

# PRISM: A PHASE ROTATOR TO OBTAIN AN INTENSE MONOCHROMATIC MUON BEAM

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## Abstract

The PRISM project, a longitudinal phase rotator using FFAG ring, is under discussion in Japan to obtain intense monochromatic low energy muons for the experiment of muon lepton flavor violation.

## 1 INTRODUCTION

Beam quality and intensity of the various secondary particle beams such as pions and muons, which are generated by high energy intense proton beams, can be improved by various beam manipulation techniques. Among them, longitudinal phase rotation, which has been proposed by Palmer and Neuffer[1], is a unique technique to make a monochromatic beam. Longitudinal phase rotation is a rotation of particle position in longitudinal phase space induced by an external time varying electric field. The energy spread of the secondary particles produced by a narrow bunched proton beam is very large. Applying an external time varying electric field, such as rf, rotates the distribution in phase space, and after twisting by a  $\pi/2$  phase rotation, the energy spread becomes small and the beam becomes monochromatic.

The PRISM (Phase Rotation Intense Slow Muons) project in Japan to consider a lepton flavor violation experiment with low energy and monochromatic muons is under discussion.[2] The intensity goal is  $10^{12}\mu^+/\text{sec}$  with high purity and small energy spread. In order to do phase rotation in PRISM effectively, a FFAG (Fixed - Field Alternating Gradient) type of ring phase rotator has been proposed as shown in Fig.1.[3].

## 2 FFAG PHASE ROTATOR

A linear system has been discussed so far for phase rotation of pion and muon beams. Since the momentum of the particles is relatively low and nonrelativistic, the external electric field requested for the phase rotation should have a long wavelength. Thus, in a linear system, an induction linac configuration [4], and a low frequency ( 1~10 MHz) rf linac configuration have been investigated intensively. The obtainable electric field gradient obtainable from such a low frequency rf system is relatively low, normally less than 0.5-1 MV/m and the length of the total system becomes very long. For example, in case of PRISM, where the central momentum and the momentum spread of the muon beam are to be 68 MeV/c and  $\pm 25\%$ , respectively, the linear system requires a total length of more than 100 m. Moreover, for transverse beam focusing, a continuous focusing channel such as superconducting solenoid

magnets surrounding the linear system is necessary to keep the beam size small. Thus, the total cost of the linear system for phase rotation becomes expensive.

If a ring configuration for phase rotation is available instead, the expected cost becomes small and reasonable because the number of turns in the ring reduce the total length of the rf system compared with the linear system. To achieve the aim for performance in PRISM, three requirements should be fulfilled.

- (1) The synchrotron oscillation frequency should be high enough to avoid an enormous beam loss due to muon decays.
- (2) The momentum acceptance should be large enough: in the case of PRISM, a momentum acceptance of more than  $\pm 25\%$  is necessary.
- (3) The transverse acceptance should be large enough: in the case of PRISM, the transverse acceptances of more than  $10000\pi\text{mm.mrad}$  and  $3000\pi\text{mm.mrad}$  for the horizontal and vertical planes, are required respectively.

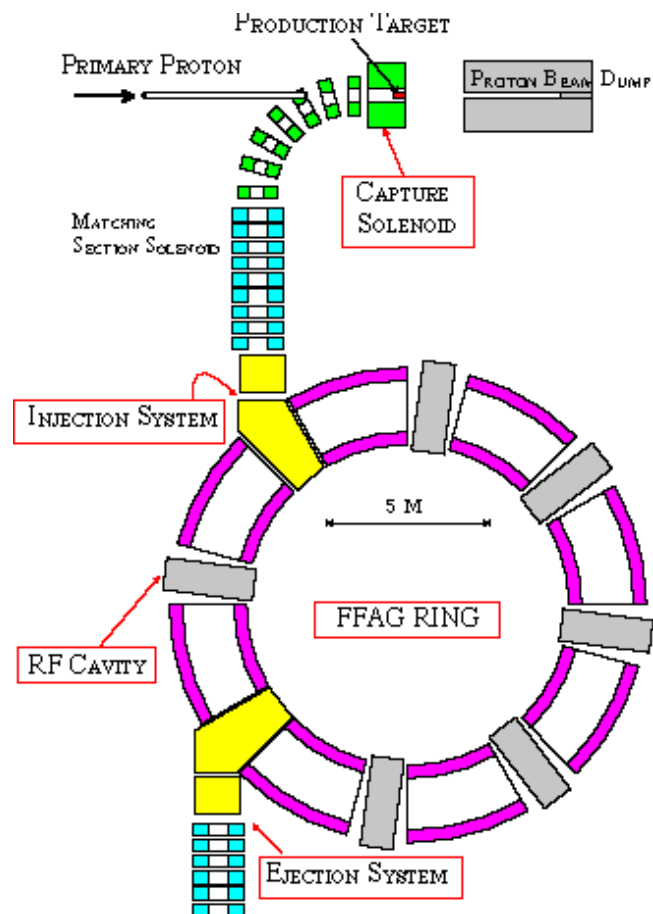


Fig. 1 Schematic layout of PRISM.

