INTRA-BEAM SCATTERING AT CELSIUS

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Abstract

Intra-beam scattering blow-ups are measured for 48 MeV and 400 MeV proton beams, and 200 MeV/*u* N^{7+} beam, at the storage ring CELSIUS, by means of the magnesium-jet beam profile monitor. The horizontal beam emittance growth with time is obtained, and is compared with the results of calculation by the extended Piwinski theory as well as the Bjorken and Mtingwa theory. The experimental measurements are in good agreement with the theoretical results.

1 INTRODUCTION

The resolution limits of a cooled ion beam in a storage ring is often dominated by intra-beam scattering (IBS), which is the multiple Coulomb scattering between charged particles in a bunched or unbunched beam. There are two general theories on IBS. One is the Piwinski model[1], generalized by Martini[2] to include the variations of the lattice functions around the ring. In this model, the Rutherford scattering between particles is calculated first. The scattering angles are then transformed to the laboratory frame and expressed as changes of emittance. The other is the Bjorken and Mtingwa (B-M) theory [3], based on the classical two-body scattering rule. Both approaches assume that the particle distribution remains Gaussian in the six-dimensional phase space. The derived IBS growth rates are expressed in complicated integral forms. Since no analytical expression for the lattice functions exists, the growth rates have to be calculated at different locations around the ring and then averaged over the whole ring. This can be done with a computer program [4].

This paper presents the results of calculation of IBS growth for different ions at CELSIUS according to the Martini and B-M formulae. The results are compared with the measurements.

2 CALCULATIONS

Following Martini expressions of the IBS growth rates in three dimensions[2], the amplitude growth rates are computed for unbunched ion beams of p (48 MeV), d (12 MeV/*u*), He^{2+} (12 MeV/*u*), N^{7+} (24.5 MeV/*u*), O^{8+}(18.8 MeV/*u*) and Ne^{10+} (17.3 MeV/*u*) at CELSIUS. The results are plotted in the figures 1-3. The number of particles is assumed to be 10^{11} for p, 10^{10} for d, He^{2+} and N^{7+}, and 10^{9+} for O^{8+} respectively in the figures 2 and 3.

Since the growth rates are generally varying with time, further calculations are performed for consecutive turns.

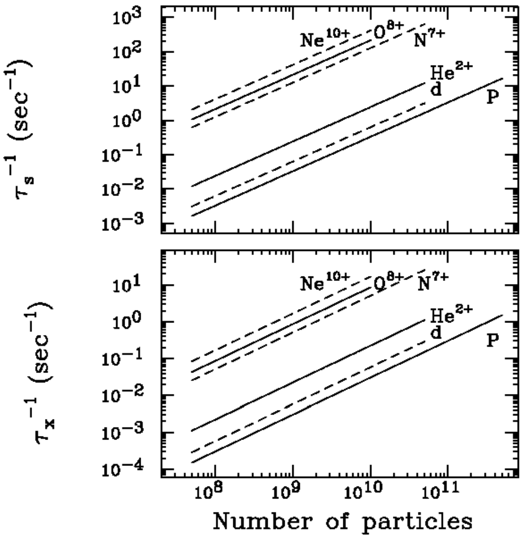

Figure 1: The longitudinal and horizontal IBS amplitude growth rates vs. the number of particles, in which $\varepsilon_x = \varepsilon_y = 1.0\pi mm.mrad$ (1σ),

$\Delta p/p = 1.0\times10^{-4}$ (1σ).

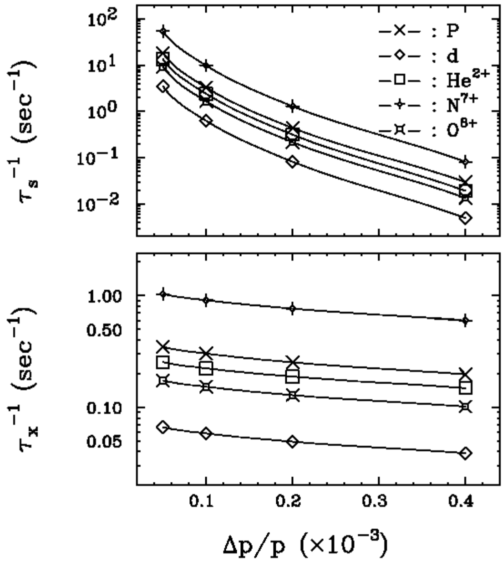

Figure 2: The longitudinal and horizontal IBS amplitude growth rates vs. the beam momentum spread(1σ), in which $\varepsilon_x = \varepsilon_y = 1.0\pi mm.mrad$ (1σ).

