NON LINEAR CUMULATIVE INTERACTION AND HALO HEATING IN MULTI CAVITY LINAC

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Abstract

Multi mode approach in cumulative beam break up theory results in number of features of beam dynamics. Besides enhanced action of induced fields, deflecting rf gradient manifests much more strong co-ordinates dependence than that in the case of single mode approximation for non resonance cavities excitation. Beam heating is a consequence of such non linear beam rf cavity interaction in transient particle dynamics. Transverse beam dynamics in induced electromagnetic fields in multi cavity linac is the subject of this paper. Multi mode approach is used for the processes modelling in multi cavity linac composed of single cell uncoupled cavities. Beam kernel coherent motion computer simulation is accompanied by trajectories calculation of halo particles. Plots presented serve to illustrate beam kernel envelope evolution along the accelerator length as well as halo emittance growth.

1 INTRODUCTION

Cumulative beam-rf-cavity interaction in multi cavity linear accelerator is a kind of convection instability in infinite media [1-4], when perturbation growths in limited area that moves to infinity. In multi cavity linac, offset head beam bunches excite non symmetric rf modes in cavities, that deflect the following bunches. Bunch transverse displacement growths with the bunch as well as with the cavity number. This process has not beam current threshold. In our previous papers we developed multi mode approach to get deflecting forces for non resonant steady state case. Of course, these forces are small, if resonance conditions do not take place for any particular mode. We remind briefly here the main features of multi mode interaction. Infinite set of deflecting modes is excited, while deflecting gradient is limited. Effect of mode saturation takes place - even for point size bunches deflecting gradient does exceed some asymptotic level with adding infinite set of TM modes with large wave vector numbers. As a rule, mode saturation takes place for the modes with smaller wave vector for the case of bunches with finite bunch sizes. As numeric calculations had shown, contribution of the modes higher symmetry order than dipole is rather small. The other very important feature of multi mode cumulative interaction is non linear character of deflecting gradient. In principle, this is a quantitative effect that manifests itself in single mode approach for large beam offset, but when many of TM modes are taken into account, non linear character of interaction becomes stronger. The strong co-ordinate dependence for deflecting gradient takes place both for transverse as well as for longitudinal direction.

Non liner beam-field interaction results as rule to beam emittance growth. A priory this statement is valid for non-linear cumulative interaction too. The paper purpose is to estimate this effect for halo particles by beam dynamics simulation. Unfortunately, we can not suggest other way than computer simulation for the reason of problem complicity. For the same reason we do this for halo only. Halo particles do not affect beam - cavity interaction due to much smaller halo intensity in comparison with beam kernel intensity, that simplifies simulation and takes smaller computing recourses.

2 THE MAIN EQUATIONS

Imagine the linac, composed of uncoupled single cell cavities with the focusing elements between them and a charged particles beam traversing this arrangement. We will assume beam to consist of point size bunches, moving with the velocity v along z-axis that coincide with linac axis, and separated by a distance L. We also suppose, that there are the particles that accompany any bunch, but do not affect cavity field (halo particles). After any cavity passage any particle acquires additional transverse momentum [5,6]

$$\Delta p_{\perp} = e\int_0^d \frac{\partial A_z}{\partial x}(x,0,z,\tau + \frac{z}{v})dz \qquad (1)$$

where $\vec{A}$ is the vector potential, e and d are particle charge and cavity length, τ stands for the moment at which particle enters the cavity (MKS units are used throughout this paper). We assume using the formula written above, that particle displacement is small inside cavity and can been neglected in transverse momentum calculation.

