	8 ns
rms beam sizes	σ_x	0.42 mm
($\beta_{xy} = 10$m)	σ_y	0.06 mm
bunch length	σ_z	4mm
rms energy spread	σ_E/E	0.0007
mom. comp. factor	α	1.8×10^{-4}
chromaticity	$Q'_{x,y}$	$4/8$
synchrotron tune	Q_s	0.015

We present a two-particle model and a coasting beam model which allow us to estimate the wake effect of the electron cloud.

2 TWO-PARTICLE MODEL

Our picture of the two-stream instability is that the cloud electrons oscillate incoherently at first, but, gradually, they and the positron bunch (the micro-bunches in the simulation) develop a coherent oscillation due to their interaction. After the bunch passage the coherence of the electrons is lost, and on the next revolution the further distorted positron bunch impresses an enhanced coherent motion on the newly formed electron cloud, which in turn increases the oscillation along the bunch.

The force from the electron cloud may be represented by an effective short range wake field with a characteristic frequency $\omega_e^2 = (2\lambda_b r_e c^2)/(\sigma_y(\sigma_x + \sigma_y))$. The strength of the wake force can be obtained by the same method as in Ref. [4]. The order of magnitude of the wake force may also be estimated analytically. For example, considering a flat beam with $\sigma_x \gg \sigma_y$, we decompose the electron cloud into infinitely thin vertical slices, each producing the same vertical electric field, and study a two-particle model with a charge of $N_b e/2$ for both head and tail. We assume that the head particle has a finite length $l_{\rm head} \approx \sqrt{2\pi}\sigma_z/2$, and a uniform charge distribution. The tail particle is considered to be pointlike and to follow immediately after the head. Head and tail are vertically displaced with respect to each other by a small offset $\Delta y \ll \sigma_y$. From the resulting force on the tail we can then estimate the effective wake field. Electrons near the beam are attracted by the field of the head and perform linear or nonlinear oscillations during its passage. Due to the relative displacement of head and tail, these oscillations induce a net electron transfer from below to above the vertical position of the trailing particle. The electron charge transfer is maximum if $\omega_e l_{\rm head}$ is equal to an odd multiple of $\pi/2$, reflecting the effect of linearly oscillating electrons within about $\pm 2\sigma_y$ from the beam. At intermediate times, the net charge transfer amounts to the number of electrons which originally occupy a vertical stripe of thickness $\sim 2\Delta y$, *i.e.*, about twice the displacement.

In this 2-particle model, the integrated wake field per revolution experienced by the tail of the bunch is of the order

$$W_0 \approx 8\pi\rho L/N_b. \tag{1}$$

On each turn the tail particle experiences a deflection of

$$\Delta y'_{\rm tail} = \frac{r_e W_0 N_b}{2\gamma}\,(y_{\rm head} - y_{\rm tail}), \tag{2}$$

where y' denotes the vertical slope of the trajectory. This estimate is valid if the distance between head and tail is large compared with $\sigma_x\sigma_y/(N_b r_e)$, where r_e denotes the classical electron radius. This is usually the case. Unlike an ordinary wake field, the wake W_0 decreases inversely with the population of the bunch considered. However, the population of the previous bunches also enters, indirectly, in the value of ρ, so that for equal bunch populations there is no dependence on N_b. Indeed, assuming that the equilibrium density ρ is equal to the average neutralization density $N_b/(\pi h_x h_y L_{\rm sep})$, where h_x and h_y are the horizontal and vertical chamber half apertures and $L_{\rm sep}$ the bunch spacing (in meters), our wake estimate can be rewritten as $W_0 \approx 8L/(h_x h_y L_{\rm sep})$, which depends only on geometric quantities.

