SIMULATION OF SHORT BUNCHED AND ACCELERATED BEAM BY AN UN-TUNED CAVITY

K. Ohtomo, S. Shibuya, Sumitomo Heavy Industries, Tokyo, Japan
T. Murakami, K. Noda, S. Yamada, National Institute of Radiological Sciences, Chiba, Japan

Abstract

Beam manipulation in a synchrotron ring by an un-tuned cavity has attracted considerable attention in recent years. An un-tuned cavity characterized by a broad band in frequency and a low Q value makes it possible to modify an RF waveform in a desired manner. Beam bunching is one of the applications of a beam manipulation technique. It uses a bunch rotation after a quarter period of a synchrotron oscillation with an adiabatically ramping RF voltage. It is expected that a short bunch is attained by high speed ramping of an RF voltage and a saw-toothed wave form, both of which are easily realized by an un-tuned cavity. As the bunch rotation proceeds, a beam intensity at a core of the bunch increases. The high intensity may harm the rotating beam through the space charge effects and the beam loading due to the wide frequency range of the un-tuned cavity. We performed numerical calculations of beam orbits including those effects for the small ion synchrotron ring; a future project at the National Institute of Radiological Sciences (NIRS). Calculated results showed the following conclusion. Firstly, no fatal effects due to the beam loading and the space charge appear up to a stored current of 100 mA during a few hundred μs which is required for a short bunch extraction. Secondly, a considerable number of beams are spilt from an RF bucket during a five hundred ms corresponding to the acceleration period.

1 SMALL ION SYNCHROTRON RING

A small ion synchrotron ring was proposed at NIRS[1]. The purpose of the new ring is to supply beams, which are characterized with a short bunch less than 10 ns and variable energies ranging from 1 MeV/u to 28 MeV/u for ions of q/A=1/2. Heavy ion beams from proton to Xe with an energy of 6MeV/u are injected into the ring with a multi-turn injection and cooling-stacking scheme up to 5x 10^9 particles or more. After the acceleration or the deceleration beam bunches are extracted by a fast kicker. The Electron Cooler (EC) is installed to realize a beam with a high intensity and small emittance. The un-tuned cavity is equipped for forming a short bunch or acceleration. The beams from the ring will be used for experiments including radiation chemistry, biology, and nuclear physics.

Design parameters of the ring are listed in Table 1, and a layout is shown in Fig. 1.

Table 1: Parameters of the small ring

Injection Energy [MeV/u]	6
Extraction Energy for q/A=1/2 [MeV/u]	1-28
Circumference [m]	23.712
Magnetic Rigidity [Tm]	0.3-1.54
Bending Radius [m]	1.1
Bending Angle / Edge Angle [deg]	90/22.5
Nominal Tune (Hori./Vert.)	2.21/1.35
Natural Chromaticity (Hori./Vert.)	-2.12/-5.61
Momentum Compaction Factor	0.106
RF Frequency for Harmonics=1 [MHz]	0.6-4
RF Voltage [V]	500-2000
Shorting Bunch Length [ns]	<10
Expected Beam Intensity [pps]	$>5\times10^9$
Momentum Spread of Cooled Beam [%]	<0.01

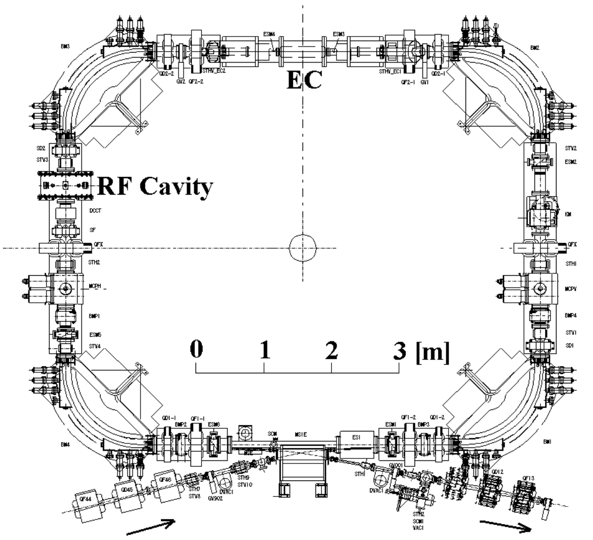

Figure 1: Layout of the ring.

