THE BEAM FORMING BY THE RF CAVITIES

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Abstract

The use of RF cavities with the field nonuniformly distributed in the transverse plane for the beam forming is considered. At installation them in the places of circle accelerator with the dispertion different from zero the longitudinal and transverse amplitude particle oscillations are changed, and consequently beam sizes are changed too. The appropriate beam formation can be used in particular for:

- increases of losses localization efficiency,
- suppression of pulsations at slow extraction,
- receptions of desirable beam parameters for modes of beam-beam interactions and with fixed target,
- increases of stochastic and electron cooling efficiency.

1 INTRODUCTION

In work the possibility of change the longitudinal and transverse amplitude oscillations with the help of special RF cavities located in the places where the magnitude of dispertion η in the appropriate plane is different from zero is considered. In a cavity the electrical field is directed along motion of particles and is distributed on a radius or vertical in such a manner that reduces or increases a momentum deviation. At the reducing of a beam momentum deviation the amplitude of appropriate betatron oscillations is increased and on the contrary. The possibility of using the special cavities for extraction and forming the beam in circle accelerators is considered. The computer simulation of swapping process of energy betatron oscillations of particles to longitudinal one and on the contrary with the help of cavities was made. This effect occurs in many circle accelerators due to existence of the parasitic field nonlinearities in the main cavities.
But this process uncontrollable and leads to magnification of beam sizes and consequently influence of this effect try to reduce.

The mechanism of change the longitudinal amplitude oscillations consists of the following. The particles of the beam have various momentum, differently will deviate from equilibrium orbit in places of the accelerator, where the dispertion is different from zero. Having a cavity with the electrical field definitely distributed on radius, it is possible selectively to influence on the particles with different momentum, reducing or increasing amplitude of longitudinal oscillations.

We research influence of a cavity with a longitudinal electrical field on the beam phase oscillations. Let's assume, that the electrical field is distributed on radius, then the field dependence versus the displacement x from the equilibrium orbit will look like:

$$E(x,t) = (E + \frac{dE}{dx}x + \frac{1}{2}\frac{d^2E}{dx^2}x^2 + ...)\cdot\cos\varphi\,, \quad (1)$$

where $\varphi = \omega t+\varphi_0$ is the phase oscillation of a field and ω is a frequency of cavity. The particle's displacement in the installation site of the cavity may represent (Fig.1):

$$x = r + \Delta r(u) = A\sin\Psi + \eta\frac{u}{p_s}\,, \qquad (2)$$

where $A = \sqrt{\varepsilon\beta}$ is the amplitude of betatron oscillations, ε is the emittance, β is the amplitude function, $\Psi = \gamma+2\pi Qn$ is the particle phase, γ is the initial phase of betatron oscillations, Q is the frequency of betatron oscillations, n is the amount of particle revolutions in the accelerator and $u = p\text{-}p_s$ is the deviation of particle momentum from the synchronous one.

Additional momentum acquired by the particle after passing through cavity:

$$\Delta u = \int_{t_1}^{t_2} eE(x,t)dt \approx eE(x)\cos\varphi\cdot\frac{l}{v},$$

Where e is the particle charge, l is the cavity gap length, v is the particle velocity, t_1 is the input time of a particle in the cavity and t_2 is the output time. That the particle momentum change did not averaging for some revolutions is necessary that the frequency of a cavity will be $\omega = q\omega_f$, where ω_f is the frequency of main accelerating field and q is multiple.

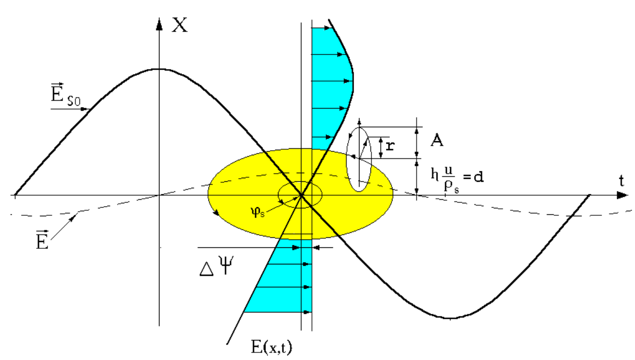

Figure 1: Motion of particles in longitudinal-transverse phase space and dependencies of electric RF fields in cavities.