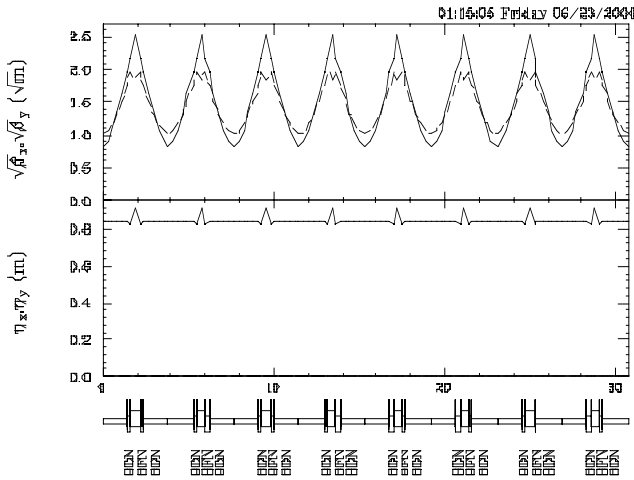


Fig., 2 Beam orbit parameters of the FFAG phase rotator.

A synchrotron type of ring has a problem in this regard because its momentum acceptance is about  $\pm 1\%$  at most. The cyclotron has a relatively large momentum acceptance, but is absolutely inadequate because of its isochronism: no synchrotron oscillation and no phase rotation.

We have proposed a FFAG (Fixed-Field Alternating Gradient) synchrotron ring for the phase rotation of low energy muons.

The first idea of the FFAG ring was proposed by Ohkawa in 1953. The FFAG ring has a fixed magnetic field and the beam orbit in the ring moves during acceleration. In other words, the FFAG synchrotron has a fairly large momentum acceptance compared with an ordinary ring. The momentum acceptance of the FFAG ring normally exceeds more than  $\pm 50\%$ .

The time period for a  $\pi/2$  phase rotation has to be small compared with the muon lifetime. Therefore,

$$1/4f_s \ll \tau_0 \gamma. \quad (1)$$

Here,  $\tau_0$  and  $\gamma$  are the muon lifetime in the rest frame (2.197  $\mu\text{s}$ ) and the relativistic constant, respectively, and  $f_s$  is the synchrotron frequency. Thus,

$$f_s \gg 100 \text{ kHz}. \quad (2)$$

The present design of the FFAG ring for PRISM has a diameter of 10m. The parameters are summarized in Table 1. We chose a radial sector type of ring configuration and each sector consists of a triplet focusing (DFD) lattice. The geometrical magnetic field index ( $k$ ) is 5, and therefore the momentum compaction factor becomes  $1/6$ . The linearized beam orbit parameters (beta function,

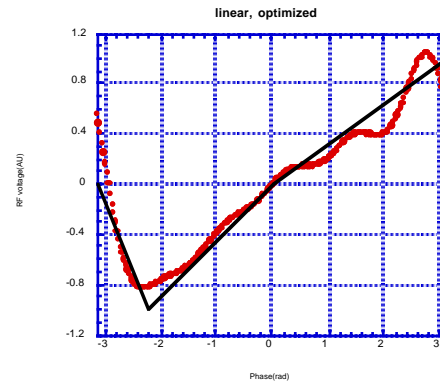


Fig. 4 A saw-tooth rf waveform.

dispersion) of the scaling type of FFAG ring can be estimated using an effective field index  $n$  with a circumference factor.

$$n = \pm (1 + \xi \cos \psi) / (1 + 2\xi \cos \psi + \xi^2), \quad (3)$$

(+:focus, -:defocus)

where

$$\xi = \zeta - 1. \quad (4)$$

Here  $\zeta$  is a so-called circumference factor and  $\zeta = r_0/\rho$ . Figure 2 shows the orbit parameters at the central beam momentum (68 MeV/c) in the PRISM FFAG ring. The betatron tunes for both planes are also shown in Table 1.