2 UN-TUNED CAVITY

2.1 Magnetic Alloy Core

Performance of the un-tuned cavity is derived from a new material : Fe-based nano-crystalline soft magnetic alloy[1], which has high permeability in MF and HF band, capability of high power dissipation and high temperature resistibility. These properties may make frequency- and amplitude-control system simple. A core was manufactured by rolling and stacking from thin magnetic

---

[1] http://www.nrim.go.jp/open/usr/hono/apfim/project/finemet.html

alloy tape. It has an outer diameter of 650mm, an inner diameter of 290mm, and a thickness of 26mm. The complex permeability of cores were measured at frequencies of 0.5, 1, 5, and 10MHz. By fitting those data, it was found that the real part of permeability is proportional to frequency to the –0.93 of power and the imaginary part are to one to the –0.7 of power. It was nearly same comparing with other core of un-tuned cavity at 2MHz[2].

2.2 *Cavity*

The un-tuned cavity for R&D was constructed. It has a length of 350mm along a beam path, and a diameter of 780mm. The cavity is a re-entrant cavity filled with magnetic core rather than push-pull λ/4 type. A maximum of 8 cores can be installed in the cavity. An RF power is fed through a coupling loop attached to the cores. Measurements of impedance were performed by changing the number of cores and a size of the coupling loop as a parameter using low level. Finally we decided to add a 1:9 transformer in an input circuit for impedance matching and installed 5 cores in the cavity because power reflection was minimum. A model circuit consisting of passive elements was established from the data. Equations expressing the model of the core are listed below. In the model, the core is expressed as an inductance and a resistance having frequency dependence connected in series, and connected to a capacitance of the cavity in parallel[3]. The measured impedance of the cavity are shown in Fig. 2, as well as those estimated with the model.

$$L = \frac{1}{2\pi}\mu'(\omega)\mu_o t \ln\frac{b}{a}$$

$$C = 2\pi\varepsilon'\varepsilon_o t / \ln\frac{b}{a}$$

$$R = \frac{1}{2\pi}\mu''(\omega)\mu_o t\omega \ln\frac{b}{a}$$

$$Z = \frac{R + j\omega L}{1-\omega^2 LC + j\omega RC} = \frac{R}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2} + j\frac{\omega L - \omega^3 L^2 C - \omega R^2 C}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}$$

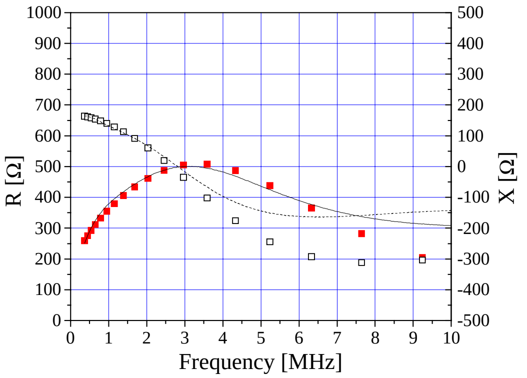

Figure 2: Impedance of un-tuned cavity.
Squares indicate calculated values from the model circuit, and lines the measured values. Red mark correspond to resistance being referred to the left scale while black one reactance to the right scale.

