# Possible resonance free lattices for the VLHC

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*Abstract*

A systematic search for resonance-free lattices has been performed. Numerous solutions exist for sets of about fifty cells. A VLHC-like machine could be made of blocks of 61 cells which are free from resonances up to tenth order. A tracking test shows a 20% improvement of the dynamic aperture with respect to standard cells, for several multipole components. It opens the possibility of relaxing the constraints on systematic multipole components for the magnet cells. The tolerances on field quality will then be dictated by chromatic and an-harmonic effects of the multipole components.

## 1 INTRODUCTION

The principle of resonance-free lattices has been established in ref. [1]. An application to the LHC has shown that it is indeed attractive for practical machines [2]. For this latter case, it was shown that an acceptable dynamic aperture could be obtained with an octupole uncertainty b4 of $1.5\times10^{-4}$ at 17mm from the dipole centre , which is about three times the realistic value estimated by the magnet builders (uncertainty means that the dipoles of each arc has a different systematic b4 component with this r.m.s. value).

The fact that the dynamic aperture is somewhat insensitive to the multipole strength opens an interesting possibility for a very large machine like VLHC. Designing arcs composed of resonance-free sets of cells, makes it possible to reduce the number of multipole correctors to the very minimum without worrying about dynamic aperture.

In what follows, we recall first the basic principles of resonance-free lattices. Then the list of possible cell arrangements with the minimum number of cell is given. A tentative machine layout is proposed.

## 2 RESONANCE-FREE LATTICES

We recall here the main arguments developed in [1] in the frame of the single resonance theory (see for instance [3]). We consider only the response in amplitude of an harmonic oscillator driven by the non-linear field associated with the unperturbed linear oscillation, **to first order in multipole strength.** The change of frequency with amplitude is not taken into account. On a resonance of order $n$ defined by $n_x.Q_x + n_y.Q_y = integer$, with $|n_x| + |n_y| = n$, the driving term, which originates from the Fourier transform of the non-linear field created by the excursion of the linear motion in the multipole $b_n$, is proportional to the circumferential integral [3] :

$$\left| \int_0^C b_n \beta_x^{\frac{n_x}{2}} \beta_y^{\frac{n_y}{2}} e^{i(n_x\mu_x + n_y\mu_y)} ds \right|$$

It can be shown that, for an ensemble of $N_c$ identical cells, the condition of cancellation of this integral is quite simple and leads to the following theorem [1] :

**" A part of a circular machine containing $N_c$ identical cells will not contribute to the excitation of any non-linear resonance, to first order in multipole strength, except those defined by**

$$n_x\mu_{x,c} + n_y\mu_{y,c} = 2k_3\pi, \tag{1}$$

**if the phase advances per cell are given the values :**

$$\mu_{x,c} = \frac{2k_1\pi}{N_c} \qquad \mu_{y,c} = \frac{2k_2\pi}{N_c} \tag{2}$$

**which cancel the one-D non-linear resonances, $k_1$ $k_2$ and $k_3$ being any integers."**

The usefulness of this theorem lies in the fact that there are much more resonances cancelled than excited. It is precisely the aim of the present paper to show that, depending on the number of cells $N_c$ and the numbers $k_1$ and $k_2$ there are no resonance excited up to a quite high order (to first order in multipole strength). For instance those associated with sextupoles are third order as well as second order (the latter are called sub-resonances as only third order resonances are traditionally associated with sextupoles). The fourth order resonances associated with sextupoles come from a second order effect which is not considered here.