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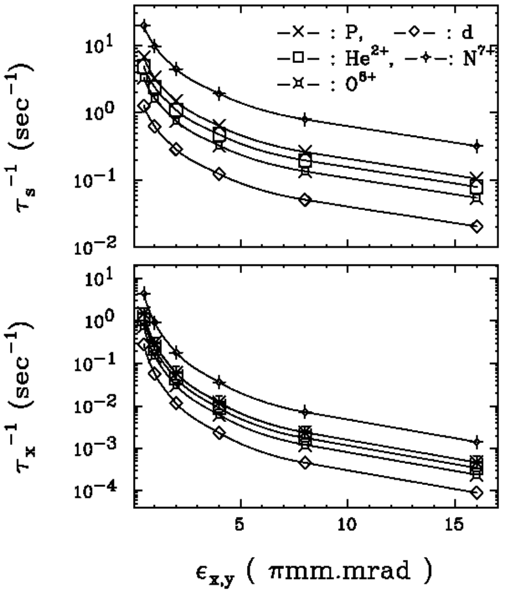

Figure 3: The longitudinal and horizontal IBS amplitude growth rates vs. the transverse emittances (1σ), in which $\Delta p/p = 1.0\times10^{-4}$ (1σ) .

Starting from the center of the e-cooler section with initial emittances ε_{x0} , ε_{y0} and relative momentum spread $(\Delta p/p)_0$, the three growth rates $\tau_j^{-1}(j = x, y, s)$ are calculated at some equally-spaced lattice locations through numerical integration to the formulae[2][5], and their average values are taken. The momentum spread and emittances grow to

$$\Delta p/p = (\Delta p/p)_0 \cdot \exp\left(\frac{t}{\tau_s}\right),\ \varepsilon_{x,y} = \varepsilon_{x0,y0} \cdot \exp\left(\frac{t}{\tau_{x,y}}\right)$$

after N_{ibs} turns. N_{ibs} is determined by

$$N_{ibs} = f \times \frac{\min\left(|\tau_x|,|\tau_y|,|\tau_s|\right)}{\tau_c}$$

where τ_c is the revolution period of the ion, and f denotes a factor (e.g. 0.1) for adjustment of the N_{ibs} value.

3 MEASUREMENTS

Beam blow-ups have been measured for unbunched proton beams of 48 MeV and 400 MeV, and a N^{7+} beam of 200 MeV/*u*, at CELSIUS, by means of the magnesium-jet beam profile monitor. These measurements were performed by first doing cooling accumulation, and then acceleration to the nominal energy, and then cooling again until the beam seemed to reach equilibrium. Thereafter, the cooling was turned off and the ion beam was left to blow up. The horizontal beam profile was scanned inwards and outwards every 1s (for the 200 MeV/*u* N^{7+} beam) or every 4s over a time interval of 40-100 s, during which the ion beam current was found to be nearly constant. Figure 4 exemplifies the recorded beam profiles.

Most of the profiles can be reproduced very well with a Gaussian curve as shown in fig.4, from which one gets the rms width σ of the distribution. The σ is correlated to the rms emittance and relative momentum spread by

$$\sigma = \sqrt{\beta_x \varepsilon_x + (D_x \cdot \Delta p/p)^2}$$

where $\beta_x = 10.5m$ and $D_x = -2.0m$ are the horizontal betatron and dispersion functions at the mg-jet point of CELSIUS respectively.

To get the evolution with time of the rms horizontal emittance, the dispersion contribution has to be subtracted from the σ , i. e.

$$\varepsilon_x = \frac{\sigma^2 - (D_x \cdot \Delta p/p)^2}{\beta_x}$$

For lack of the measurements of the longitudinal beam profiles, the relative momentum spread versus time is calculated following the above theory, supposing a starting value of $(1-4)\times10^{-4}$ and a starting vertical emittance identical to the horizontal one.

Figures 5-7 demonstrate the results of measurements and calculations by the Martini as well as B-M theories, using the standard lattice of CELSIUS. In these calculations, coupling of the transverse degrees of freedom is not taken into account. It turns out that the experimental results are in good agreement with the theoretical predictions, and a comparison between the two models yields a deviation below 10%.

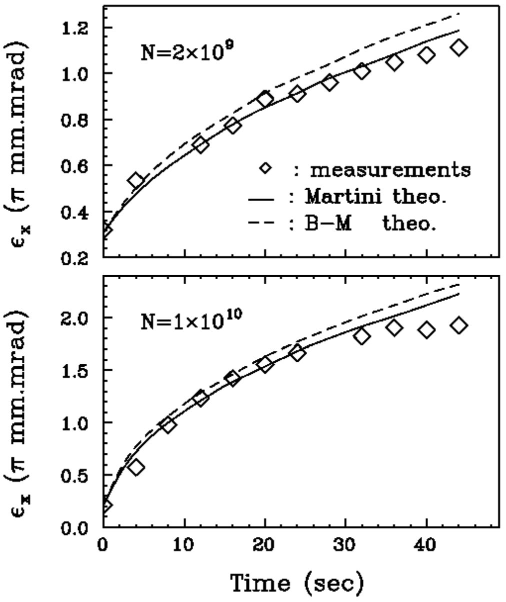

Figure 5: IBS growth of 48 MeV proton beam.