Following our previous papers [7,8] and [9], let us represent vector potential of excited electromagnetic field in any cavity as the sum of eigenvectors $\vec{A}_\lambda(\vec{r})$:

$$\vec{A}(\vec{r},t) = \sum_\lambda q_\lambda(t)\vec{A}_\lambda(\vec{r}) \qquad (2)$$

with the time dependent amplitudes $q_\lambda(t)$ satisfying the differential equation

$$\ddot{q}_\lambda + \frac{\omega_\lambda}{Q_\lambda}\dot{q}_\lambda + \omega_\lambda^2 q_\lambda = \frac{1}{\varepsilon_0}\frac{\int_V \vec{j}\,\vec{A}_\lambda\,dV}{\int_V \vec{A}_\lambda^{\,2}\,dV}, \qquad (3)$$

and with $\vec{A}_\lambda$ satisfying the condition $div\,\vec{A}_\lambda = 0$ as well as the Helmholtz equation:

$$\Delta\vec{A}_\lambda + \frac{\omega_\lambda^2}{c^2}\vec{A}_\lambda = 0. \qquad (4)$$

Here, ω_λ and Q_λ are frequency and quality factor of any particular mode respectively, ε_0 is electrical permeability of free space, c is the light velocity. The solution for amplitude $q_\lambda(k,N,t)$ for the point size bunch of charge q can be represented in the form:

$$q_\lambda(k,N,t) = \frac{q}{\omega_\lambda\varepsilon_0\int_V A_{\lambda,z}^2 dV}\mathrm{Im}\exp\left[\omega\left(\frac{1}{2Q_\lambda} - i\right)(\tau_{k,N} - t)\right]\times \qquad (5)$$
$$\int_0^d A_{\lambda,z}(x_{k,N},0,z)\exp\left[\frac{\omega_\lambda z}{v}\left(\frac{1}{2Q_\lambda} - i\right)\right]dz$$

Here indexes (variables) k and N denote, that appropriate values have to be taken for bunch number k in N-th cavity. The formula is valid only for a moment when bunch has already left the cavity. To get the expression for the transverse momentum that any particle with charge e in k-th bunch acquirers in N-th rf cavity, we have to calculates the sum of the fields, induced in this cavity by all previous bunches of bunch train:

$$\Delta p_\perp(k,N) = eq\ \mathrm{Im}\sum_\lambda\sum_{j=1}^{k-1} Z_\lambda(x_{j,N})Y_\lambda(x_{k,N})\times \qquad (6)$$
$$\exp\left[\omega_\lambda((k-j)T)\left(i - \frac{1}{2Q_\lambda}\right)\right]$$

where the following designations are used:

$$Z_\lambda(x) = \frac{\int_0^d A_{\lambda,z}(x,0,z)\exp\left[\frac{\omega_\lambda z}{v}\left(\frac{1}{2Q_\lambda} - i\right)\right]dz}{\omega_\lambda\varepsilon_0\int_V \vec{A}_\lambda^2\,dV} \qquad (7)$$

$$Y_\lambda(x) = \int_0^d \frac{\partial A_{\lambda,z}}{\partial x}(x,0,z)\exp\left[\frac{z\omega_\lambda}{v}\left(i - \frac{1}{2Q_\lambda}\right)\right]dz \qquad (8)$$

For cylindrical cavities the components for eigenvectors of deflecting TM modes are:

$$A_r = -\frac{k_z}{k_c}J'_n(rk_c)\cos n\varphi \sin k_z z,$$
$$A_\varphi = \frac{k_z n}{k_c^2}\frac{J_n(rk_c)}{r}\sin n\varphi \sin k_z z, \qquad (9)$$
$$A_z = J_n(rk_c)\cos n\varphi\cos k_z z,$$

and the formula for particle angle change $\Delta\alpha_k = \Delta p_{\perp,k}/p_k$ looks like

$$\Delta\alpha_k = \frac{eI}{mc^3\varepsilon_0}\frac{1}{\beta\gamma\rho^3}\mathrm{Im}\sum_{j=1}^{k-1}\sum_{n,m,p}\frac{\nu_{n,m}^3 W_{n,m,p}}{k_{n,m,p}J_{n,m,p}}\times \qquad (10)$$
$$J_n\left(\frac{\nu_{n,m}\xi_{k,N}}{\rho}\right)J_n\left(\frac{\nu_{n,m}\xi_{j,N}}{\rho}\right)\exp\left[k_{n,m,p}(k-j)\left(i - \frac{1}{2Q_{n,m,p}}\right)\right]$$