Applications of resonance-free lattices have been already worked out. As said above, it has been shown that an LHC optics using this principle is rather insensitive to a large octupole uncertainty [2]. A restricted version of the principle has also been tested experimentally in LEP a long time ago, when there was a serious coupling problem associated with unexpected skew quadrupole components in the dipoles. The linear coupling resonance defined by $n_x = 1, n_y = -1$ is cancelled provided $\mu_{x,c} - \mu_{y,c} \neq 2k'\pi$. Taking $k_2 = k_1 \pm 1$, fulfils this condition provided $\mu_{x,c}$ and $\mu_{y,c}$ take the specified values. The solution retained for LEP was $\mu_{y,c} = 60°$ and $\mu_{x,c} = 71.3°$. The value of $60°$ of the vertical phase was needed for the non-linear chromaticity correction, it does not fulfil the condition for the cancellation of the vertical resonances for a number of 32 cells which is that of LEP arc. This number of cells per arc requires a value of the horizontal phase advance equal to the vertical one plus $k_1 \times 11.25°$ to cancel the linear coupling resonance alone. The value retained for $\mu_{x,c}$ corresponds to $k_1 = 1$. It was shown experimentally that the importance of the linear coupling resonance was dramatically reduced [4].

## 3 RESONANCE-FREE ENSEMBLES OF FODO CELLS

There is no need for a sophisticated algorithm to find values for the three parameters $N_c, k_1, k_2$ which do not satisfy equation 1. The simplest method for doing this is to find $n_x, n_y$ and $k'$ which satisfy equation 1 for given $N_c, k_1, k_2$ up to a given order of resonance, simply discard those values of $N_c, k_1, k_2$. The values which are not discarded constitute the solutions for resonance-free lattices.

We start by putting the values of $\mu_x$ and $\mu_y$ given by equation 2 in equation 1. The solution to our problem consists then of solving the Diophantine equation :

$$n_x k_1 + n_y k_2 = k' N_c \tag{3}$$

The values of $k_1$ and $k_2$ are constrained to provide reasonable values of the phase advances. As the phase advance of FODO cells is always smaller than $\pi$, the value of $k_1$, associated arbitrarily with the horizontal plane, is constrained to be smaller than $N_c/2$. As we want to have values of the phase advance leading to reasonable $\beta$-values, it is also constrained to be larger than $0.13N_c$ (lowest value corresponding to a phase advance of 45°).

Similarly the value of $k_2$, associated arbitrarily with the vertical plane has to be comprised between $0.13N_c$ and $0.31N_c$. These values determine phase advances such that the maximum $\beta$-functions are not far from the minimum they take when the phase advance is varied. Furthermore we impose $k_1 > k_2$ as the focusing must be stronger in the horizontal plane for aperture reasons. The solutions are listed in the tables 1 to 9. Each table contains the smallest values of $N_c$ as well as possible $k_1$ and $k_2$, associated with a given resonance order.

Table 1: Lowest values of $N_c$ , $k_1$ and $k_2$ with no resonance up to second order only.

| | | | | |
|---|---|---|---|---|
| $k_1$ | 2 | 4 | 6 | 6 |
| $k_2$ | 1 | 2 | 3 | 3 |
| $N_c$ | 7 | 15 | 22 | 23 |

Table 2: Lowest values of $N_c$ , $k_1$ and $k_2$ with no resonance up to third order only.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k_1$ | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 |
| $k_2$ | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 3 | 4 |
| $N_c$ | 10 | 11 | 12 | 14 | 15 | 16 | 17 | 18 | 18 |

Table 3: Lowest values of $N_c$ , $k_1$ and $k_2$ with no resonance up to fourth order

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k_1$ | 3 | 3 | 3 | 4 | 5 | 4 | 4 | 5 | 4 |
| $k_2$ | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 |
| $N_c$ | 13 | 14 | 15 | 17 | 17 | 18 | 19 | 19 | 20 |

Table 4: Lowest values of $N_c$ , $k_1$ and $k_2$ with no resonance up to fifth order

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k_1$ | 4 | 4 | 5 | 4 | 5 | 5 | 7 | 8 | 5 |
| $k_2$ | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 7 | 4 |
| $N_c$ | 21 | 22 | 22 | 23 | 26 | 27 | 27 | 27 | 28 |