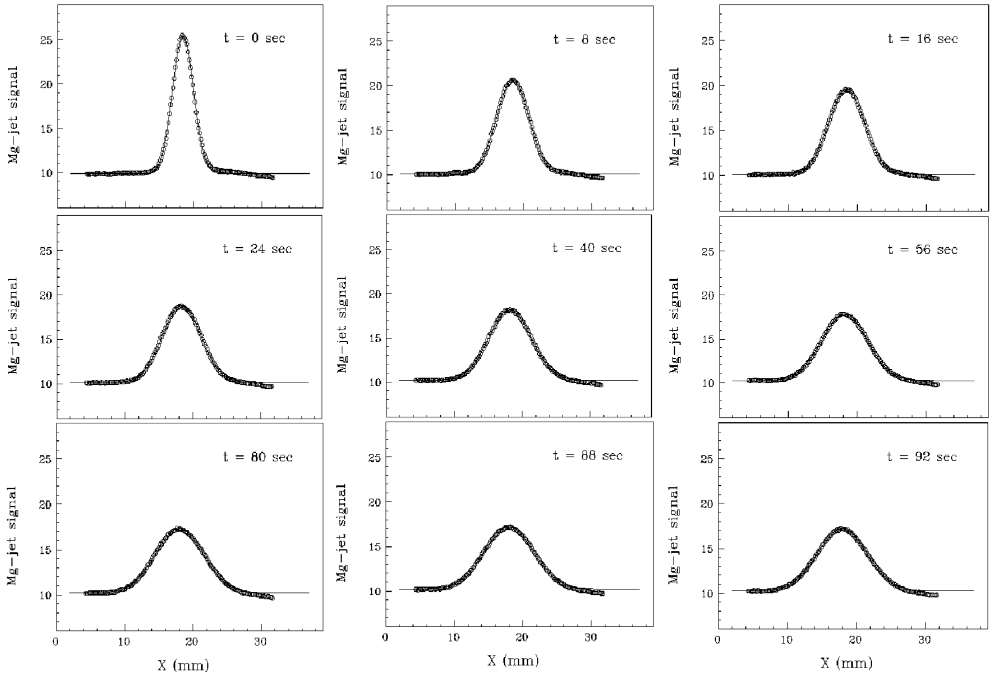

Figure 4: Profiles of 400 MeV proton beam during blow-up (averaged over several scans), t=0 corresponds to when the cooling was turned off, thin lines are aussian fittings.

4 CONCLUSION

The experimental measurements of IBS blow-ups agree satisfactorily with the theoretical results with an assumed starting relative momentum spread of $(1-4)\times10^{-4}$.Both Martini and B-M theories exhibits a deviation of less than 10%.

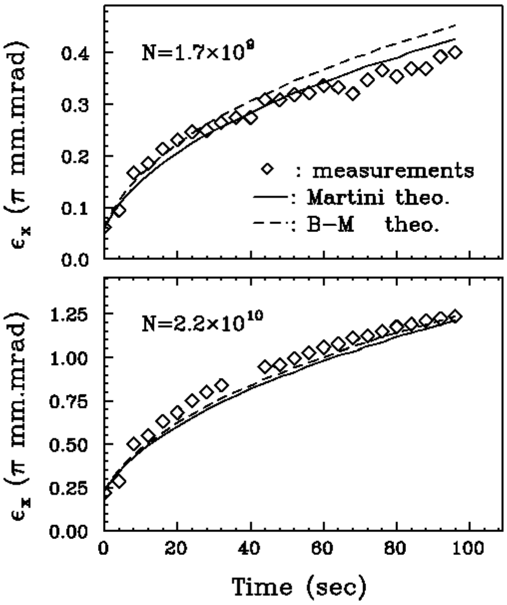

Figure 6: IBS growth of 400 MeV proton beam.

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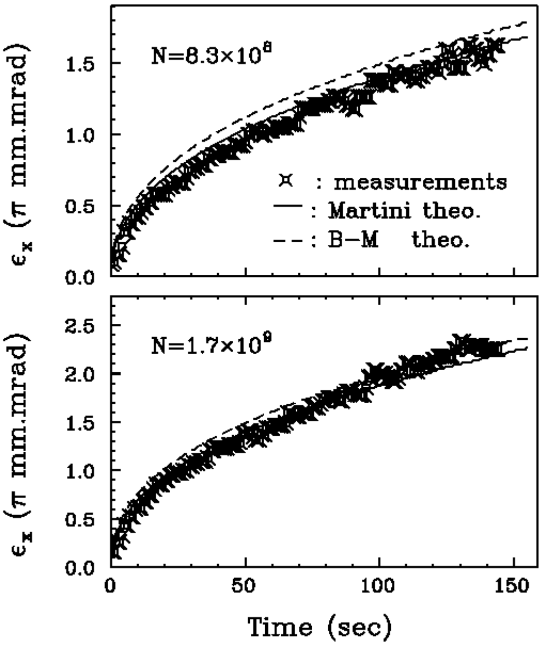

Figure 7: IBS growth of 200 MeV/u N^{7+} beam.

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