SPACE CHARGE PROBLEMS IN THE TESLA DAMPING RING

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Abstract

The proposed TESLA linear collider will use long bunch trains. Damping of these bunch trains at once requires a damping ring with a large circumference of 17 km and a moderate energy of 3-5 GeV. This unusual large ratio of circumference to energy leads to a considerable incoherent space charge tune shift, which in turn leads to an increase of the vertical emittance. Possible solutions to reduce the space charge force will be discussed. Their benefits and problems are investigated through numerical simulations.

1 INTRODUCTION

The TESLA linear collider will use super-conducting $1.3\,\mathrm{GHz}$ cavities to accelerate electrons and positrons up to an energy of $400\,\mathrm{GeV}$ per beam. The rf-pulse length is $\approx 1\,\mathrm{ms}$ allowing to accelerate a long bunch train with many well separated bunches [1]. The present reference design assumes a bunch train of 2820 bunches spaced by $337\,\mathrm{ns}$ [2]. This bunch train will be stored and damped in the damping ring in a compressed mode where each bunch has to be injected and ejected separately. The damping ring bunch spacing of $\approx 20\,\mathrm{ns}$ is then given by the achievable kicker rise/fall times and the damping ring length has to be $\approx 17\,\mathrm{km}$. Most part of the ring will consist out of two straight sections built into the main linac tunnel while short arcs provide the return connections. The main parameters of the damping ring are summarized in table 1 and further information can be found in [3].

Table 1: TESLA damping ring parameters

Circumference	L	$17\,\mathrm{km}$
Arc length	L_{arc}	$2\,\mathrm{km}$
Straight length	L_{str}	$15\,\mathrm{km}$
Energy	E_0	$5.0\,\mathrm{GeV}$
Hor. extracted emittance	$\gamma\varepsilon_x$	$9\times10^{-6}\,\mathrm{m}$
Ver. extracted emittance	$\gamma\varepsilon_y$	$2\times10^{-8}\,\mathrm{m}$
Transverse damping time	$\tau_{D;x,y}$	$28\times10^{-3}\,\mathrm{s}$
Equilibrium bunch length	σ_z	$6\,\mathrm{mm}$
Equilibrium momentum spread	$\frac{\sigma_p}{P_0}$	$0.13\,\%$
Longitudinal damping time	$\tau_{D;s}$	$14\times10^{-3}\,\mathrm{s}$
Particles per bunch	N_e	2×10^{10}

2 INCOHERENT SPACE CHARGE TUNE SHIFT

Assuming a Gaussian distribution of particles within the bunch the direct space charge force is in linear approximation ($x < \sigma_x, y < \sigma_y$):

$$F_{sc;x|y}(x|y,z) \approx -\frac{2r_eN_e e^{\frac{-z^2}{2\sigma_z^2}}}{\sqrt{2\pi}\gamma^3\sigma_{x|y}(\sigma_x+\sigma_y)\sigma_z}x|y \qquad (1)$$

with r_e the classical electron radius, γ the Lorentz factor, $\sigma_{x|y}$ the horizontal/vertical beam size, σ_z the longitudinal beam size, and N_e the number of electrons per bunch. The resulting space charge tune shift follows from the integral $\Delta Q_{sc} = \frac{1}{4\pi}\oint \beta F_{sc}$ and becomes for flat beams ($\varepsilon_x \gg \varepsilon_y$):

$$\Delta Q_{sc;x|y} \approx \frac{Lr_eN_e e^{\frac{-z^2}{2\sigma_z^2}}}{(2\pi)^{\frac{3}{2}}\gamma^2\sqrt{\varepsilon_{x,n}\varepsilon_{x|y,n}}\sigma_z} \qquad (2)$$

where L denotes the ring circumference and $\varepsilon_{x,n}, \varepsilon_{y,n}$ the normalized emittances. For larger amplitudes the space charge force becomes non-linear and can be calculated in terms of the complex error function.

The space charge force depends on the longitudinal position z of the particle within the bunch. A particle performing synchrotron oscillations will therefore experience a tune modulation with twice the synchrotron frequency f_s. For a flat beam in which the vertical space charge force dominates this leads to the following extended resonance condition:

$$nQ_x + m(Q_y \pm 2Q_s) = l \qquad (3)$$

with $n, m, l = \mathrm{integer}$.