As can be seen from Fig.2, the required beam half apertures in the magnet for the both horizontal and vertical directions are 480 mm and 104 mm, respectively, when the horizontal and vertical emittance are 10000  $\pi\text{mm.mrad}$  and 3000  $\pi\text{mm.mrad}$ , respectively, and the momentum spread is  $\pm 25\%$ . At the straight section, beam half apertures for horizontal and vertical directions are 280 mm and 55 mm, respectively. Non-linear effects caused by a sinusoidal rf waveform are an impediment to realizing a small energy spread after phase rotation. Figure 3 shows the longitudinal emittance configurations after phase rotation by a sinusoidal rf voltage of 2.5 MV applied in the FFAG ring. As can be seen in the figure, the longitudinal emittance has largely deteriorated and the energy spread of the beam after five turns is still more

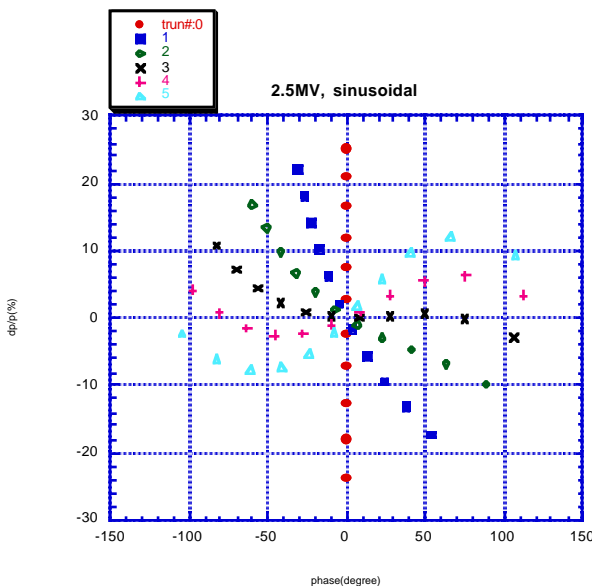


Fig. 3 Phase rotation with sinusoidal rf.

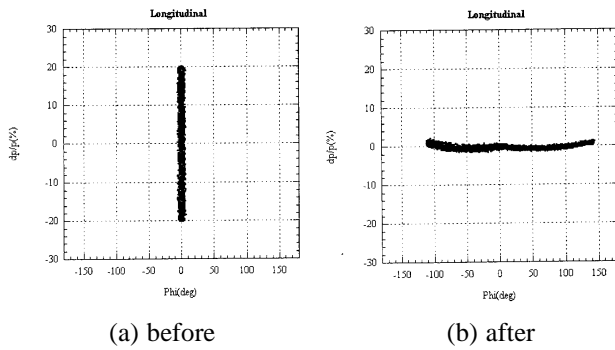


Fig. 5 Longitudinal emittance configurations before and after the phase rotation.

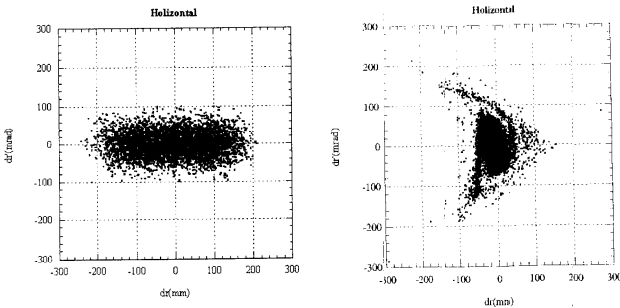


Fig.6 Transverse emittance configurations before and after the phase rotation.

than  $\pm 10\%$ , which is marginal for this application. A brute force method to overcome this difficulty is to use a high rf voltage of more than 10MV in total.

This could be realized with a low frequency and high gradient rf cavity whose field gradient should be at least 0.5 MV/m or more. This type of rf cavity is under development at KEK and FNAL in US-Japan high energy collaborations. Another solution is to make a linear rf force with a saw-tooth rf waveform as shown in Fig. 4. The saw-tooth wave form can be generated by mixing higher harmonic rf components. A broad band magnetic alloy (MA) rf cavity, where the Q-value is less than one, could be used for this purpose.[5] Figure 5 shows the results of a particle tracking simulation for the longitudinal emittance before and after the phase rotation with the saw-tooth rf waveform. The momentum spread of  $\pm 25\%$  becomes less than  $\pm 5\%$  after the phase rotation. In the horizontal phase space, the beam distributions before and after the phase rotation are also obtained by a simulation and the results are shown in Fig. 6.

### 3 SUMMARY

A phase rotator to obtain monochromatic low energy muons seems to be feasible with a FFAG ring in the PRISM project. We hope to open a new era in the field of muon lepton flavor violation. Further, the techniques and experience of the PRISM project would be useful for the future neutrino factory based upon a muon storage ring.

Table 1 Parameters of the FFAG phase rotator for PRISM

Circumference @ 68 MeV/c	30.708 m
Geometrical field index	5
Magnetic field strength	0.402 T : focus 0.0661 T : defocus
Averaged orbit radius in the magnet @68 MeV/c	5m
Number of sectors	8
Magnet angle	52 degree :focus 3.5 degree: defocus
Betatron tunes	3.13 : Horizontal 2.78 : Vertical
Momentum compaction factor	0.167

### ACKNOWLEDGEMENT

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