3 NUMERICAL SIMULATION

3.1 *Formula*

Evolution of the synchrotron oscillation was simulated in the longitudinal phase space including the space charge effect and the beam loading from the un-tuned cavity using the Runge-Kutta Method. Particle distribution in the phase space is calculated for each revolution. Then it is supposed that the derivative of particle density induces the space charge force while the impedance multiplied by the current up to 5th component of the Fourier Transform of particle distribution does the beam loading. Formulas of equations are listed below[4].

$$\frac{d\phi}{dt} = \frac{\omega h\eta}{\beta^2 E_0}(E - E_0)$$

$$\frac{dE}{dt} = \frac{\omega q}{2\pi}\left(V_{RF}(t)(\sin\phi - 1/9 * \sin 3\phi) + \cdots + V_{SC}(\phi) + V_{BL}(\phi)\right)$$

$$V_{SC}(\phi) = \left| -j\frac{Z_0}{2\beta\gamma^2}\left(1 + 2\log\left(\frac{b}{a}\right)\right)\right| \frac{\left(I(\phi_{i+1}) - I(\phi_i)\right)}{d\phi}$$

$$V_{BL}(\phi) = \sum_{0}^{5} Z(i\omega)I_i \cos(i\phi) + \sum_{-5}^{-1} Z(-i\omega)I_i \sin(i\phi)$$

Here *a* and *b* are beam and duct radii, supposed to a constant throughout the ring, 20mm and 30mm, respectively. *Z* means the impedance of the un-tuned cavity, which was described in the last section. In the initial stage, 10000 particles are distributed uniformly in a phase axis and gaussian-like in a momentum axis. The particles are assumed to be fully-stripped carbon ions having an average energy of 6MeV/u.

3.2 *Bunching*

In simulation of shortly bunched beams, an amplitude of RF voltage (V_{RF}) increases from 0 to a top voltage (normally set to 1000V) during 10µs. The top voltage continues several hundred µs equal to a quarter period of synchrotron oscillation. Fundamental RF frequency is set to 1.428 MHz that is equal to the frequency at the injection. The 3rd harmonic component is added to the fundamental one in order to produce a wave form approximately linear between -90 and 90 degrees. Initial momentum spread is supposed to $\pm 10^{-4}$.

A wave form, top RF voltage, ramping time, and stored currents were varied to fine a best condition to make the bunch length shortest. In a case of a wave where a 3rd harmonic component was added, about a half of particles were contained in a short bunch, while in a case of a simple sine wave a third are contained in a bunch. It was conjectured that the peak of the distribution became sharper as the top voltage increased. When the top voltage was changed from 500 to 2000V, FWHMs of the peak reduced 10 and 6 degrees, corresponding 20 and 12ns, respectively. There was a difference of a few percent whene varying the ramping time from 10 to 100 µs. The influence caused by stored current up was slight up to 1A, as shown in Fig. 3.

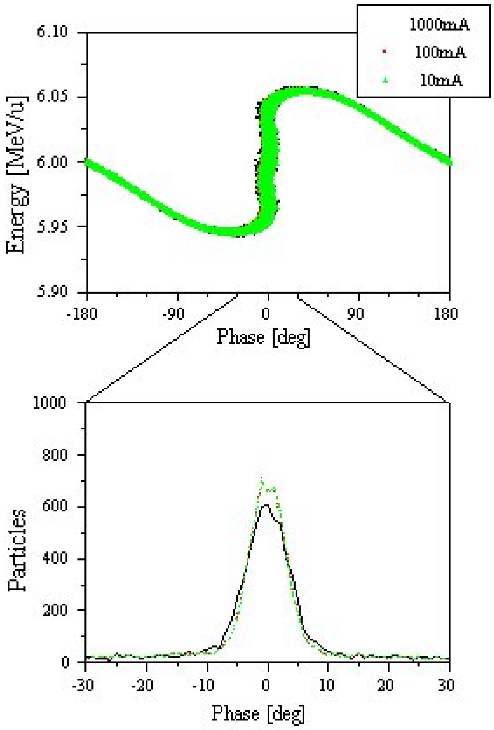

Figure 3: Distribution of a short bunch. Green, red, and Black correspond to cases of 10, 100, and 1000mA, respectively.