On the other hand, if the bunch length is short compared with $\sigma_x\sigma_y/(N_b r_e)$, so that only electrons in the linear part of the beam field contribute to the charge transfer, the wake field can be estimated as

$$W_0 \approx 4\pi\rho L r_e l_{\rm head}/(\sigma_x\sigma_y). \tag{3}$$

The growth rate for the BBU mode, without synchrotron motion, can be estimated in the two-particle model using the saturated wake field of Eq. (1):

$$\frac{1}{\tau} \approx \frac{2\pi\rho r_e c < \beta_y >}{\gamma} \tag{4}$$

For KEKB parameters the BBU growth time evaluates to about 100 μs, in good agreement with macro-particle simulations [3]. Modifying the theory for the single-bunch instability due to ionization electrons [1] can give an alternative estimate [2].

Inserting our wake field estimate, Eq. (1), into the standard expression for the regular head-tail growth time [7], we estimate the growth rate of the $l = 1$ head-tail mode as

$$\frac{1}{\tau^{(1)}} \approx \frac{64}{3}\frac{\rho < \beta_y > r_e\sigma_z Q'_y}{T_0\alpha\gamma} \tag{5}$$

where α is the momentum compaction factor. For $Q'_y = 8$ this equation predicts a growth time of about 0.5 ms, again in reasonable agreement with simulations [3].

Finally, we can calculate the threshold of the strong head-tail instability for the two-particle model. Following Ref. [7], the threshold is reached when the parameter $N_b r_e |W_0| \beta_y c/(8\gamma L Q_s)$ is equal to 2. This translates into a threshold value for the electron-cloud density of

$$\rho_{\rm thr} = \frac{2\gamma Q_s}{T_0 r_e c \beta_y}, \tag{6}$$

which evaluates to 7×10^{11} m^{-3}, and agrees well with the simulated threshold [3].

3 WAKE FORCE FOR COASTING BEAM MODEL

We next discuss the wake force using a coasting beam model. The beam is assumed to have an uniform charge density in longitudinal direction. The beam is characterized by its vertical coordinate at a longitudinal position $y_b(s, z)$. We consider motion of center of mass of electron cloud.

The equation of motion for the beam and cloud particle are expressed as follows,

$$\frac{d^2 y_b(s,z)}{ds^2} + \left(\frac{\omega_\beta}{c}\right)^2 y_b(s,z) = -\frac{\omega_b}{c}^2 (y_b(s,z) - y_e(s,(s+z)/c)) \tag{7}$$

$$\frac{d^2 y_e(s,t)}{dt^2} = -\omega_e^2(y_e(s,t) - y_b(s, ct-s)) \tag{8}$$

where ω_β is a betatron angular frequency. ω_b and ω_c characterize the linearized force between beam and cloud,

$$\omega_b^2 = \frac{2\lambda_e r_e c^2}{\gamma k \sigma_x \sigma_y} \qquad \omega_e^2 = \frac{2\lambda_b r_e c^2}{k\sigma_x\sigma_y} \tag{9}$$

where λ_e and λ_b are line densities of cloud and beam, and σ_x and σ_y are horizontal and vertical beam sizes, respectively. The parameter k characterizes the coupling between beam and cloud. If cloud is represented by a rigid Gaussian distribution with the same rms size as the beam, we have $k = 2$.

Using the initial condition $y_e(s, -\infty) = 0$, the solution is

$$y_e(s,t) = \omega_e \int_{-\infty}^{t} y_b(s, s-ct')\sin\omega_e(t-t')dt'. \tag{10}$$

Substituting the solution into Eq.(7),

$$\frac{d^2 y_b(s,z)}{ds^2} + \left(\frac{\omega}{c}\right)^2 y_b(s,z) = \\ = \frac{\omega_b^2\omega_e}{c^3}\int_z^\infty y_b(s,z')\sin\frac{\omega_e}{c}(z-z')dz' \tag{11}$$

where $\omega^2 = \omega_\beta^2 + \omega_b^2$. The right-hand side of Eq.(11) can be represented by an effective wake function, which depends only on the longitudinal distance,

$$W_1 = \frac{\gamma \omega_b^2 \omega_e}{\lambda_b r_e c^3} \sin \frac{\omega_e}{c}(z - z'). \qquad (12)$$

In our parameter, the wake force is written as

$$W_1 = 3.6 \times 10^5 \sin \frac{\omega_e}{c}(z - z') \qquad \omega_e = 2\pi \times 29 GHz. \qquad (13)$$

