THEORETICAL VERIFICATION OF COULOMB STRINGS OF IONS

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Abstract

We discuss and verify theoretically two effects which confirm that in heavy ion rings such low temperatures have been attained that the ions cannot pass each other but rather are aligned in the form of strings:
(1) At very low density with average particle distances of the order of centimeters and very low longitudinal temperature there exists an anomalous longitudinal temperature reduction of electron cooled ions, i.e. a jump in the momentum spread to very low values. Intrabeam scattering is completely inhibited.
(2) In the experiments of Schottky mass measurements of stored and cooled projectile fragments there exist correlations between two nearby isobars which can drive their frequencies to the same value. I.e. if two independent strings come close to each other in the horizontal direction they may lock into a common Coulomb string.
These effects are explained by calculations of the reflection probability of two charged ions under the given longitutinal and transverse thermal velocities. The probabilities rise sharply when the average particle distances are in the vicinity of the experimental values. Furthermore, two nearby strings of charged ions lock into a common string if the average thermal radii overlap.
It is also shown that under the given experimental conditions in the NAP-M experiment of 1980 and in the recent laser cooling experiments at the Heidelberg TSR linear Coulomb order could not have been reached.

1 INTRODUCTION

Schiffer and Kienle [1] have speculated on the possible existence of ordered structures in cold ion beams. However, a storage ring is far from being ideal in the sense that the ideal constant radial focussing is only approximately achieved and that it has free drift sections and bending magnets. They create strong shearing forces in the crystal which break it and which increase the temperature irreversibly. Computer simulations with realistic bending and focussing-defocussing [2], however, indicate that stable three dimensional structures indeed might be stable under special circumstances.

On the other hand, if all particles run on or close to the central orbit bending forces cannot build up. One therefore expects that a linear string of particles can survive in a storage ring more easily provided the longitudinal and transverse temperatures can be maintained low enough.

By kinematical reasons longitudinal electron cooling is more effective than the transverse one. This manifests itself also in a lower ion temperature in the beam direction as compared to the transverse one.

2 ESR AND SIS EXPERIMENTS

In 1996, Steck *et al.* [3] reported on measurements with very low density and extremely electron cooled heavy ions in the Experimental Storage Ring (ESR) of GSI. By Schottky noise measurements they found a sharp drop of the longitudinal momentum spread $\delta p/p$ by an order of magnitude from 5×10^{-6} down to 5×10^{-7} if the particle number decayed due to radiative electron capture from 10^3 down to 3 in the ring of 108 m circumference. Thus, arranged in linear strings the average distances between the ions would be between 10 cm and 33 m. Due to machine limitations $\delta p/p$ could not fall below this lower value. The typical example U^{92+} at 360 MeV/u is shown in Fig. 1.

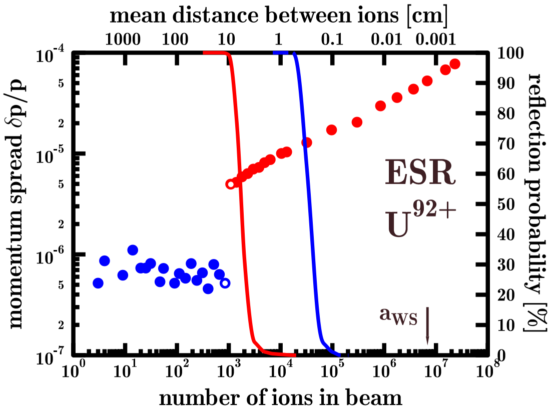

Figure 1: Exp. momentum spreads and calculated reflection probabilities *vs.* number of stored ions in the ESR. for electron cooled U^{92+} ions at 360 MeV/u (after refs. [3,4])

The beam radius for the heavy beams could be determined to about 30 μm and, hence, the transverse temperature was limited to about 1.5 eV. This anomaly resembles a strong suppression of intrabeam scattering below a certain threshold. Since heating of the beam is caused by intrabeam scattering, also heating is strongly inhibited, thus reaching the very low $\delta p/p \approx 5\times10^{-7}$. The data for $C^{6+}, Ne^{10+}, Ar^{18+}, Ti^{22+}, Ni^{28+}, Kr^{36+} Xe^{54+}$ and Au^{79+} exhibit the same feature.