where $I = q/T$ is average beam current and

$$W_{n,m,p} = -\left(i\frac{k_{n,m,p}}{\beta} - \frac{k_{n,m,p}}{2\beta Q_{n,m,p}}\right)^2\times$$
$$\frac{\left\{(-1)^p\exp\left(\frac{\delta k_{n,m,p}}{2\beta Q_{n,m,p}} - i\frac{\delta k_{n,m,p}}{\beta}\right) - 1\right\}}{\left(-\frac{ip\pi}{\delta} + \frac{ik_{n,m,p}}{\beta} - \frac{k_{n,m,p}}{2\beta Q_{n,m,p}}\right)^2}\times \qquad (11)$$
$$\frac{\left\{(-1)^p\exp\left(-\frac{\delta k_{n,m,p}}{2\beta Q_{n,m,p}} + i\frac{\delta k_{n,m,p}}{\beta}\right) - 1\right\}}{\left(\frac{ip\pi}{\delta} + \frac{ik_{n,m,p}}{\beta} - \frac{k_{n,m,p}}{2\beta Q_{n,m,p}}\right)^2}\times$$

Here, lower case Greek letters designate normalised values, $R = \rho\Lambda, d = \delta\Lambda, x = \xi\Lambda$ Λ and R are the wavelength of the TM_{010} mode and cavity radius, $\Lambda = cT$, $\nu_{n,m}$ is the m-th null of Bessel function of the n-th order $J_n(x)$, $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$, $k_z = \pi p/d, k_c = \nu_{n,m}/R, p = 0,1,...,$ m, n=1,2... , and

$$J_{m,p} = \frac{\pi^3\rho^2 p^2}{2\delta\nu_{n,m}^2}(A_{n,m} + B_{n,m}) + \pi\delta C_{n,m}\times\begin{cases}1, p=0\\ \frac{1}{2}, p\neq 0\end{cases} \qquad (12)$$

$$A_{n,m} = \int_0^{\nu_{n,m}} J_n'^2(x)x\,dx,\quad B_{n,m} = \int_0^{\nu_{n,m}}\frac{J_n^2}{x}dx,$$
$$C_{n,m} = \int_0^{\nu_{n,m}} J_n^2(x)x\,dx, k_{n,m,p} = \sqrt{\frac{\nu_{n,m}^2}{\rho^2} + \frac{\pi^2 p^2}{\delta^2}} \qquad (13)$$

3 DYNAMICS SIMULATION

To make simulation, the main equation for additional particle momentum angle change in rf cavity has to be supplement by the appropriate relationships for transverse motion in the space between cavities. We shall assume that single quadrupoles are installed in the mid plane of drift space, focusing sign being changed periodically every cavity period. Thus,

$$m_{1,1} = 1 + PD,\quad m_{1,2} = D(2 + PD),$$
$$m_{2,1} = P,\quad m_{2,2} = 1 + PD. \qquad (14)$$

where P is quadrupole strength, $P=1/F$, F is lens focusing distance, and D is the distance between cavity exit and next cavity entrance.

Recursive expressions (10) were used to simulate self consistent beam kernel transverse motion in induced non symmetric rf fields and focusing channel. Step by step one can calculate co-ordinate and angle at the entrance of N-th cavity for k-th bunch in a train, if appropriate values are already known (calculated) for (N-1) cavity as well as co-ordinates for bunches with numbers 1,2…k-1 are calculated, too. Fig. 1 represents the dependence of maximum train displacement on cavity number for two different cases.

