PROPERTIES OF SURFACE ROUGHNESS WAKE FIELDS*

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Abstract

The surface roughness of the vacuum chamber, that is caused by the production process, gives rise to wake fields while the bunch passes through. They mainly consist of the fundamental tube mode, which runs synchronously with the beam, because its phase velocity is decreased to the speed of light by the rough boundary. The wavelength of the rough tube mode is proportional to the square root of the tube radius and the roughness depth. For usual beam pipes the frequency of this mode is in the THz regime. Longitudinal as well as transverse wake fields have to be expected, They will strongly affect the beam dynamics in Linear Colliders and Free Electron Lasers(FEL).

1 INTRODUCTION

In the design of the new 4th generation light sources very short bunches bring up an additional source of energy spread, that threatens the SASE-FEL process.

The beam samples the surface of the tube and excites wake fields. As a consequence of the modification of the wave guide boundary, the phase velocity of the fundamental mode is decreased to the speed of light. The *rough tube mode* now runs synchronously with the beam. The effect of the surface roughness can be described very comprehensively if the roughness is replaced by a thin dielectric layer [1]. In addition to the increase of energy spread the transverse effect has to be taken into account[2].

A decrease of the undulator leads also to reduction of costs of the undulators. At the same time the surface roughness wake field effect is increasing. But it is not the only limiting effect, also resistive wall wake fields have to be considered [3].

2 THE MODEL OF THE SURFACE ROUGHNESS WAKE FIELDS

In the dielectric layer model one single mode is excited, that runs synchronously with the beam. Its wave number in a circular tube with radius a and roughness depth δ reads

$$k_0^2 = f \cdot \frac{4}{a\delta}, \tag{1}$$

where all the details of the roughness are summarized in the form factor f. The wake function is harmonic

$$w_0^{\|}(s) = \widehat{W}_c \cos(k_0 s) \tag{2}$$

where $\widehat{W}_c = Z_0 c/(\pi a^2)$ is the wake amplitude, Z_0 the impedance of free space and c the speed of light.

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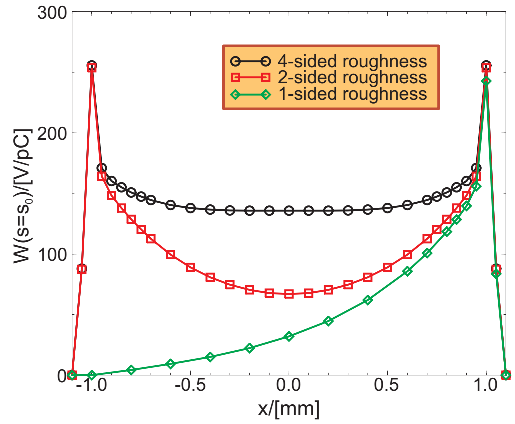

Figure 1: Transverse field profile of the wake potential of a bunch in the center of a rectangular waveguide due to surface roughness

The field of the rough tube mode excited by a bunch moving in the center of the wave guide is constant along the cross section of the wave guide (circles on Fig. 1). It increases rapidly at the boundaries to the surface structure and then drops to zero.

It is useful to introduce a normalized loss factor $\mathrm{H}(k_0\sigma_z)$ and energy spread $\Delta(k_0\sigma_z)$ so that the absolute loss factor $\langle W\rangle$ and energy spread ΔE can be calculated as

$$\langle W\rangle = W_{0c}Q \cdot \mathrm{H}(k_0\sigma_z), \quad \Delta E = W_{0c}Q \cdot \Delta(k_0\sigma_z) \tag{3}$$

where Q is the bunch charge [3] and $W_{0c} = \widehat{W}_c/2$ the maximum loss factor in a circular waveguide.

Gaussian Bunches The loss factor and energy spread of a Gaussian bunch to the rough tube mode are already discussed in [3, 5]. They both depend on the product of wave number and bunch length $k_0\sigma_z$.

Rectangular Bunches In case of a rectangular bunch, the wake potential can be calculated directly in the bunch region $|s| \leq \sqrt{(3)}\sigma_z$

$$W(s) = -\frac{\widehat{W}_c}{2\sqrt{3}k_0\sigma_z} \sin\left(k_0\left(s + \sqrt{3}\sigma_z\right)\right). \tag{4}$$

The normalized lossfactor as well as the normalized energy spread can be derived directly from the wake potential (Eq. 4) [4].