Table 5: Lowest values of $N_c$ , $k_1$ and $k_2$ with no resonance up to sixth order

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k_1$ | 8 | 8 | 9 | 5 | 8 | 9 | 8 | 9 | 8 |
| $k_2$ | 6 | 6 | 8 | 4 | 6 | 7 | 6 | 7 | 7 |
| $N_c$ | 27 | 29 | 30 | 31 | 31 | 31 | 33 | 33 | 34 |

Table 6: Lowest values of $N_c$ , $k_1$ and $k_2$ with no resonance up to seventh order

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k_1$ | 10 | 11 | 10 | 11 | 9 | 10 | 10 | 10 | 11 |
| $k_2$ | 6 | 6 | 8 | 7 | 8 | 6 | 7 | 8 | 9 |
| $N_c$ | 37 | 37 | 37 | 38 | 38 | 39 | 39 | 39 | 39 |

Table 7: Lowest values of $N_c$ , $k_1$ and $k_2$ with no resonance up to eighth order

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k_1$ | 10 | 11 | 10 | 10 | 12 | 11 | 10 | 13 | 10 |
| $k_2$ | 8 | 8 | 8 | 9 | 7 | 7 | 8 | 8 | 8 |
| $N_c$ | 41 | 42 | 43 | 43 | 44 | 45 | 45 | 46 | 47 |

Table 8: Lowest values of $N_c$ , $k_1$ and $k_2$ with no resonance up to ninth order

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k_1$ | 13 | 11 | 15 | 14 | 12 | 13 | 17 | 12 | 16 |
| $k_2$ | 7 | 9 | 7 | 8 | 10 | 9 | 9 | 10 | 15 |
| $N_c$ | 50 | 50 | 55 | 55 | 55 | 55 | 57 | 57 | 57 |

Table 9: Lowest values of $N_c$ , $k_1$ and $k_2$ with no resonance up to tenth order

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k_1$ | 12 | 18 | 17 | 14 | 18 | 12 | 12 | 13 | 18 |
| $k_2$ | 10 | 15 | 8 | 9 | 15 | 11 | 10 | 10 | 15 |
| $N_c$ | 61 | 61 | 62 | 62 | 62 | 63 | 63 | 64 | 64 |

## 4 EXAMPLES OF LATTICES FOR THE VLHC

The total number of cells of the VLHC presently envisaged is between 400 and 500 [5]. The lattice contains two interaction regions, which leads naturally to a race-track layout.

Using sets of 61 cells to build the arcs, the simplest solution consists of using 4 sets per arc (about half a machine) with $k_1 = 18$, i.e. $\mu_{x,c} = 106.23°$ and $k_2 = 15$, i.e. $\mu_{y,c} = 88.52°$ (see table 9). At the end each arc there must be a dispersion suppressor which is made from two cells with missing dipoles and two independently powered horizontally focusing quadrupoles to make the horizontal dispersion exactly zero at the exit of the arc. In the empty spaces associated with the missing dipoles, multipole correctors have to be provided to complete the resonance-free scheme according to the most important systematic multipoles in the dipoles.

The cell length, which is an important optimisation parameter [5], is not constrained by the scheme. It can probably be increased compared with a conventional scheme, thanks to its larger dynamic aperture, in order to reduce the number of quadrupoles. This length will be probably limited by the non-linear chromaticity and the anharmonicities associated with the multipoles. In order to cope with these problems, one set of multipole compensators could be installed in each sequence of 61 cells, which is compatible with the resonance-free scheme. For the above case, there is room for four different multipoles, e.g. sextupoles, octupoles, decapoles and do-decapoles per half ring. As each set of corrector does not excite its associated non-linear resonance, it can be probably be excited enough to compensate for the rest of the half ring. This is particularly true for the octupole scheme which might be needed for landau damping at high energy [6]. Of course the exact range of excitation of such correctors have yet to be determined as the resonance-free system is associated with first order multipole strength only.

