ORBIT CONTROL ADVANCES AT ELETTRA

E. Karantzoulis
Sincrotrone Trieste, Italy

Abstract

Advances in orbit control and concepts of a new algorithm are described that allow the simultaneous minimisation of the global orbit and correction kicks in a systematic way especially for a non-optimised machine. Although the algorithm is not SVD dependent, using it with SVD renders flexibility and transparency ensuring thereafter optimised solutions that are intrinsic to SVD. The algorithm is implemented in an orbit correction program that is further used in studying the feasibility and best strategy for a closed orbit fast feedback system.

1 ORBIT STABILITY

Low emittance machines (such as 3rd generation light sources) have a large amplification factor for closed orbit distortions against quadrupole misalignment, while the presence of strong sextupoles generate a large limitation on the motion stability and high sensitivity to optical distortions. Other than the usual stability issues related to ground motion, girder response to vibrations and the spectral components of the mains that are common to all machines, the mismatch between injection energy (1 GeV) and actual operation energy (2 GeV / 2.4 GeV at 22% of the time) gives additional orbit distortion problems at ELETTRA. These problems are connected with magnetic hysterisis and the radiation induced thermal load on the vacuum vessel. While the first one slightly compromises the reproducibility of the global orbit the later induces orbit shifts of up to 100 μ horizontally and 30 μ vertically peak to peak as measured in the middle of the straight sections, reaching a maximum 5 hours after ramping 320 mA to 2 GeV. For these reasons efficient orbit control and correction schemes are of paramount importance.

Global and local orbit correction programs have to take care of the final orbit that has to be kept globally below 500 μm rms and locally below 2 (μm and μrad) in the middle of the straight sections. For the orbit acquisition a beam position monitor (bpm) system consisting of 96 (four button) multiplexed detectors, attached to the quadrupoles with a bandwidth for the closed orbit of 1 kHz, a resolution of 2 μ and an absolute accuracy of < 150 μm rms is used. For the corrections, a Beam Steering system is employed consisting of 82 combined H+V correctors 0.22 m long with a 140-130 Gaus m maximum field strength.

2 ORBIT CORRECTION ALGORITHMS

2.1 Introduction

The orbit is corrected globally once or a few times per run whereas locally it is corrected via a simple local slow feedback every 5 minutes. From 1994 to 1997 global orbit corrections were performed by means of the program Orbit[1] equipped with the COCU package developed at CERN [2] that comprises well known correction schemes like MICADO, SIMPLEX, BUMPS and was associated with a unique data structure management system (MOPS). The program performed well but it could not be easily handled by non-experts and quite often correction schemes had to be interchanged in order to achieve a better correction. A global orbit and dispersion correction scheme was added afterwards (1995) using the SVD (Singular Value Decomposition) method completely de-coupled from COCU and MOPS. “Orbit” was finally (1997) withdrawn to be replaced by “TOCA” [3] a very user friendly SVD based program conceptually written for non experts. The program globally corrects the orbit and/or dispersion as well as tune and chromaticity.

All these methods however are orbit minimisation oriented and do not consider minimising the corrector strengths. It is true that SVD keeps the strength’s change at minimum but does not minimise the total strengths. Minimal corrector strength is desirable for many reasons such as better orbit reproducibility, reduction of local distortions and/or dispersion, lesser stress for the correctors and their power supplies and strength margin in case of need. Thus one should aim to the best orbit correction with the minimal absolute corrector strengths

2.2 Eigenvalues and the Reduction Algorithm

It is well known that the orbit change due to a change in the corrector strengths can be expressed as:
where θ is the kick vector i.e. the m corrector settings, Y

$$A\vec{\theta} = \vec{Y} \quad (1)$$

is the orbit change vector as seen by the n bpm readings and A is the response matrix (theoretical or measured). This matrix A is rectangular, over/under constrained and sometimes numerically unstable. To solve Eq. 1 one multiplies with the transpose of A and solves instead the equation:

$$A^T A\vec{\theta} = A^T \vec{Y} \quad (2)$$

The matrix A^TA is symmetric, non negative and the solution of eq. 2 is the least square approximation of eq.1. To better understand the physical meaning of the solutions one diagonalises the A^TA matrix finding thus the eigen values λ_j and the unitary and orthogonal eigen vectors ϑ_j of eq. 2. Each of the m eigenvectors represents a set of corrector values and by substitution in eq. 2 one obtains a corresponding set of orbit changes y_j. Since the eigenvectors are unitary it is easy to prove that:

$$< y_j^2 >_{rms} = \lambda_j / n \qquad (3)$$

Thus, corrector eigen vectors that correspond to small eigen values do not appreciably change the orbit [4]. This is also very useful when treating singularities of the matrix A as we shall see later. On the other hand eigen vectors corresponding to large eigenvalues do change appreciably the orbit; but what can be said about the eigenvectors themselves?

