SUPERPOSITION OF MULTIPLE HIGHER ORDER MODES IN A CAVITY*

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Abstract

A higher order mode has often been treated as a nuisance. For some applications, it can be used to enhance performance of an accelerating cavity such as high peak-field-gradient or low power dissipation. A narrow peak of the resulted wave form is suitable for a short bunched beam. Such pulse-like wave form may also benefit in raising spark limit and gradient for low duty applications: energy compression for short-lived pions or muons. Complicated wave forms can also be synthesized by such a superposition.

1 INTRODUCTION

In order to reduce the energy spread of the secondary particles such as muons, some schemes are under investigation at many laboratories. The principle is 1) muons are produced by short bunched energetic protons, 2) time spread of the muon bunch arises after a drift space from the production target in accordance with the energy spread, 3) high gradient RF field decelerates early arriving high energy muons and accelerates late low energy muons. Because the muons have short life time and the central energy is rather high for better yield, the electric filed gradient should be very high.

2 USE OF HIGHER ORDER MODES[1]

A higher order mode, which has often been treated as a nuisance, can be used to enhance performance of an accelerating cavity such as high peak-field-gradient or low power dissipation. Although harmonics superpositions have been used for bunchers or cyclotrons, they seem only aim at wave form modifications. Because all resonant modes in a cavity are orthogonal each other, electric field strength and power dissipation in a cavity are just sums of those for each mode, respectively. That is, if there are two modes with same shunt impedance in a cavity and we apply same power, we can get twice of peak electric field with twice of power input. Because the resulted wave form has a narrow peak, bunching factor of a beam should be high. Such pulse-like wave form may also benefit in raising spark limit and be suitable for high gradient but low duty applications: the energy compression for short-lived pions or muons.

Limitations on achieving such very high gradient field are heat and spark limits. The former may be overcome by the superposition of the orthogonal higher order modes. The latter is conventionally given by so-called Kilpatrick's criterion: [2]

$$f[\mathrm{MHz}]=1.64\left(E[\mathrm{MV/m}]\right)^2 Exp\left(\frac{-8.5}{E[\mathrm{MV/m}]}\right) \qquad (1)$$

This is an asymptotic limit to a high frequency of the original form, which assumes the sinusoidal waveform. The Kilpatrick's model is a three-step-process: 0) electrons generated by cold emission from a cathode, 1) electrons hit an anode to produce positive ions, and 2) the positive ions with "possible maximum energy" hit a cathode to make an electron emission. The overall gain should be less than one for a stable operation. The impulse-shape-waveform should decrease the probability of each step.

2.1 Multi-Harmonic Impulse Cavity (MHIC)

Figure 1a shows the superposition of three harmonics of a cosine wave with amplitude of one. It should be useful to accelerate a very short bunch beam. Figure 1b shows a single-sine-like wave form generated by superposition of three harmonics of a sine wave, which is required for the rebunching operation or energy compression. The peak amplitude in this case, however, is slightly less than proportional to the number of modes applied. This kind of wave form is also useful for a double gap resonator that has two gaps with opposite signed electric field.

A simple example of such resonant cavity with integral harmonics is a $\lambda/2$ coaxial cavity. Among the integral harmonics, those of even harmonics are not useful because they have nodes of electric field at the center.

2.2 Offsetting Multi-Harmonic Impulse Cavity

Because a $\lambda/2$ coaxial resonator has two acceleration gaps, it has restrictions on designing a system. Figure 2 shows the mode spectra of TMmn0 in a simple cylindrical cavity, which has only one gap. Unlike the coaxial cavity, frequencies of higher order modes in a cylindrical cavity are not integral multiple of that of a fundamental mode. Thus, adjustments are needed to satisfy such requirements.

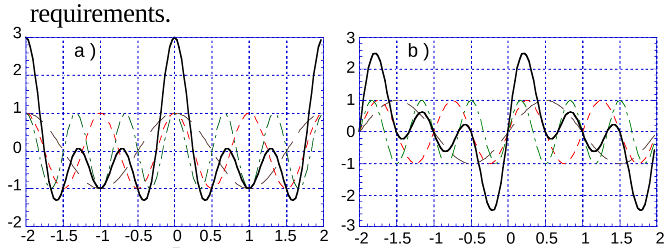

Fig.1 Superposition of three harmonics of a) a cosine wave and b) sine wave.

*Work supported in part by Grant-in-Aid for Scientific Research from Ministry of Education, Science, Sorts and Culture Government of Japan.