With the nominal TESLA damping ring parameters the incoherent vertical space charge tune shift before ejection (when the beam is smallest) becomes $\Delta Q_{sc;y} \approx 0.23$. The effect of this tune shift has been simulated through tracking calculations. The computer code MAD [4] has been used with an extension simulating the non-linear space charge kick at each element in the ring [5]. The space charge force is calculated based on the unperturbed transversal dimensions of the beam. Particles are launched with an initial amplitude of $1\,\sigma$ in the transverse planes and various initial energy offsets. The particles are followed over 512 turns (approximately one transverse damping time) and their positions are recorded each turn. The magnets in the accelerator model are misaligned to give a residual dispersion and emittance coupling leading to the design value of the vertical emittance. The result is given as the average single particle emittance normalized with the initial particle emittance, indicating a beam blow-up. Figure 1 shows the average particle emittance for the cases with and without space charge forces. Figure 2 shows the corresponding tunes. One can see the behavior of the vertical tune with its maximum tune shift for a particle in the longitudinal bunch center and decreasing tune shifts for a particle further away from the longitudinal center. Note that a particle further away from the center will experience tune oscillations with larger amplitudes at twice the synchrotron frequency. Figure 1 shows a large beam blow-up for certain initial longitudinal positions, corresponding to resonance crossings especially of the $Q_x - 2Q_y$ resonance. To avoid third order resonances and their sidebands at twice the synchrotron frequency the tune shift has to be restricted to values of $\Delta Q_{sc;y} < 0.1$.

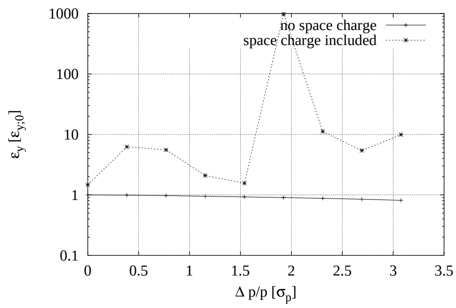

Figure 1: Increase of vertical beam size for various initial energy offsets in the case of no space charge forces (solid line) and with space charge forces included (dashed line). The linear unperturbed tune was $Q_x = 72.32$, $Q_y = 39.30$.

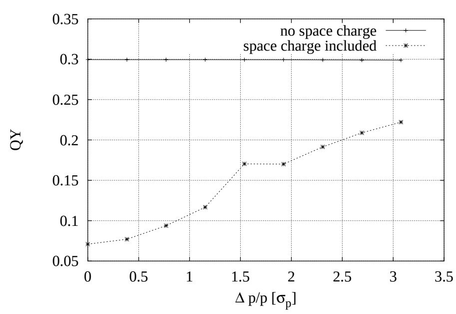

Figure 2: Particle tune calculated from Fourier transformation of turn-by-turn data for the case with no space charge forces (solid line) and with space charge forces included (dashed line).

3 REDUCTION OF THE INCOHERENT SPACE CHARGE TUNE SHIFT

3.1 Decrease of the Ring Circumference

Decreasing the ring circumference is only possible if the kicker bandwidth can be increased. We feel that the kicker system for the TESLA damping ring is already very demanding, requiring a stable kicker pulse to be switched with a raise/fall time of $20\,\mathrm{ns}$ [6]. For this reason we try to find alternative means to reduce the incoherent space charge tune shift.

3.2 Energy Increase

An energy increase is the most straight forward way to reduce ΔQ_{sc} as it scales with $1/\gamma^2$. On the other hand the normalized horizontal emittance of a storage ring scales as γ^3 and in the case of a wiggler dominated damping ring the emittance contribution of the damping ring arcs goes even as γ^6. To counteract this strong energy dependence one can decrease the bending angle θ_C per cell, with the emittance scaling like θ_C^3. The TESLA damping ring arc consists of so-called minimum emittance cells tuned to an emittance four times higher than the theoretical achievable one in favor of improved non-linear properties of the lattice. The momentum compaction factor α_c of these lattice scales approximately with $l_C\theta_C$, with l_C the cell length. The bunch length will thus decrease linearly with decreasing bending angle if no other actions are taken. These leads to a rather weak dependence of ΔQ_{sc} with γ.

Nevertheless the energy of the TESLA damping ring has been increased from its originally proposed value of $3.2\,\mathrm{GeV}$ to $5\,\mathrm{GeV}$. Together with a change of the lattice and an increase of the cell length l_C, $\Delta Q_{sc;y}$ is reduced from 0.45 to 0.23. Due to the above mentioned unfavorable scalings further reductions along this route seem not be feasible.

3.3 Local Beam Size Increase

The long straight sections of the damping ring consist of a FODO structure with a cell length of $100\,\mathrm{m}$. These sections provide most of the space charge defocusing due to their length. An increase of the beam sizes in the straight sections will reduce the space charge forces [7]. There are two possibilities to increase the vertical beam size in the arcs: introduction of vertical dispersion and local coupling of the horizontal and vertical betatron motion.

To increase the vertical beam size by a factor of 10 one would need an average vertical dispersion of $\approx 0.13\,\mathrm{m}$. Vertical dispersion in the long straight sections would in turn have a negative influence on the intra-beam scattering growth rate. We have therefore ruled out this solution for the time being.