To answer this question one has to see how vectors are decomposed to eigenvectors. Let Θ be a correctors' actual vector associated to its corresponding Y orbit vector via the operation $A\Theta=Y$. . This correctors' vector can be decomposed in terms of its orthogonal and unitary eigen vectors according to:

$$\vec{\Theta} = \sum_{j=1}^{m} c_j \vartheta_j \text{ with } c_j = \vec{\Theta}\hat{\vartheta}_j \qquad (4)$$

Similar decomposition can be performed for the orbit Y. In terms of the orbit orthogonal but not unitary eigenvectors:

$$\vec{Y} = \sum_{j=1}^{m} k_j y_j / \lambda_j \text{ with } k_j = \vec{Y} y_j \qquad (5)$$

Taking the norm of the orbit vector it can be shown that:

$$\left\|\vec{Y}\right\| = \sum_{j=1}^{m} k^2{}_j / \lambda_j \;\; => \;\; k^2{}_j / \lambda_j \le \left\langle Y^2_{rms} \right\rangle \qquad (6)$$

From eq. 1,2,3,4 one can be easily arrive at the relation:

$$c_j = k_j / \lambda_j \qquad (7)$$

which connects the orbit and corrector eigen coefficients. Since the norm of the corrector vector $\|\theta\|$ is the sum of the squares of its eigen coefficients, by virtue of the right part of equation 6 the following relation can be written:

$$c^2{}_j \le \left\langle Y^2{}_{rms} \right\rangle / \lambda_j \qquad (8)$$

Eq. 8 toghether with eq. 3 indicates that if we choose eigen vectors corresponding to large eigen values we have the maximum orbit change with the minimum strength rms of the correctors. Thus in general a corrector minimisation algorithm could be as such: Decompose the corrector's vector in its eigen vectors and choose a sum of them corresponding to the higher eigenvalues until the orbit is either as before or even minimised. Note that the actual orbit data are not essential to this algorithm.

2.3 The Reduction Algorithm and SVD

The SVD is a well known, old and robust method for dealing with singular (or quasi so) matrices. SVD is based on a linear algebra theorem whereby any A nxm matrix (with $n \ge m$) can be decomposed as a product of a column orthogonal (U) a diagonal (W) and an orthogonal (V) matrix:

$$A = UWV^T \;\Rightarrow\; A^{-1} = V\left[1/W_{jj}\right]U^T \qquad (9)$$

The degree of singularity is given by the W matrix, which being diagonal its inverse is the reciprocal of its diagonal matrix elements and to render the solution regular one has to replace the 1/Wj by "zero" if Wj is "zero".

$$A^T A = V\left[\lambda_{jj}\right]V^T \qquad (10)$$

In order to understand the physical meaning of the three matrices U, W and V one has to consider again the eigenvalue problem. From eq. 2 and the SVD decomposition it can be readily shown that the elements of the W matrix are equal to the square root of the eigenvalues λi. Furthermore since: multiplying from the right hand side by V we see that the columns [V] of the V matrix satisfy the eigen value equation: $AA^T [V] = \lambda i\ [V]$. It can be further proven that the columns of the orthogonal matrix V are the same with the corrector eigenvectors ϑj obtained in the previous section, while the columns of the column-orthogonal matrix U are the eigen orbits. Furthermore due to the strong correlation of eigen-strenghts and eigen-orbits, in the inverse matrix of A the elements in the diagonal or about it have large values (since there, each eigen corrector is multiplied and summed by its eigen orbit (VU^T)).