Very similar results have been found with experiments at the synchrotron SIS with partially stripped ions and at low energy. The data for U^{73+} at 11.4 MeV/u is shown in Fig. 2. Here due to stronger focusing with the tunes $Q_{x,y} = 3.3, 4.3$ one order of magnitude larger momentum spreads $\delta p/p \approx 5\times10^{-6}$ suffice as compared to the ESR.

3 STRINGS IN ESR AND SIS

It has been speculated that the final beam structures might be the storage ring analogues of Coulomb crystals as they were calculated in ref. [5]. Here we confirm with the methods applied in ref [4] that indeed the beams resemble strings with particles which move slowly in the beam direction but, however, cannot pass at each other any more. This type of order of a liquid caused by the nearest neighbours only we call Coulomb order in contrast to a Coulomb crystal which is generated by long range Coulomb interaction over many neighbours.

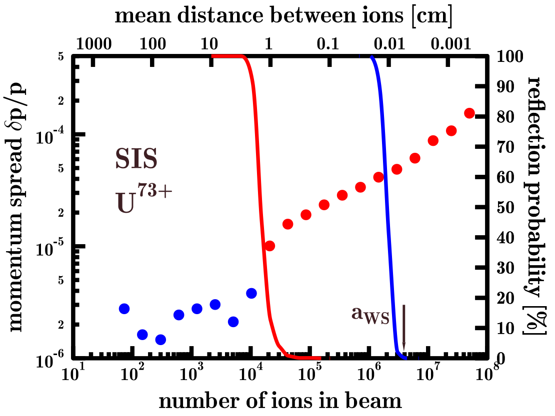

Figure 2: As Fig. 1 but experiment in SIS

In order to explain this effect we perform classical Monte-Carlo trajectory calculations of two charged particles heading at each other with constant focusing with the corresponding betatron frequencies and calculate the probability of these two particles being reflected at each other. It is sufficient to consider the interaction of two particles only since their mutual Coulomb repulsion acts only considerably at near distance of the order of tens of micrometers. To have a constant beam radius for all masses the experimental transverse kinetic energy is distributed among the two transverse degrees of freedom according to a Boltzmann distribution in harmonic potentials with equal betatron frequencies $\omega_\beta = 2\pi Q\beta c/L$, where βc is the beam velocity, Q=2.3 is the average tune, and L is the circumference of the ring. The longitudinal kinetic energy is obtained from $M(c\beta\ \delta p/p)^2/(8\ln 2)$, where M is the mass, see ref. [3].

In order to systematize the calculations, three dimensionless parameters are introduced: The relative transverse, $\Theta_{\rm trans}$ and longitudinal, $\Theta_{\rm long}$, kinetic energies measured in units of the mutual Coulomb energy of two particles at the average distance d, $e_{\rm C} = q^2\gamma/d$, where q is the charge and γ is the relativistic parameter. These relative temperatures, thus, are the reciprocal gamma parameters in Wigner crystal theory; i.e. a one-component plasma is in the gaseous state for $\Gamma \ll 1$, in the liquid state for $1 < \Gamma < 100$, and in the crystalline state for $\Gamma \geq 170$. Note, however, that here Γ does not play a decisive role since distances involved are much larger than the Wigner-Seitz radius. Furthermore, the linear string density $\lambda = a_{\rm WS}/d$ is the axial number of particles within a Wigner-Seitz radius $a_{\rm WS} = \left(3q^2/2M\omega_\beta^2\right)^{1/3}$. Note that at zero temperature $\lambda = 0.709$ is the limiting value for a Coulomb string turning into a zigzag and $\lambda \approx 4$ would give a 3D structure [5].