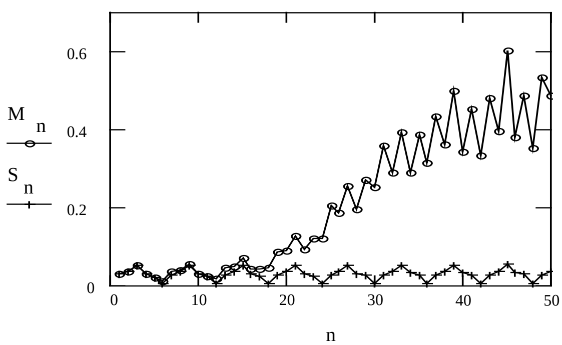

Fig.1:Beam envelope along the linac for multi (M) and single (S) mode approach

Larger displacement corresponds to the case when the modes up to 5 radial and 2 longitudinal variations were taken into account for excited field calculation, while lower curve corresponds to the single mode (TM110) approximation. For simplicity we considered the costing electron beam with the energy 5 MeV and average beam current 10 A, focusing parameter being equal PD = 1 and Q=1000 for all modes. The train was considered to consist of 300 bunches. MATLAB code was used for simulation. The same main equations were used for halo particles transverse dynamics simulation, but in this case the simulation reduced to particle trajectories calculation in external fields of focusing channel and time dependent field, excited by beam kernel. MATLAB interface makes it easy to save workspace in file and restore it later, that allows to make this separation process of simulation comfortable.

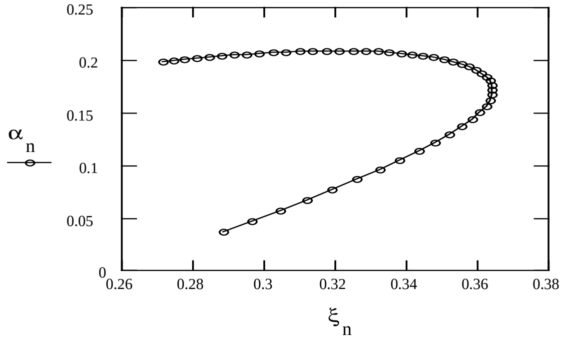

Fig.2: Final halo density distribution

Fig. 2 represents the halo particles final distribution in transverse phase space for the bunch number 250 with the initial line uniform distribution within the normalised co-ordinate range 0.03 – 0.1. Cumulative interaction resulted in emittance growth, that manifests itself in transformation of phase segment into significantly curved line.

4 CONCLUSION

Non-linear effects of transient multi mode cumulative interaction in rf linac may result in enhanced influence on beam dynamics, including process of halo heating. These previous results of computer simulation have to be continued to see more detail picture of such an interaction.

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REFERENCES

[1] W.K.H.Panofsky and M.Bander, "Asymptotic Theory of Beam Break Up in Linear Accelerators", The Review of Scientific Instruments, vol. 39, pp. 206-212, February, 1968.

[2] A.W.Chao, B.Richter and C.Y.Yao, "Beam emittance growth caused by transverse deflecting fields in a linear accelerator", Nuclear Instruments and Methods, vol. 178, pp. 1-8, 1980.

[3] R.L.Gluckstern, R.K.Cooper and P.J.Channel, "Cumulative beam breakup in rf linacs", particle Accelerators, vol. 16, pp. 125-153, 1985.

[4] C.L.Bohn and J.R.Delayen, "Cumulative beam breakup in linear accelerators with periodic beam current", Physical Review A, vol. 45, No 8, pp. 5964-5993, April, 1992.

[5] W.K.H.Panofsky and W.A.Wenzel, "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", The Review of Scientific Instruments, vol. 27, No 11, p. 967, November, 1956.

[6] M.Jean Browman, "Using the Panofsky-Wenzel Theorem in the Analysis of Radio-Frequency Deflectors", in Proceedings of 1993 Particle Accelerator Conference, Washington, May 17 - 20, 1993, pp. 800 - 802.

[7] V.G.Kurakin, "Multi Mode Approach in Cumulative Beam Break up Theory", in Proceedings of Fourth European Particle Accelerator Conference, London, 27 June to 1 July, 1994, pp. 1123 -1125.

[8] V.G.Kurakin, “Deflecting Forces for the Case of Multi Mode Beam – RF Cavity Interaction in Linear Accelerators” in Proceedings of the 1995 Particle Accelerator Conference and International Conference on High Energy Accelerators, May 1-5, 1995, Dallas, USA, 3049-3051.

[9] V.M.Lopuchin, The excitation of electromagnetic oscillations and waves by electron beams, Moscow: The publisher of technical and theoretical literature, 1953, 324 pp., in Russian.