If one assumes full coupling in the damping ring straight section ($\varepsilon_{straight;x} = \varepsilon_{straight;y} = \frac{\varepsilon_x}{2}$) the vertical space charge tune shift will be decreased by the ratio of the length of the uncoupled section to the circumference of the ring, yielding in this case a reduction by approximately a factor of 6 (this takes into account the $2\,\mathrm{km}$ of arcs and the wiggler sections, matching etc., yielding approximately $14\,\mathrm{km}$ length available for full coupling).

An elegant way to achieve full coupling has been pointed out recently [8]. By means of a skew quadrupole triplet the horizontal distribution is transformed in a vortex distribution with the particles rotating in the x-y plane. The resulting vertical and horizontal beam sizes are equal and $\sqrt{2}$ time smaller than the initial horizontal beam size. The rotation is taken out of the beam with a similar triplet with the only condition that the phase advances between the two insertions have to be equal.

Figure 3 shows the vertical amplitude growth for the cases with and without the coupling insertion, while figure 4 shows the respective behavior of the vertical tune. The space charge tune shift is reduced by a factor of 6, as predicted. One can still observe a growth of the vertical amplitude when crossing sidebands of third order coupling resonances. A working point which avoids these resonances shows better behavior (see figure 5). This working point should allow to decrease the horizontal emittance even further up to an incoherent space charge tune shift of 0.1 . Results for a beam with 10 times smaller emittance are displayed in figure 5. An vertical emittance increase by a factor of 10 is still observed. In case of full local coupling the horizontal and vertical space charge forces become equal, leading to similar tune modulation in both planes. This will in turn change the resonance condition eq. 3 by adding a term $\pm 2Q_s$ to the horizontal tune. A detailed analysis of excited resonances and dedicated correction of some of them can lead to further reduction of the vertical emittance increase.

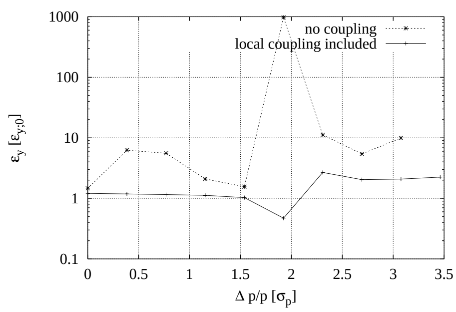

Figure 3: Increase of vertical beam size for various initial energy offsets in the case of no insertion (dashed line) and with coupling insertion included (solid line). The linear unperturbed tune was $Q_x = 72.32$, $Q_y = 39.30$.

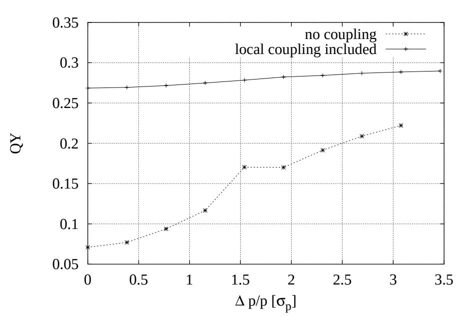

Figure 4: Particle tune calculated from Fourier transformation of turn-by-turn data for the case with no insertion (dashed line) and with coupling insertion included (solid line).

The tolerances to achieve the desired small vertical emittance do not change in presence of the coupling insertion.

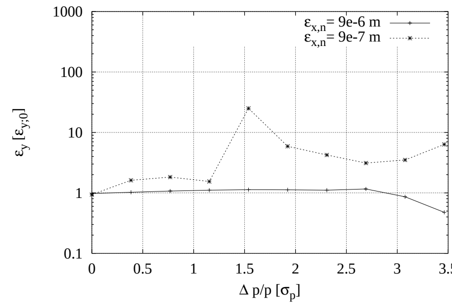

Figure 5: Increase of vertical beam size for various initial energy offsets including space charge and local coupling with the nominal horizontal emittance of $9 \times 10^{-6}\,\mathrm{m}$ (solid line) and with a smaller horizontal emittance of $9 \times 10^{-7}\,\mathrm{m}$ (dashed line). The linear unperturbed tune was $Q_x = 72.32$, $Q_y = 39.27$.

The additional requirement of identical phase advances can be relaxed with additional skew quadrupole circuits and empirical tuning of the coupling insertions.

4 CONCLUSION

The effects of the direct space charge forces on the vertical beam size of the TESLA damping ring have been investigated. An increase in energy to $5\,\mathrm{GeV}$ and the local increase of the vertical beam size in the long straight sections of the ring have led to a reduction of $\Delta Q_{sc;y}$ from 0.45 to 0.04 leaving enough safety margin in the tune space for a reliable operation of the damping ring. A further increase of the bunch current or decrease of the beam sizes seems possible as long as the condition $\Delta Q_{sc;y} < 0.1$ is fulfilled.

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