As can be seen from Fig. 3, for given distance the reflection probability varies very slowly with $\Theta_{\rm trans}$, i.e. it goes from 10% to 90% about within a factor 100 in $\Theta_{\rm trans}$, but more rapidly, with a factor of 5 only, in $\Theta_{\rm long}$. As a rule of the thumb $\Theta_{\rm long}\Theta_{\rm trans}^{1/3}$ stays constant for given distance and fixed reflection probability. In the analysis of the experiments, hence, the results are little sensitive to the assumed transverse temperature of $7.5\times$ A meV.

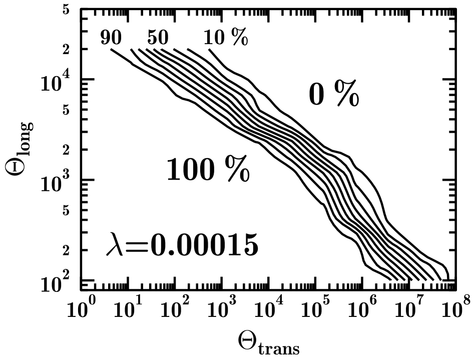

Figure 3: Contour plot of the calculated reflection probabilities *vs.* relative transverse and longitudinal temperature at fixed density $\lambda = 0.00015$

With the help of these tools the ESR and SIS experiments were analysed with the results shown in Figs. 1, 2. The reflection probability rises sharply in the vicinity of the last upper data point, thus indicating that for larger particle distances the ions cannot pass each other any more (left curves). Also shown are the reflection probabilities for the first lower data point (right line). They rise at much smaller interparticle distances which could be achieved if there were no mechanism of heating in the lattice.

4 LOCKED STRINGS IN THE ESR

The ESR in connection with the fragment separator (FRS) can be run as a high precision mass spectrometer for relativistic exotic nuclei [6]. Less than 5000 isotopes in the region $57 \leq Z \leq 84$ are produced by projectile fragmentation of 930 MeV/u bismuth ions, separated in-flight by the FRS and stored and cooled in the ESR down to a velocity spread of $\delta v/v \approx 7 \times 10^{-7}$. The mass m is obtained from the Schottky frequency f via the relation:

$$\frac{\delta f}{f} = \eta\frac{\delta p}{p} = \eta\gamma^2\frac{\delta v}{v} = -\alpha\frac{\delta(m/q)}{(m/q)}$$

Here $\eta = 0.39$, $\gamma = 1.37$, $\alpha = 0.14$ are machine and beam parameters. If the signal is mixed with 30 MHz the thermal frequency spread is just $\delta f_{\rm th}$ ≈15 Hz. The masses of most of the isotopes are already known. Nevertheless, there are characteristic deviations in the vicinity of peaks which are close in frequency, namely that isotopes with slightly heavier (lighter) masses, i.e. slightly smaller (larger) frequencies, are systematically shifted to higher (lower) frequencies, see Fig. 4.

This effect is explained by a simple model as follows: Due to the average dispersion function of the ring, $\bar{D}$=3 m two isotopes with slightly different masses δm and, hence, different frequencies δf run on different horizontal trajectories with different circumferences δC and, hence, on different radii δr, viz. $\delta r = \bar{D}\delta f/\eta f$. This gives 4 μ m for $\delta f_{\rm th}$ and 25 μ m for δf =90 Hz. Since the momentum spread and number of particles are as in sections 2,3, each set of isotopes forms a separate Coulomb string. These strings are independent of each other if their transverse radii do not overlap. If they come closer they lock into the same frequency, i.e. a higher frequency is shifted to a lower one and *vice versa*. With the assumption that the probability of lock is proportional to the overlap area of the two transverse cross sections and folding it with the thermal radius gives the theoretical curve of Fig. 4. It reproduces nicely the experimental deviations.

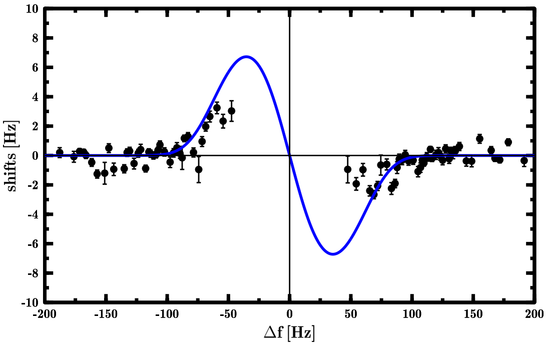

Figure 4: Average deviations of the frequency shift *vs.* the distance to the nearest neighbouring frequency (after [6]).