We can estimate the wake force by a simulation. This calculation is performed following the same procedure as was used for studying the multi-bunch electron-cloud instability in Ref. [4]. We consider an electron cloud with a transverse size represented by macro-particles and a micro-bunch train with a very narrow spacing. Note that the micro-bunch train represents a coasting beam. The motion of macro-particles in electron cloud is expressed by

$$\frac{d^2 \boldsymbol{x}_{e,a}}{dt^2} = -\frac{2N_+ r_e c}{N_b} \sum_{i=1}^{N_p} \boldsymbol{F}_G(\boldsymbol{x}_{e,a} - \bar{\boldsymbol{x}}_{p,i}; \boldsymbol{\sigma}) \delta(t - t(s_b)), \qquad (14)$$

where the force $\boldsymbol{F}_G(\boldsymbol{x})$ is expressed by the Bassetti-Erskine formula [5] normalized so that $\boldsymbol{F}_G \to \boldsymbol{x}/|\boldsymbol{x}|^2$ as $\boldsymbol{x} \to \infty$.

When beam passes through the center of cloud, beam does not affect by the cloud and the center of mass of cloud is kept. If a micro-bunch with a small transverse displacement pass through the cloud, the cloud is perturbed and center of mass start to move, and following micro-bunches are affected by the perturbation of cloud. Beam

$$\Delta \bar{\boldsymbol{x}}'_{p,i} = -\frac{2r_e}{\gamma} \sum_{a=1}^{N_e} \boldsymbol{F}_G(\bar{\boldsymbol{x}}(s)_{p,i} - \boldsymbol{x}_{e,a}; \boldsymbol{\sigma}), \qquad (15)$$

The wake force is calculated by the response for a small displacement of a micro-bunch $\bar{\boldsymbol{x}}_{p,i} = \Delta \boldsymbol{x}$.

Figure 2 shows the wake force obtained by the analytic formula and the simulation. The wake force obtained by the simulation is close to sinusoidal function with an angular frequency $\omega = 2\pi \times 35$GHz. The magnitude of the wake force is consistent with Eq.(13).

4 SUMMARY

We discuss the wake force induced by electron cloud. Using a two-particle model, we estimated the growth time of the $l = 1$ weak head-tail mode and the threshold of the strong head-tail instability. In a costing beam model, we calculated the wake force. The two models give the same wake force in the limit of short bunches, *i.e.*, for $\sigma_z < \sigma_x \sigma_y / (N_b r_e)$ (or $\omega_c \sigma_z < c$). For longer bunches they deviate. The wake field of the coasting beam model is the green function wake from a point source displacement, whereas the two-particle model gives the wake field from a displaced head particle which is of finite length (it includes however electrons which are initially at larger amplitudes, whereas the coasting beam model does not). So the two wake fields cannot be exactly the same.

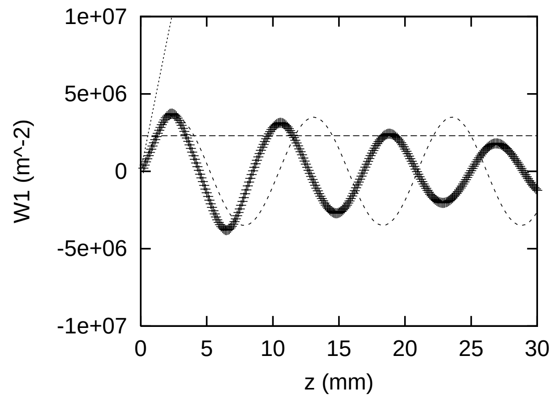

Figure 1: Wake force (W_1) induced by an electron cloud. The beam has a line density of $3.3 \times 10^{10} cm^{-1}$, which is the same as that for KEKB-LER. Two straight lines are short (Eq.(3)) and long range wake (Eq.(1)) obtained by the two particle model. A sinusoidal curve is obtained by coasting model (Eq.(12)). The wake obtained by simulation is plotted by '+'. Micro-bunches are put with a population 10mm^{-1} along longitudinal direction in the simulation.

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