Thus SVD provides all the ingredients needed to solve the problem. But it does even more since it can discard small eigen values in a very elegant way. It is easy to show that exclusion of an eigen value is equivalent to singling out the corresponding (corrector) eigen vector from the solution set i.e. subtracting the particular eigen vector from the original (corrector) vector as expressed in eq. 4. However in this case the diagonally biased form of A^{-1} might be lost since certain eigen-strengths do not combine with the corresponding eigen orbits. Discarding those eigen values still the solution θ minimises in the least square sense the expression $r = A\theta - Y$ where $r \neq 0$. Furthermore it can be proven that the solution vector θ has the smallest possible length [5]. This solution is added to the already existing corrector strengths and although it is minimal the original strengths might not be. To achieve a total minimisation the actual corrector setting, the vector $\Theta 0$ is decomposed by using the basis column vectors of matrix V (which span the corrector strength space). The re-composition of the vector is made by a step-wise approach where eigen vectors corresponding to the larger eigen-values W^2 (i.e large orbit changes with small corrector strength change as seen from eqs. 3, 8) are chosen until the predicted orbit rms has its initial value. It can be shown that setting to zero all eigenvalue coefficients c_j for $m1 \le j \le m$ any arbitrary vector can be decomposed as follows:

$$\vec{\Theta} = \vec{V}\begin{bmatrix} \vec{c}\, j_{\ for\, j \ge m1} \\ \vec{0} \end{bmatrix} \qquad (11)$$

and the rms orbit change squared equals to the sum of the squares of the eigenvector coefficients:

$$\left\langle \Delta\vec{Y}^T \Delta\vec{Y} \right\rangle = \left[\vec{c}_j \quad \vec{0} \right] \left[\begin{matrix} \vec{c}_j \\ \vec{0} \end{matrix} \right] \approx \sum_{j=m1}^{m} \frac{\left\langle Y^2 \right\rangle}{w_{jj}^2} \quad (12)$$

The number of non zero eigen-values m-m1 is therefore the free parameter to be adjusted, usually the first 10-20 larger ones. The recomposed vector has remarkably low strengths i.e. ≤ 0.2 mrad and thus reducing the rms original correctors strengths by more than 70% and up to 90% maximum. Note that to this point the bpm readings are not essential. Adding to those reduced original corrector strengths the SVD solution θ of corrector strength changes needed to further correct the orbit (here the bpm readings are essential) the final strengths are kept always at minimum. Alternatively (and this is the adopted method here) one first finds the solution as Θ0+θ and then decompose it to the V eigen vectors, both ways give similar results. In the next table orbit and the correctors strenghts reduction results using the Reduction algorithm alone as well as in combination with the SVD orbit correction scheme is shown for ELETTRA:

Table 1: reduction algorithm and SVD correction results

method	Corrector settings (Amps)			Orbit (mm)	
	mean	rms	ptp	rms	ptp
initial config.	0.065	1.422	9.926	0.376	2.42
Reduction Alg.	0.078	0.158	0.631	0.315	2.28
Reduction, relaxed Correction	0.074	0.193	1.078	0.258	1.74
Reduction, stronger Correction	0.081	0.466	2.5	0.177	1.0
Full Correction	0.059	1.211	8.0	0.123	0.9

A program called “Gloc” has been developed [6] that comprises all the above features. Programmed on a modular structure that helps in developing and testing new ideas, the program includes many features such as: measure and analyses of the response matrix, orbit corrections under various constraints such as: de-select certain correctors or bpms, zeroes, scales or reduces the setting of certain correctors while conserving the orbit. It also includes many other options such as local global correction that is discussed briefly below.

2.4 Global and/or local corrections

Local corrections minimise the orbit variations at each experiment individually involving only the correctors needed for the bump. The main disadvantage is the bump leakage that can lead to a cross talk between experiments or even to a deterioration of the global orbit. However in many cases it can be very useful if particular experiments require certain conditions of local orbit as in the case of ELETTRA where a slow local feedback is operating to both give at experiments the wished conditions (position and angle of the beam) as well as to keep the global orbit from diverging due to thermal motion of the chamber. Alternatively correcting globally minimises the orbit at all experiments. Fast global feedbacks can be made [7] if one divides the bpm system in subsystems per sector and take advantage of the strong correlated form of A^{-1} around the diagonal as discussed previously, meaning that to set the steerers only local orbit information is required. However as we have seen this is true if not many cancellations of the eigen vectors are required since then the matrix loses its diagonally biased form. In this case it could be much simpler to ignore the corresponding corrector, which however would decrease the correction efficiency. At ELETTRA currently a 7 corrector local bump correction scheme is investigated whereby at each section the beam can be set at specific position and/or angle to 3 specific points connected to experiments (i.e long straight, bend and short straight). The bump has been incorporated into Gloc [6] (using SVD for the inversion) and tested with beam in the horizontal plane with good results correcting simultaneously all sections. This will be combined with a global slow feedback system based on the described algorithm.

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