Parabolic Bunches The distribution function of a parabolic bunch is chosen dependent on the RMS value σ_z. The wake potential in the bunch region $|s| \leq \sqrt{(5)}\sigma_z$ can be expressed analytically by

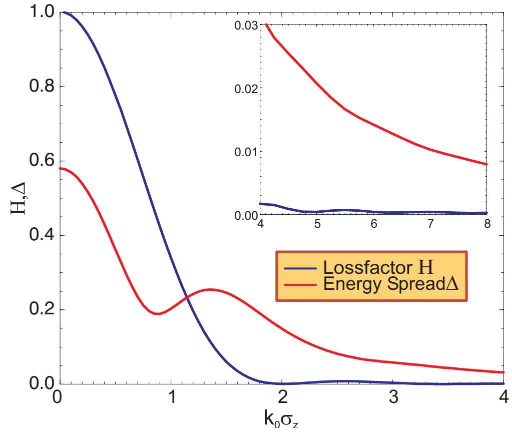

Figure 2: Normalized loss factor H and energy spread Δ per unit length of a parabolic bunch due to the rough tube mode. k_0 is the wave number of the rough tube mode and σ_z the bunch length.

$$W(s) = -\frac{3\widehat{W}_c}{10(k_0\sigma_z)^2} \cdot \cdot \left[\frac{s}{\sqrt{5}\sigma_z} + \cos\left(k_0\left(s+\sqrt{5}\sigma_z\right)\right) - -\frac{1}{\sqrt{5}k_0\sigma_z} \sin\left(k_0\left(s+\sqrt{5}\sigma_z\right)\right)\right]. \quad (5)$$

The loss factor and energy spread can again be calculated directly from Eq.5.

2.1 *High Frequency Approximation*

For a given bunchlength σ_z the term $k_0\sigma_z$ is increased if either the radius a or the roughness depth δ is decreased (Eq. 1). Depending on the radius, the roughness depth and the bunch length the frequency of the rough tube mode may become large in comparison to the frequencies in the bunch spectrum ($k_0\sigma_s \gg 1$). If the spectrum of the Bunch is limited to wave numbers $k_b < k_0$ the loss factor of the bunch is zero. The wake potential becomes purely inductive, i.e. proportional to the derivative of the (smooth) charge distribution

$$W(s) = \widehat{W}_c \int_{-\infty}^{\infty} \lambda(s'-s)\cos(k_0 s')ds' \approx -\frac{\widehat{W}_c}{k_0^2}\frac{\partial}{\partial s}\lambda(s). \quad (6)$$

Gaussian Bunches In case of a Gaussian Bunch the high frequency approximation agrees very well to the results of the full description if $k_0\sigma_z > 6$. The normalized energy spread is calculated as

$$\Delta_G = \frac{2}{(k_0\sigma_z)^2}\sqrt{\frac{1}{\pi 6\sqrt{3}}}. \quad (7)$$

Rectangular Bunch The expression for a rectangular bunch can be derived from the analytical wake potential (Eq. 4)

$$\Delta_R = \frac{1}{\sqrt{6}k_0\sigma_z} \quad (8)$$

Parabolic Bunch Using Eq. 6 the wake potential of a parabolic Bunch is replaced by a straight line. The high frequency approximation for the energy spread gives

$$\Delta_P = \frac{2}{(k_0\sigma_z)^2}\frac{3}{10\sqrt{5}} \quad (9)$$

The complete energy spread can be derived by evaluation of the wake potential (Eq. 5)

$$\Delta_P = \frac{2}{(k_0\sigma_z)^2}\frac{3\sqrt{7}}{10\sqrt{10}} \quad (10)$$

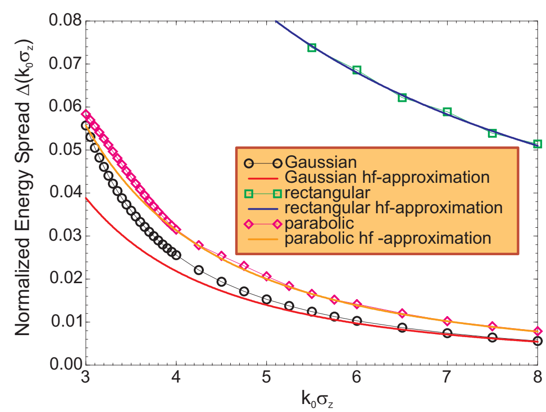

Figure 3: Normalized energy spread of bunches with 0 shapes due to the surface roughness in a tube (the plotting symbols) and the approximations for $k_0\sigma_z \gg 1$ (the continuous lines)

Comparison The Gaussian and the parabolic bunch experience nearly the same energy spread. If only the linear part is taken the parabolic spread is even lower ($\Delta_P/\Delta_G \approx 0.69$), otherwise higher($\Delta_P/\Delta_G \approx 1.43$). The energy spread of the rectangular bunch drops only proportional to $1/k_0\sigma_z$ and is much higher hence. The approximations show very good agreement to the computed values of the energy spread (Fig. 3).