In Fig.1 the characteristic sinusoidal dependence of the accelerating electric RF field versus the time is shown (E_{s0}). The closed curves show motion of particles in the longitudinal phase space. If the maximum of special cavity field is displaced from the equilibrium phase φ_s appropriate to bunch centre on $\Delta\psi$ for the phase of synchrotron oscillations, the phase of the particle flyover through cavity will be $\varphi = \Delta\varphi + q\psi$, where $\Delta\varphi = q\Delta\psi$, $\psi = \varphi\text{-}\varphi_s$, and change of momentum:

$$\Delta u = \frac{el}{v}\cos(\Delta\varphi + q\psi)\times(E + \frac{dE}{dx}x + \frac{1}{2}\frac{d^2E}{dx^2}x^2 + ...).$$

The constant component a little changes the amplitude and phase of the main RF field and in case q = 1: $\vec{E}_s = \vec{E}_{s0} + \vec{E}$. Even nonlinearities also influence to the main RF field by only nonlinear manner depending on the amplitude oscillations.

2 CAVITY WITH GRADIENT RF FIELD

Let's consider the influence of the first nonlinearity degree of field to the dynamics of synchrotron oscillations. The change of particle momentum at passing through cavity:

$$\Delta u = \frac{el}{v}\cos(\Delta\varphi + q\psi)\times\frac{dE}{dx}(A\sin(\gamma + 2\pi Qn) + \eta\frac{u}{p_s}). \quad (3)$$

As the influence of betatron oscillations for the some time averages, then

$$\frac{du}{dt} = \frac{du}{dn}\cdot\frac{dn}{dt} = C\cos(\Delta\varphi + q\psi)\cdot u, \quad \text{where}$$

$C = \frac{el\eta}{Lp_s}\cdot\frac{dE}{dx}$, such as $\frac{dn}{dt} = \frac{v}{L}$, L is the accelerator's perimeter. At the availability of accelerating RF field, the equations of small phase oscillations will look like:

$$\begin{cases} u' = -c_1\psi + C\cos(\Delta\varphi + q\psi)\cdot u, \\ \psi' = c_2 u, \end{cases} \quad \text{where}$$

$c_1 = \frac{eV\sin\varphi_s}{L}$, $c_2 = \frac{\omega_f\eta_\alpha}{p_s}$. From here we receive:

$$\psi'' - \psi' C\cos\Delta\varphi\cos q\psi + \psi' C\sin\Delta\varphi\sin q\psi + \Omega^2\psi = 0, \quad (4)$$

where $\Omega = c_1\times c_2$ is the frequency of small phase oscillations. The second member at ψ' averages for one phase oscillation. From the equation (4) it is visible, that the change of amplitude was greatest, it is necessary that $\Delta\varphi = 0,\pi n$, then

$$\psi'' - C\cdot\cos q\psi\cdot\psi' + \Omega^2\psi = 0. \quad (5)$$

At the multiple cavity q = 1 the magnitude of coefficient at the member ψ' will be greatest and, therefore, there will be a maximum effect of the cavity to the beam. In case of small phase oscillations $\cos\psi \sim 1$ and

$$\psi = \psi_0 e^{Ct/2 + i\omega t}, \; u' = \frac{\psi}{c}, \; where \; \omega = \sqrt{\Omega^2 - \frac{1}{4}C^2}. \quad (6)$$

That is the amplitude of phase deviation ψ and momentum are changed exponential during the time, with the decrement $C/2$.

Let's consider what happens with the transverse radial betatron oscillations in this case. The particle position in radial plane remains former after passing the cavity. The changing of angle will be $\Delta r' = -r'\Delta u/p$, but for some time that influence on the amplitude averaging if not accelerating process. The equilibrium orbit of particle displaces only due to change of the momentum and consequently on the same size changing coordinate of betatron oscillations $\Delta r = -\eta\Delta u/p_s$. We may fined the changing of amplitude by using the Courant and Snyder invariant $\varepsilon_r = A^2/\beta = \gamma r^2 + 2\alpha rr' + \beta r'^2$, then $\Delta A^2 = 2\Delta rA(\sin\Psi + \cos\Psi)$. The influence to a change of amplitude of the second member averages, that is visible at the substitutions significance Δu from the equation (3) in Δr. Then $\Delta A^2 = 2\Delta r\cdot r$. The influence of synchrotron oscillations on the change of betatron one averages for some revolutions and: $\Delta A^2 \approx -CA^2$, $A \approx A_0 e^{-Ct/2}$. That is, in how many times the amplitude of phase oscillations increases, in so much times the amplitude of radial betatron oscillations is decreased and on the contrary.