3.3 Acceleration

In simulation of acceleration, the amplitude of RF voltage was changed so as to keep an area of separatrix constant. Particles were accelerated up to 28 MeV/u in 450ms. IA momentum spread at the injection was set to $\pm 10^{-3}$. Other parameters except beam current were same as those used in the previous calculations.

At first the space charge and beam loading effects were estimated independently. Fig. 4 shows a particle distribution in the longitudinal phase space at the beginning stage of acceleration with each effect only when current is 100mA. Comparing with the cases including both effects or no effect, it was found that the space charge effect was dominant to spill particles out of RF bucket. After acceleration, all particle in RF bucket were lost with the space charge effect only, while 87.9% of the particles survived with the beam loading effect only and 89.4% with no effect.

As a next step, calculations including both effects were performed for beam currents of 1mA, 5mA, 10mA, and 50mA. Particle distributions at the end of the acceleration period are shown in Fig. 5. Although no particles were accelerated to the final energy at a current of 50mA, 44%, 64%, and 84% of particles were accelerated at currents of 10mA, 5mA, and 1mA, respectively. From these results, it is speculated that there may be a threshold of current accumulated in the small ring and it might be 3~4mA.

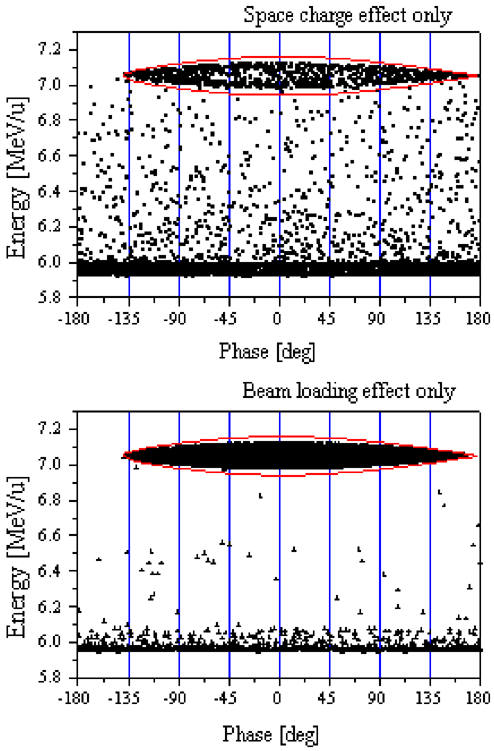

Figure 4: Comparison between space charge effects and beam loading effects.

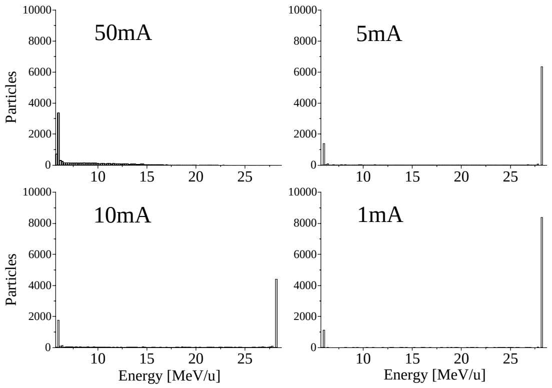

Figure 5: Particle distributions after the acceleration for the currents of 50, 10, 5, and 1mA, respectively.

REFERENCES

[1] K. Noda et al., Proc. Workshop on S-ring, NIRS, 1997, HIMAC-report-017, p.1-33.
[2] C. Ohmori et al., “A Wide-band RF Cavity for JHF Synchrotrons”, PAC97, Vancouver, May 1997.
[3] K. Ohtomo, Y. Kumata, “Un-tuned Cavity for Small Ion Synchrotron”, Sumitomo Heavy Industries Technical Review, vol.47 No.141, Dec. 1999.
[4] S. Y. Zhang, W. T. Weng, “Beam loading effect in bunch leakage at the PSR”, NIM-A, p.420 (1999),12-19