ANALYTICAL FORMULA FOR MAGNETIC FIELD COMPONENTS OF IRON ROAD IN SOLENOID FIELD

Mihaela Dan, Univ. "Babes-Bolaiy", Physics Faculty, R-3400 Cluj-Napoca, Romania,
e-mail: nena@personal.ro
V. Babin, National Institute for Optoelectronics-2000, Bucharest-Magurele, MG-20, Romania
V.I.R. Niculescu, National Institute for Lasers, Plasma and Radiation Phys., Electron Accelerators Laboratory, Bucharest-Magurele, MG-36, Romania, e-mail: nicval@ifin.nipne.ro

Abstract

A system (FEL wiggler) composed of many roads that are placed in an induction magnetic field was studied. In a cylinder coordinates system the total magnetic field in the exterior space of a road was computed. Using Maxwell and Poisson equations analytical expressions for magnetic potential and magnetic field components was obtained. The dependence $r^{-3/2}$ of the magnetic field was also obtained analytically. So the analytical complete magnetic field components were deduced for any angle between the road axis and the induction field direction.

1 INTRODUCTION

The use of free electron lasers (FELs) need the development of different types of wigglers more practical and chiper. One the improvement for solenoid-derived wigglers was given in the paper [1]. The solenoid-derived wigglers is a staggered array of high-permeable materials situated inside the bore of a solenoid. There the approximative formulas for magnetic field of roads placed in a solenoidal magnetic field was given.

2 CALCULATION OF THE MAGNETIC FIELD

We take more general system :

A metalic cylinder with the radius r_0 and the length L_c is placed in an uniform magnetic field $\vec{H}_0$, α_0 is angle between the road axis and the induction field direction (metalic magnetic permeability $\{\mu = \mu_r \mu_0\}$;medium permeability $\{\mu = \mu_0\}$).

We compute the total magnetic field in the cylinder exterior domain $\{r \geq r_0\}$. The cylindrical coordinate system is defined:

$$\Sigma : \{X = X, Y = r_0 \cos\varphi, Z = r_0 \sin\varphi\}$$

The magnetic field intensity in cylindrical coordinates is given by:

$$\vec{H}_0 = \vec{i}\, H_0 \cos\alpha_0 + \vec{e}_r \{H_0 \sin\alpha_0 \cos\varphi\} - \\ -\vec{e}_\varphi \{H_0 \sin\alpha_0 \sin\varphi\}$$

The magnetic field equations [2] are:

$$\vec{\nabla} \times \vec{H} = 0, \vec{\nabla}\vec{B} = 0, \vec{B} = \mu_0 \vec{H} + \vec{M} \quad (1)$$

From the equations (1) we obtained :

$$\vec{H} = -\vec{\nabla}\varphi_m, \vec{\nabla}\vec{H} = -\frac{1}{\mu_0}\vec{\nabla}\vec{M} \quad (2)$$

where $\{\varphi_m\}$ is the magetic potential.

Also from equations (2) we obtained the Poisson equation for $\{\varphi_m\}$:

$$\Delta\varphi_m = \frac{1}{\mu_0}\vec{\nabla}\vec{M}, \vec{M} = \mu_0 \vec{H}(\mu_r - 1) \quad (3)$$

So the magnetic potential [3] is given by:

$$\varphi_{Am} = -H_0[r \sin\alpha_0 \cos\varphi + x\cos\alpha_0] + \rho_1\sqrt{\frac{2}{\pi\kappa r}} * \\ * \sin\left[\kappa r - \frac{\pi}{4} - n\frac{\pi}{2}\right]\cos[n\varphi + \varphi_0]\, e^{-\kappa x} \quad (4)$$

The field components $\{\vec{H}_A\}$ take the form:

$$H_{A_x}(x, r, \varphi) = H_0 \cos\alpha_0 - \rho_1\sqrt{\frac{2}{\pi\kappa r}} * \\ * \sin\left[\kappa r - \frac{\pi}{4} - n\frac{\pi}{2}\right]\cos[n\varphi + \varphi_0]\, e^{-\kappa x}$$

$$H_{Ar}(x, r, \varphi) = H_0 \sin\alpha_0 \cos\varphi - \rho_1\kappa\sqrt{\frac{2}{\pi\kappa r}} * \\ * \left\{\cos\left[\kappa r - \frac{\pi}{4} - n\frac{\pi}{2}\right] - \frac{1}{\kappa r}\sin\left[\kappa r - \frac{\pi}{4} - n\frac{\pi}{2}\right]\right\} * \\ * \cos[n\varphi + \varphi_0]\, e^{-\kappa x}$$

$$H_{A\varphi}(x, r, \varphi) = -H_0 \sin\alpha_0 \sin\varphi + \rho_1\frac{n}{r}\sqrt{\frac{2}{\pi\kappa r}} * \\ * \sin\left[\kappa r - \frac{\pi}{4} - n\frac{\pi}{2}\right]\sin[n\varphi + \varphi_0]\, e^{-\kappa x} \quad (5)$$