Although radially folded cylindrical cavities[3] are adequate for some applications because of the smaller cavity radius, it is not easy to adjust the frequencies to multiples of the fundamental frequencies. Fortunately, the TM_{0n0} mode frequencies in a cylindrical cavity are almost linear with offset. Fitting five frequencies by a line, the frequency $F(n)$ normalized by the frequency separation is approximated by,

$$F(n)=n\text{-}0.235\,. \tag{2}$$

Adjustment of the mode frequencies to modify the value 0.235 to 1/4 or 1/5 should be easier than that of all the mode frequencies to multiples of the fundamental one. Let the value is adjusted to 1/4. Then a wave form that is a superposition of five modes is

$$f(t)=\sum_{n=1}^{5} cos\left(n\text{-}\frac{1}{4}\right)t\,. \tag{3}$$

Figure 3 shows the wave form superposing up to five modes. This scheme is thought to be a partial set of the harmonics series missing many harmonics including fundamental one(ω=1/4). If we rescale the frequency four times, the included frequencies are ω=3,7,11,15,19,... (4i-1). This Offsetting Multi-Harmonic Impulse Cavity (OMHIC) still has periodical high peaks and is practical compared with the MHIC, although the repetition cycle becomes long and extra pulses appear.

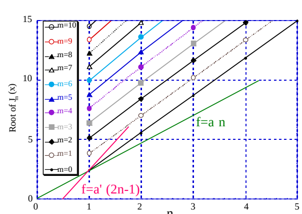

Fig. 2 Mode spectra of TM_{mn0} in a cylindrical cavity.

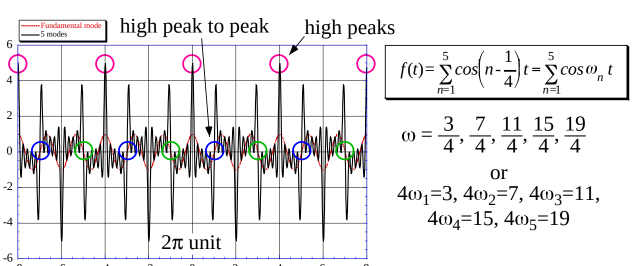

Fig. 3 Superposition of offsetting harmonics (OMHIC implementation)

2.3 Multi-Higher Order Mode Impulse Cavity

For a very low duty factor application, even the periodicity may not be needed. If the cavity has short filling time enough to work for a pulse operation and only one short bunch comes in one pulse, the phase of each mode needs to coincide only at one moment. Starting to feed RF power to each mode with proper RF phase so that every mode has peak at t=0, one can obtain a high peak (see Fig. 4). This scheme can utilize any modes that can accelerate charged particles, which seems more flexible than previous schemes when the duty factor is very small. The maximum CW mode operation is also possible for this scheme. The highest frequency that can be used is limited by the bunch length.

We can relax the harmonic condition on the MHIC or the OMHIC for applications with small repetition rate. Supposing that Q-value is 10^4 and the lowest frequency used is 10MHz, the band width is 1kHz. If the repetition rate is less than 1kHz, the frequency tune will not be needed. A condition required for the 1kHz repetition is that the mode frequencies are the multiples of 1kHz. The resulted wave form will have the same property around the necessary time point and become different otherwise. For higher lowest frequencies, the repetition rate can be increased.

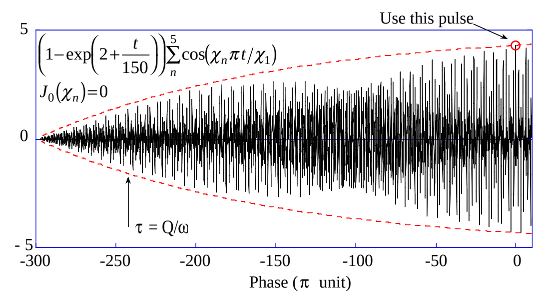

Fig. 4 Conceptual wave form for MHOMIC.

3 COMB PULSE ENERGY COMPRESSION (CPEC)

Very low frequency less than 10MHz is required for an energy compression (so-called a phase rotation) of muons. Because of the nonlinearity between a time-of-flight and an energy, we need a somewhat complicated wave form to manipulate muon bunches with wider energy spread. Thick solid line in Fig. 5 shows a required wave form at 30m downstream from a production target for muons with energy of 20±10MeV. Thin broken curve is fitted with 6.5MHz and 13MHz sine curves. The fitted curve is roughly expressed as follows:

$$20+12.5\left(\sin(\omega t-\phi_1)-\sin(2\omega t-\phi_2)/4\right) \tag{4}$$

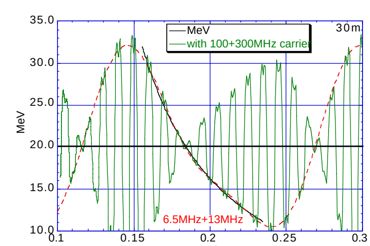

Fig. 5 Approximated waveform for energy compression by superposition of harmonics and that with amplitude modulation.