5 OTHER EXPERIMENTS

Finally, we analyse the cooling experiment with protons in the then existing Novosibirsk NAP-M storage ring [7] and the laser cooling experiments in der Heidelberg TSR storage ring [9]. The authors of the NAP-M experiment suggested that order has been reached based on the fact that if the proton current fell below 10 μA the noise power dropped to unmeasurable levels and thereafter stayed constant, see Fig. 5.

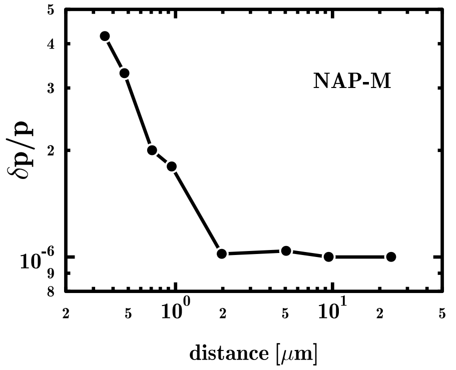

Figure 5: Momentum spread *vs.* longitudinal distance between protons of the NAP-M experiment (after ref. [7])

With the given data of circumference 47.25 m, average tune 1.29, energy 65 MeV, the number of particles in the ring for 10 μA current was $N = 2.5\times10^7$ from the relation $I_{\rm p} = eNf_{\rm rev}$, where $f_{\rm rev}$ is the revolution frequency. An average transverse kinetic energy of 25 meV was derived from the measured beam radius of 100 μm and an average longitudinal kinetic energy of 10^{-4} eV was obtained from Schottky noise measurements. ¿From this one gets the momentum spread of Fig. 5 with a critical $\delta p/p = 10^{-6}$. The linear density is $\lambda \approx 4$ indicating that the system is no longer in the linear regime and the average *axial* distance is 2 μm, much smaller than the Wigner-Seitz radius of 8 μm. The average *spatial* particle distance is about 40 μm.

Full molecular dynamics calculations with 1000 particles with periodic boundary conditions as in ref. [8] reveal that the collision time and transverse period are about the same. Our predictions, on the other hand, yield Coulomb ordered strings for an interparticle distance of 0.2 cm, i.e. proton currents below 10 pA.

Similarly, Eisenbarth *et al.* [9] recently also observed a sharp drop in the longitudinal temperature from 20 K to 0.2 K if the number of laser cooled $\rm Be^{1+}$ ions with energy 7.3 MeV decayed to about 10^6 in the Heidelberg TSR storage ring, see Fig. 6. Here the beam is bunched on the third harmonic with a filling factor of about 1:10. With the circumference of 55.4 m this gives an average particle spacing of about 5 μm within the bunches. This, in turn, again yields $\lambda \approx 4$, far out the string region. Hence no string like order can be expected.

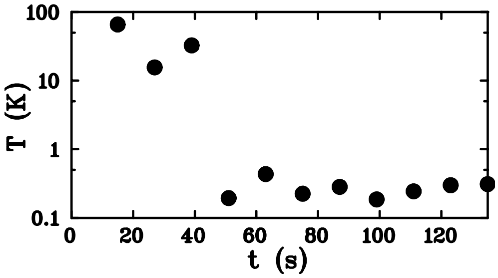

Figure 6: Temporal development of the observed longitudinal temperature of the TSR experiment (after ref. [9]. The transition ocurs at 10^6 ions in the ring.

6 SUMMARY

In summary, we have shown that with the ESR and SIS experiments for the first time Coulomb order has been established in heavy ion storage rings. This order is of liquid type where the particles still move slowly against each other but cannot pass any more. Also, the mass measurements in the ESR establish the existence of at least two interlocked strings.

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