The parameters $\{\rho_1, \kappa, n\}$ were computed using the limit conditions. After some evaluations we obtained:

$$\kappa = \frac{1}{r_0}\left\{\left[\frac{\pi}{4} + \arccos\frac{(-1)^{k+1}}{\sqrt{1+\dfrac{(\mu_r-1)^2(\mu_r r_0 I_3 - I_1)^2}{(\kappa r_0)^2\left[4\pi r_0 + (\mu_r - 1)I_1\right]^2}}}\right]\right\} \quad (6)$$

an implicit relation for $\{\kappa\}$.

Computing example: Like in [4] we choose the approximation $\kappa r_0 = \delta\pi ; \delta < 1$. So the total magnetic field intensity in the cylinder exterior $\vec{H}_A(H_{Ax}, H_{Ar}, H_{A\varphi})$ is given by:

$$H_{Ax} = H_0\left[\cos\alpha_0 - \sin\alpha_0\frac{\cos[(2k+1)\varphi + \varphi_0]}{\cos\varphi_0} * G(r,x)\right]$$

$$H_{Ar} = H_0 \sin\alpha_0\left[\cos\varphi + \frac{\cos[(2k+1)\varphi + \varphi_0]}{\cos\varphi_0} * R(r,x)\right]$$

$$H_{A\varphi} = H_0 \sin\alpha_0\left[-\sin\varphi + \frac{\sin[(2k+1)\varphi + \varphi_0]}{\cos\varphi_0} * (2k+1)F(r,x)\right] \quad (7)$$

Where:

$$G(r,x) = \frac{\delta}{r_0}\frac{4\pi r_0 + (\mu_r - 1)I_1}{4\cos\left(\delta\pi - \frac{\pi}{4}\right)}\frac{\cos\left(\pi\delta\frac{r}{r_0} - \frac{\pi}{4}\right)}{\sqrt{\frac{r}{r_0}}} * e^{-\pi\delta\frac{x}{r_0}}$$

$$F(r,x) = \frac{1}{r}\frac{4\pi r_0 + (\mu_r - 1)I_1}{4\cos\left(\delta\pi - \frac{\pi}{4}\right)}\frac{\cos\left(\pi\delta\frac{r}{r_0} - \frac{\pi}{4}\right)}{\sqrt{\frac{r}{r_0}}} * e^{-\pi\delta\frac{x}{r_0}}$$

$$G(r,x) = \left(\frac{r}{r_0}\right)\delta F(r,x)$$

$$R(r,x) = \frac{\delta}{r_0}\frac{4\pi r_0 + (\mu_r - 1)I_1}{4\cos\left(\delta\pi - \frac{\pi}{4}\right)}\left[\frac{\sin\left(\pi\delta\frac{r}{r_0} - \frac{\pi}{4}\right)}{\sqrt{\frac{r}{r_0}}} + \frac{\cos\left(\pi\delta\frac{r}{r_0} - \frac{\pi}{4}\right)}{\pi\delta\left(\frac{r}{r_0}\right)^{3/2}}\right] * e^{-\pi\delta\frac{x}{r_0}} \quad (8)$$

We rewrite the intensity of magnetic field components (7) in rectangular coordinates $\{x, y, z\}$ for $\delta \neq 0$ in the form:

$$H_{Ax} = H_0 \cos\alpha_0,$$

$$H_{Ay} = H_0 \sin\alpha_0 * \left[1 - \rho_0\left(\frac{r_0}{r}\right)^{3/2}\sin\varphi\sin(\varphi + \varphi_0)\right]$$

$$H_{Az} = H_0 \sin\alpha_0 \rho_0\left(\frac{r_0}{r}\right)^{3/2}\cos\varphi\ \sin(\varphi + \varphi_0) \quad (9)$$

where :

$$\rho_0 = \frac{1}{4\pi r_0}\sqrt{(\mu_r - 1)^2 I_2^2 + \left[4\pi r_0 + (\mu_r - 1)I_1\right]^2},$$

$$\varphi_0 = -\arctan\left[\frac{(\mu_r - 1)I_2}{4\pi r_0 + (\mu_r - 1)I_1}\right] \quad (10)$$

In this way the dependence $r^{-3/2}$ of the magnetic field was obtained analytically.

3 CONCLUSIONS

So in a cylinder coordinates system the total magnetic field in the exterior space of a road was computed. Also the explicite magnetic field components in rectangular coordinates were obtained and a computing model for many cylinders was constructed. In this way the analytical complete magnetic field components for any angle between the road axis and the induction field direction were deduced.

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