Three-Ring FFAG Complex for H^{+} and C^{6+} Therapy

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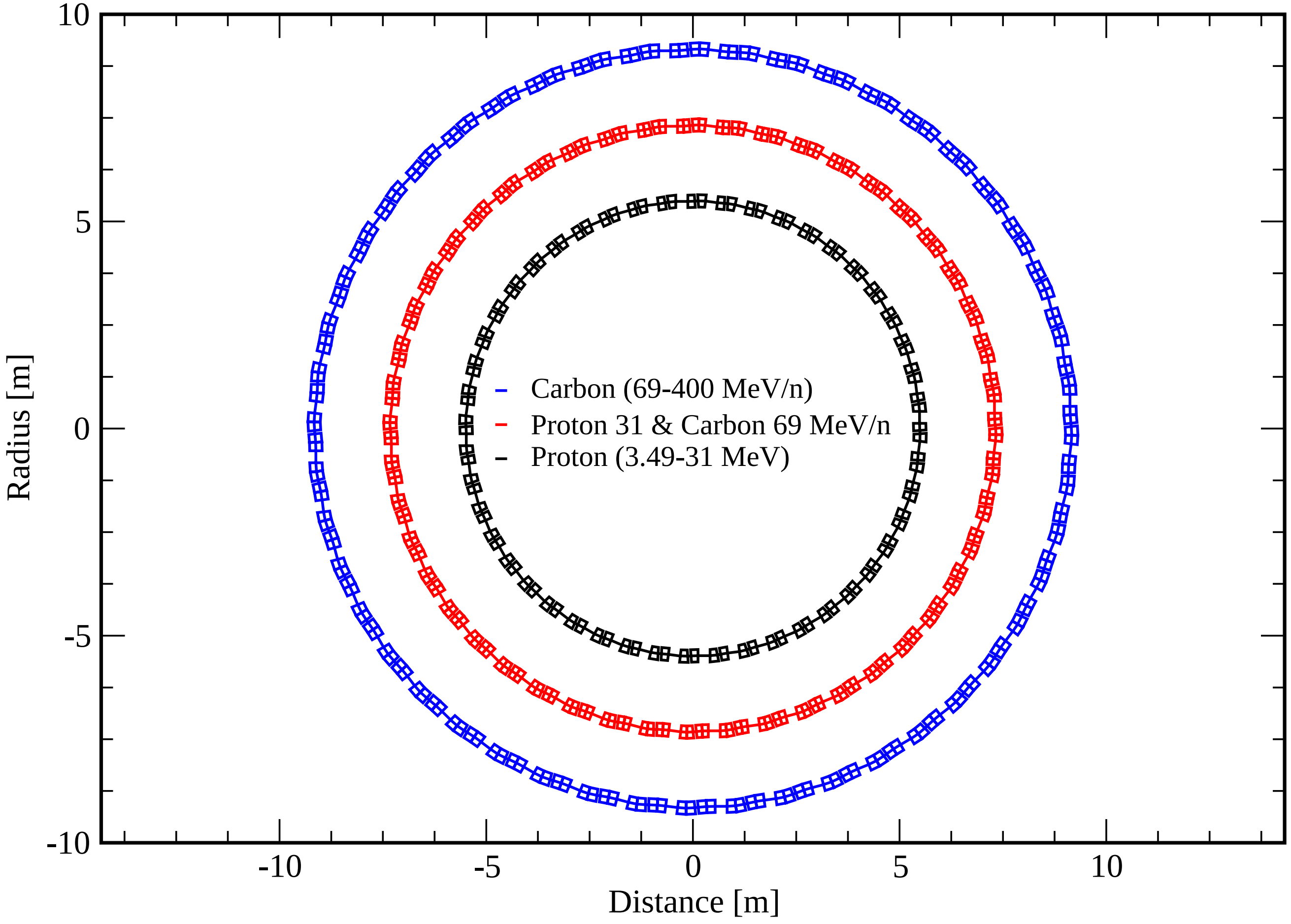
`http://keil.home.cern.ch/keil/Conferences/07Cyclotron/update.pdf`

Schematic layout of the rings



Design Principles and H^+ Beam Parameters

- Accelerate H^+ **and** C^{6+} ions in 3 non-scaling small-aperture FFAG rings
- Ring 1 only for H^+, Ring 2 for both H^+ and C^{6+}, Ring 3 only for C^{6+}
- Maximum H^+ kinetic energy fixed at 250 MeV
- Ring 2 accelerates by a factor 3 in momentum and is the most difficult one
- Ring 1 accelerates by smaller factors
- In Ring 2 H^+ and C^{6+} have equal rigidities $B\rho$ and equal magnet excitation
- H^+ and C^{6+} have equal β at injection into Ring 1 and Ring 2, respectively
  $\Rightarrow$ equal RFQ and linac for H^+ and C^{6+}
- All other relativistic beam parameters follow from the design parameters