A sorting of the magnets by sets of 61 can be envisaged in order to reduce the values of the most important random multipole components since they will be dominant once the effect of the systematic components is attenuated.

## 5 TRACKING TEST

Tracking was done on a race-track ring with two superperiods which could simulate a VLHC. There are two arcs made from four times 61 cells. Transfer matrices are added at both end of each arc to simulate an insertion and to set both $\beta$ values to 1m at the end of each super-period. The circumference is of the order of 50km. No dispersion suppressor has been included as the scheme has a zero dispersion in the insertions, due to the integer phases in the arcs. The cell phase advances are defined by $k_1$=12 and $k_1$=10 which makes the $\beta$-functions a littler large but avoids resonances up to 10th order. The cell length is 100m.

The dynamic aperture has been computed for three different arc cells : 60$\circ$ in both planes (labelled 60/60), 60$\circ$ in both planes (labelled 90/90) and the resonance-free lattice with $k_1$=12, $k_2$=10. Some optics parameters associated with these optics are given in table 10. The dynamic aperture is defined for a given ratio of the initial coordinates, e.g. $\{$x,y=x/10$\}$, $\{$x,y=x$\}$, $\{$x,y=10x$\}$, by the largest value of the larger coordinate for which the betatron oscillations remain stable over $10^4$ turns. The chromaticity sextupoles are included but not the synchrotron oscillations. The results are given in the tables 11 and 12 for different multipole contents of the dipoles.

Table 10: Characteristics of the optics used for tracking.

| | Qx | Qy | $\hat{\beta_x}$/m | $\hat{\beta_y}$/m | $\hat{Dx}$/m |
|---|---|---|---|---|---|
| 60/60 | 122.28 | 122.31 | 170 | 170 | 1.74 |
| 90/90 | 82.28 | 82.31 | 173 | 173 | 3.22 |
| Res. free | 96.28 | 80.31 | 160 | 182 | 2.41 |

Table 11: Dynamic aperture results for b4=b5=10$^{-4}$ at 0.01m from the dipole centre, for three different optics. Both $\beta$-functions are equal to 1m and both $\alpha$ are zero at the starting point. The coordinates are in millimetre, the resolution is 1%. The anharmonicities are in $10^5 m^{-1}$.

| optics | y=x/10 | y=x | x=y/10 | $\frac{\partial Q_x}{\partial E_x}$ | $\frac{\partial Q_y}{\partial E_x}$ | $\frac{\partial Q_y}{\partial E_y}$ |
|---|---|---|---|---|---|---|
| 90/90 | 0.64 | 0.65 | 0.87 | 4.12 | -8.47 | 4.22 |
| 60/60 | 1.11 | 0.65 | 1.17 | 7.65 | -15.3 | 7.65 |
| Res. free | 0.92 | 0.80 | 1.15 | 5.65 | -13.6 | 8.09 |

Table 12: Dynamic aperture results for b10=10$^{-4}$ at 0.01m from the dipole centre. Same optics and same units as for fig.11.The anharmonicities are in $m^{-1}$

| optics | y=x/10 | y=x | x=y/10 | $\frac{\partial Q_x}{\partial Q_x E_x}$ | $\frac{\partial Q_y}{\partial E_x}$ | $\frac{\partial Q_y}{\partial E_y}$ |
|---|---|---|---|---|---|---|
| 90/90 | 0.89 | 0.58 | 0.86 | -579 | -3485 | 196 |
| 60/60 | 0.71 | 0.50 | 0.66 | -295 | -930 | -219 |
| Res. free | 1.01 | 0.70 | 1.00 | -472 | -1757 | 91 |

## 6 CONCLUSION

A VLHC lattice could be built from sets of 61 cells. With suitable phase advances, these sets do not excite systematic resonances of order up to 10 to first order in multipole strength.

Limited numbers of multipole correctors can be used, typically one multipole type per set of 61 cells .

## 7 REFERENCES


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