Although the nonlinearity can be synthesized well, the frequency of 6.5MHz is rather low, which requires a very large cavity. A heavily capacitive loaded cavity can reduce its size with less shunt impedance and narrower gap distance[4], which may not be compatible to the high gradient requirement. On the other hand, cavities for a wave form shown as the thin solid curve in Fig. 5, whose amplitude is modulated by an approximated square wave, have better properties such as a smaller size, higher shunt impedance and higher field gradient. The wave form is expressed by

$$\left(\sin(\omega t-\phi_1)-\sin(2\omega t-\phi_2)/4\right)\left(\cos\omega_c t-\cos 3\omega_c t/4\right). \qquad (4)$$

Using a trigonometric reduction, it can be expressed by a sum of eight frequency components. Because the 13MHz component has a half wave length and one quarter amplitude of the fundamental component, we can fabricate it as is. The trigonometric reduction for the wave form is:

$$\begin{aligned}&\sin(\omega t-\phi_1)(\cos\omega_c t-\cos 3\omega_c t/4)\\&=(-\sin((\omega-3\omega_c)t-\phi_1)+4\sin((\omega-\omega_c)t-\phi_1)\\&\quad+4\sin((\omega+\omega_c)t-\phi_1)-\sin((\omega+3\omega_c)t-\phi_1))/8\end{aligned} \qquad (5)$$

Thus, four high frequency cavities and one 13MHz cavity can manipulate about 30% of the muons.

3.1 Double Frequency Cavity

Because it uses the beats, phase slips between cavities have to be small. If two components are generated in a cavity, the phase slip between the two components is neglected. Figure 6 shows a rough example of such cavity that has resonant frequencies of 93MHz and 108MHz and shunt impedance of about 6MΩ each. The diameter and the length are ø2.3m and 1m, respectively

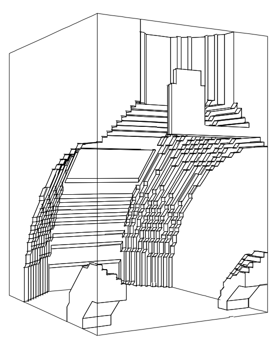

Fig. 6 Double Frequency Cavity

3.2 Modulating Buncher

The resulted muons are bunched with 100MHz carrier, which is suitable for post acceleration, while more muons are desired to be treated; the muons have to be bunched before the energy compression. A required wave form for a buncher located at a middle point between the production target and the CPEC is

$$cL^2 M_\mu \omega_c^{-1}\left(c^2t^2-L^2\right)^{-3/2}\sin 2\omega_c t, \qquad (6)$$

where L is the distance between the production target and the buncher, M_μ is muon mass in eV unit and ω_c is the carrier frequency (see Fig.7). The amplitude term before the sine function is dE/dt at the buncher location, where E is a muon energy at the buncher location. Equation 6 can be approximated by

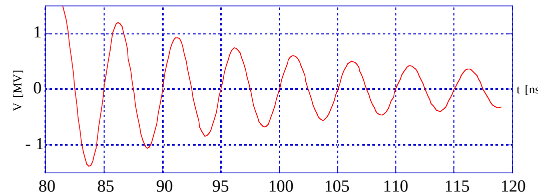

Fig. 7 Modulated Bunching Waveform

$$\left\{a_0-a_1\sin(\omega_b(t-t_1))-a_2\sin(2\omega_b(t-t_2))\right\}\sin 2\omega_c t, \qquad (7)$$

where ω_b is a buncher frequency (usually ~few MHz). Again, the required wave form can be synthesized by

$$\begin{aligned}&a_0\sin 2\omega_c t-a_1\cos(\omega_b-2\omega_c)t+a_1\cos(\omega_b-2\omega_c)t\\&\quad-a_2\cos(2\omega_b-2\omega_c)t+a_2\cos(2\omega_b+2\omega_c)t\end{aligned}, \qquad (8)$$

where phase factors are omitted. Thus, five components will generate the wave form, which can be achieved by two DFC and one single mode cavity. Fig. 8 shows a rough CPEC simulation for 20MeV muons with 30m decay channel and the modulating buncher at the center.

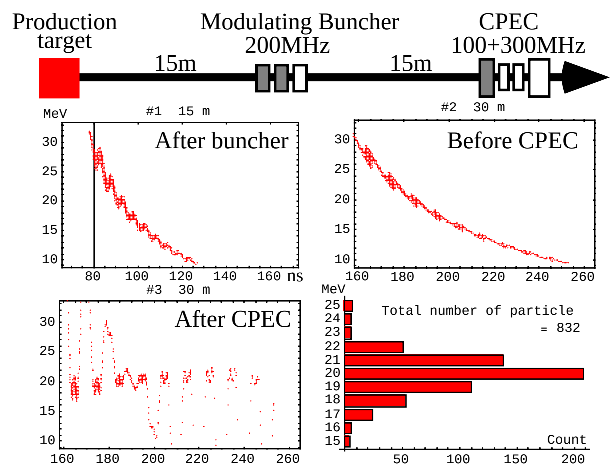

Fig. 8 Rough simulation with modulating buncher.

4 DISCUSSION

Because these schemes need power for multiple frequencies, power feeding scheme should be established to realize them. Phase slips between cavities should be kept small for a CPEC installation.

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