3 COMMON CASE

Generally the influence of m nonlinearity degree of RF field to the momentum change and transverse coordinate of the particle at passing it through the cavity:

$$\begin{cases} \Delta u = C_m\cdot(r+d)^m, \\ \Delta r = -C_m\cdot(r+d)^m\cdot\frac{\eta}{p}, \end{cases} \quad (7)$$

where $C_m = \frac{el}{v}\cdot\frac{1}{m!}\frac{d^mE}{dx^m}\cdot\cos(\Delta\varphi + \psi)$ and $d = \eta\frac{u}{p}$.

Change of small area ΔS_0 enveloping some of particles in a longitudinal - transverse phase space depending on the amount of their revolutions in the accelerator n:

$$\frac{\partial(\Delta S)}{\partial n} = \left[\frac{\partial}{\partial u}\left(\frac{\partial u}{\partial n}\right) + \frac{\partial}{\partial r}\left(\frac{\partial r}{\partial n}\right)\right]\cdot\Delta S_0.$$

Substituting in expression the above-stated significances of momentum and coordinate change is discovered that $\partial(\Delta S)/\partial n = 0$ for any degree of nonlinearity m, that is for any distribution of cavity's field the beam density in the three-dimensional phase space is saved.

Let's proceed to the new coordinates frame A, U, where $U = u_m\cdot\eta/p$, u_m is the momentum amplitude of phase oscillations. Then the influence of nonlinearities on those coordinates will be

$$\begin{cases} \Delta U^2 = 2C_m\cdot\frac{\eta}{p}d\cdot(r+d)^m, \\ \Delta A^2 = -2C_m\cdot\frac{\eta}{p}r\cdot(r+d)^m, \end{cases} \quad (8)$$

where $r = A\sin\Psi$ and $d = U\sin\xi$ for small phase oscillations. Averaging at first on to faster betatron, and then on to synchrotron oscillations decides the equations:

$$\begin{cases} \Delta U^2 = \frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{2\pi} C_m\cdot\frac{\eta}{p}d\cdot(r+d)^m\,d\Psi\,d\xi, \\ \Delta A^2 = -\frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{2\pi} C_m\cdot\frac{\eta}{p}r\cdot(r+d)^m\,d\Psi\,d\xi, \end{cases} \quad (9)$$

The influence of even nonlinearities for synchrotron oscillation averages. The relative change of momentum and amplitude for some odd RF field nonlinearities in the cavity: linear, cubic and fifth degrees, after averaging will be:

$$\begin{cases}\dfrac{\Delta U}{U}=\dfrac{1}{2}\dfrac{\eta}{p}\cdot C_1,\\ \dfrac{\Delta A}{A}=-\dfrac{1}{2}\dfrac{\eta}{p}\cdot C_1.\end{cases}\qquad \begin{cases}\dfrac{\Delta U}{U}=\dfrac{3}{8}\dfrac{\eta}{p}\cdot C_3\cdot(2A^2+U^2),\\ \dfrac{\Delta A}{A}=-\dfrac{3}{8}\dfrac{\eta}{p}\cdot C_3\cdot(A^2+2U^2).\end{cases}$$

$$\begin{cases}\dfrac{\Delta U}{U}=\dfrac{5}{16}\dfrac{\eta}{p}\cdot C_5\cdot(3A^4+12A^2U^2+U^4),\\ \dfrac{\Delta A}{A}=-\dfrac{5}{16}\dfrac{\eta}{p}\cdot C_3\cdot(A^4+12A^2U^2+3U^4).\end{cases}$$

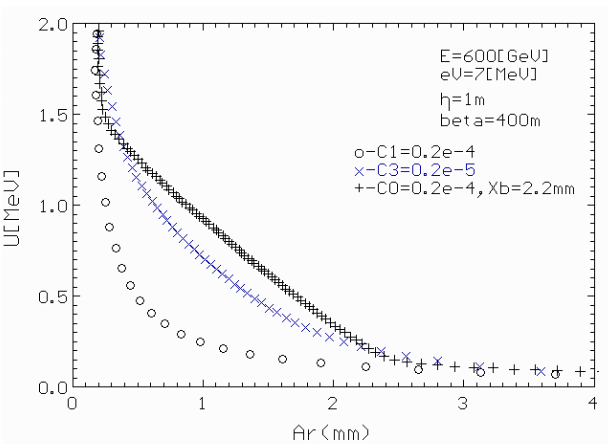

Figure 2: Motion of the particle on the phase plane (A,U_m) at the any field nonlinearities.