3 THREE DIMENSIONAL SIMULATION

Three dimensional simulations are performed to demonstrate the validity of this model [5]. Here two more aspects are studied.

3.1 *Waveguide Aperture*

The wave number of the rough tube mode and the maximum loss factor in a rectangular waveguide is dependent

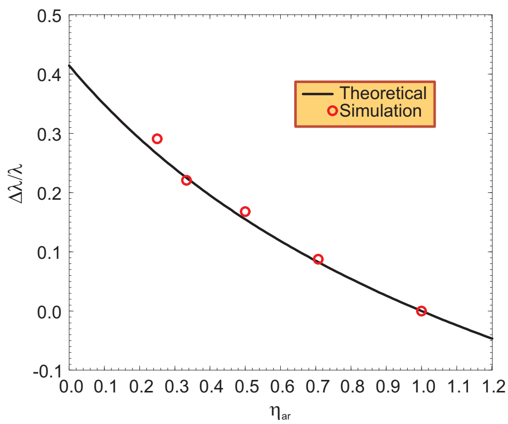

Figure 4: Relative deviation of the wavelength of the rough tube mode in a rectangular waveguide from the quadratic case $A = B$ dependent on the aspect ratio η_{ar}.

on the the aspect ratio of the side lengths $\eta_{ar} = A/B$ [5]

$$k_0^2 = f\frac{4}{A\delta}\left(1+\eta_{ar}\right), \quad W_{0r} = \frac{Z_0 c}{2AB}. \tag{11}$$

The relative deviation of the wavelength λ of a waveguide with aspect ratio η_{ar} from that of a quadratic waveguide with the same base side length A is

$$\frac{\Delta\lambda}{\lambda} = \sqrt{\frac{2}{(\eta_{ar}+1)}} - 1 \tag{12}$$

The results of the simulation (Fig. 4) show good agreement with the theoretical values (Eq. 12)

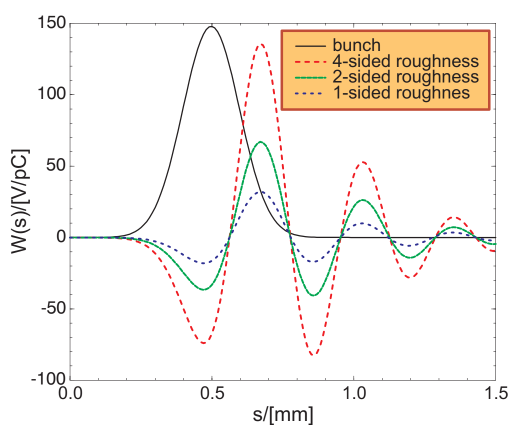

Figure 5: Wake potentials of a bunch in the center of a partially rough, quadratic ($A = B = 2$mm) waveguide. Simulated structure length 3cm, $\sigma_z = 100\mu$m, $\delta = 100\mu$m. One side, two sides or four sides of the waveguide are covered with a chequered structure

3.2 *Partially Rough Tubes*

A rectangular waveguide, that is covered on one, two or four walls with a surface structure is chosen to study the dependencies.

On Fig. 5 the wake potentials in the center $x = 0, y = 0$ of the waveguides are displayed. The frequency does not change with the number of disturbed walls. Obviously it is determined by the structure of the walls. The amplitude of the wake potential is directly proportional to the number of disturbed walls.

The profile along the x-axis is studied next (Fig. 1). The 4-sided roughness shows nearly no transverse dependency. In case of the 2-sided roughness there is a significant sag in the course of the profile. The amplitude in the center of the waveguide is exactly one half of the amplitude of the 4-sided roughness. They come to the same value at the transition to the rough boundary. The wake potential of a waveguide with only one disturbed boundary finally is vanishing at the plain wall. In the center the value is one forth of the 4-sided case.

The dependency on the direction perpendicular to the shown axis is sinusoidal in case of the 2- and 1-sided roughness.

4 CONCLUSION

It is shown, that the case of a purely inductive impedance is included in the more general thin dielectric layer model. It is an approximation for a high frequency rough tube mode. This model is also applicable for different bunch shapes.

The study in the influence of different bunch shapes on the additional energy spread in a rough tube shows, that switching to not smooth bunch shapes can lead to energy modulations inside the bunch and a significant increase of energy spread.

The wake potential of a waveguide covered all over the circumference with a surface structure is the result of a superposition of the wake potentials of the single walls or sectors with a disturbed boundary. This allows the application of the thin dielectric layer model also on arrays of pumping holes.

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