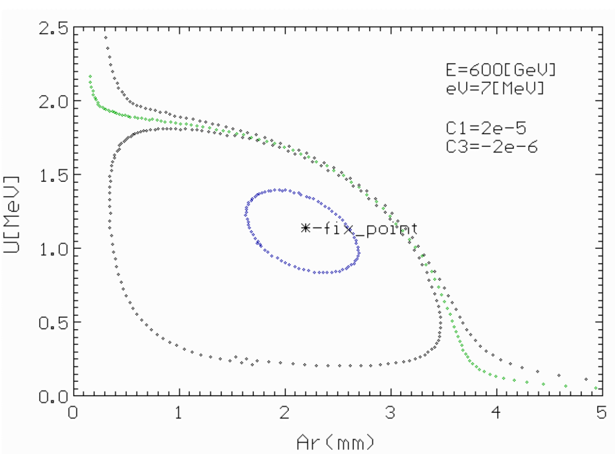

Figure 3: Motion of the particle on the phase plane with availability of the first and cubic degree nonlinearities in the cavity.

At the availability of odd nonlinearities the particle's motion on the phase plane (A,U) can be described by a Hamiltonian $H(A^2,U^2)$, that is fined from the equations

$$\frac{\partial H}{\partial A^2}=\frac{dU^2}{dn},\quad \frac{\partial H}{\partial U^2}=-\frac{dA^2}{dn}.$$

The motion of particles on that phase plane at the different nonlinearities in cavities is shown on Fig.2. All represented curves got by the computer simulation of real motion particle in the accelerator. Due to the influence of the linear RF field to the beam the particles move on the hyperbolas, so as $H_1=\eta/p\cdot C_1A^2U^2$ and $A\cdot U$ = constant. At the availability of two odd nonlinearities of the RF field with different signs there are stable areas of particle motion on the phase plane (A,U). So at availability of the first and cubic degree of nonlinearities in the cavity:

$$H=H_1+H_3=\frac{\eta}{p}C_1U^2A^2+\frac{3}{4}\frac{\eta}{p}C_3\cdot(U^2A^4+U^4A^2).$$

If the nonlinearities have a different sign, the particles move on closed curves around stable fixed point on phase plane (A, U), or at large amplitudes the motion is unstable Fig.3. The position of fixed point is defined from the equations ΔU = 0, ΔA = 0. In considered case $A_0 = U_0 = -2C_1/3C_3$.

The relative change of longitudinal - transverse phase volume, with consideration that $\varepsilon = \varepsilon_p\varepsilon_r$:

$$\frac{\Delta\varepsilon}{\varepsilon}=\frac{\Delta\varepsilon_p}{\varepsilon_p}+\frac{\Delta\varepsilon_r}{\varepsilon_r}=\frac{2\Delta U}{U}+\frac{2\Delta A}{A}.$$

At availability of the first degree of nonlinearity RF field in the cavity, how see from the above mention equations, the change of full phase volume will be $\Delta\varepsilon/\varepsilon = 0$, that is it is saved. For another odd nonlinearities it's not saved.

4 USING OF THE EFFECT

For forming a beam it is possible to use cavities with stepped distribution of an electrical field (Fig.2 (+)) which are installed in a place with the dispertion η distinct from zero. The change of a field relatively equilibrium orbit will look like:

$$\begin{cases}E(x,t)=E\cos\varphi, & x>x_b,\\ E(x,t)=0, & x<x_b,\end{cases}$$

where $\varphi = \omega t+\varphi_0$ is the oscillation phase of the electrical field in cavity. At the small exceeding by particle of the coordinate x_b it hits in cavity field them distinct from zero and acquires the additional change of momentum Δu, and the change of transverse particle coordinate will be $\Delta r=-\Delta u\cdot\eta/p$. The use of cavity such type enables to influence mainly on the beam halo. That it is possible to use for forming and extraction beam with added help of scattering target, bent crystal or betatron resonance. Then may considerably decrease the inevitable losses of particles and emittances of extracted beam.

Applying a cavity with the distributed transverse field it is possible considerably reduce the current pulsations of extracted beam at slow resonance extraction by increase of amplitude betatron or phase oscillations of beam particles. As there is a possibility to influence only on a small part of beam, it is possible considerably to increase the velocity of beam guiding at resonance on a comparison with a usual method of guiding, and, therefore, in as much time to reduce current pulsations of the extracted beam. In such a way suppressed not only betatron frequency pulsations but and main field pulsations. The cavities with the linear field distribution can be used for increase of luminosity in a mode of beam-beam interaction by reducing of the transverse size at the expense of increasing longitudinal.