Ring	1-Inj	1-Extr	2-Inj	2-Extr
Kin. En.(MeV)	7.951	30.97	30.97	**250**
β	**0.1294**	0.2508	0.2508	0.6136
$B\rho$(Tm)	0.4083	0.8107	0.8107	**2.432**
$\delta p/p$	-0.3301	$+0.3301$	$\mathbf{-0.5}$	$\mathbf{+0.5}$

Design Principles and C^{6+} Beam Parameters

- Maximum kinetic energy fixed at 400 MeV/u for C^{6+}
- Ring 3 accelerates by less than a factor 3 factors in momentum
- In Ring 2 H^{+} and C^{6+} have equal rigidities $B\rho$ and equal magnet excitation
- H^{+} and C^{6+} have equal β at injection into Ring 1 and Ring 2, respectively
  $\Rightarrow$ equal RFQ and linac for H^{+} and C^{6+}
- All other relativistic beam parameters follow from the **bold** design parameters

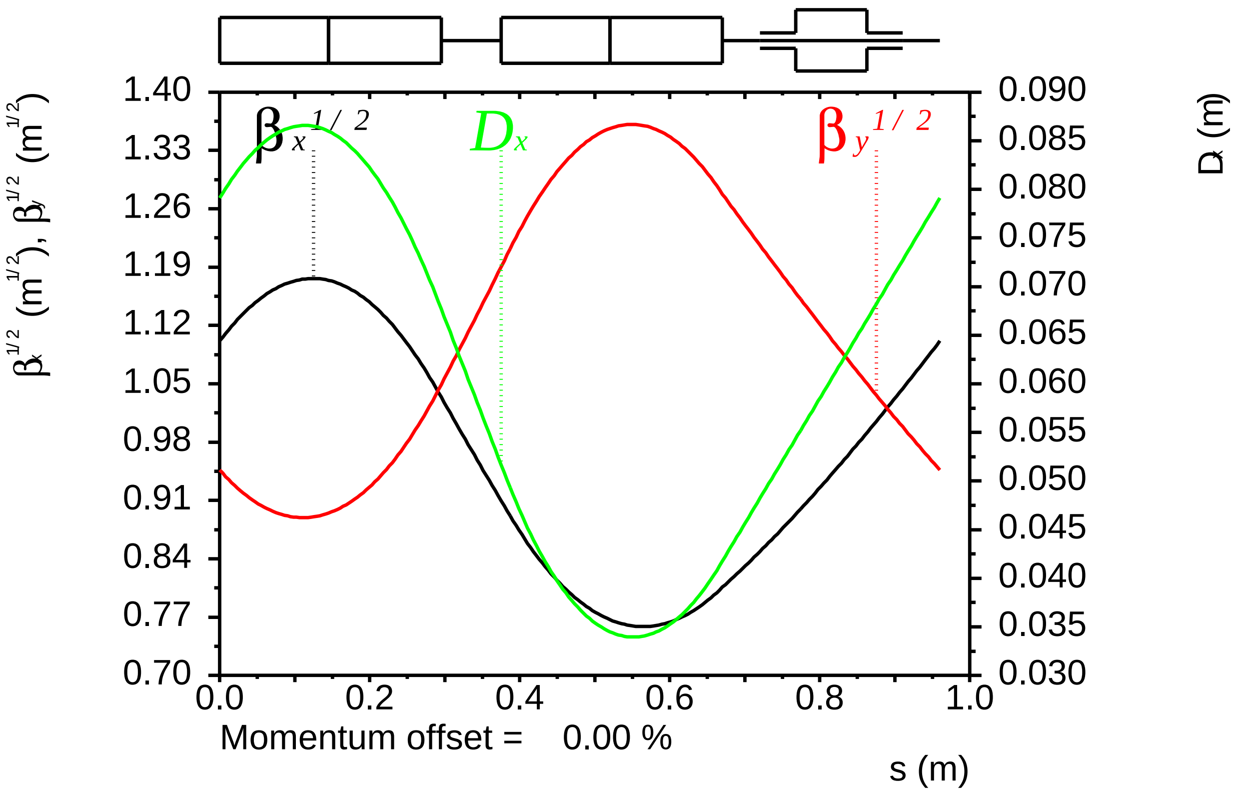
Ring	2-Inj	2-Extr	3-Inj	3-Extr
Kin. En./u(MeV)	7.8934	68.801	68.801	**400**
β	**0.1294**	0.3645	0.3645	0.7145
$B\rho$(Tm)	0.8107	**2.432**	2.432	6.3472
$\delta p/p$	$\mathbf{-0.5}$	$\mathbf{+0.5}$	-0.4459	$+0.4459$

- Ring 3 for C^{6+} ions can be added later
- Ring 1 can be replaced by a cheaper source of H^{+}
- Replace concentric layout by rings on either side of straight beam line?

Lattice Issues

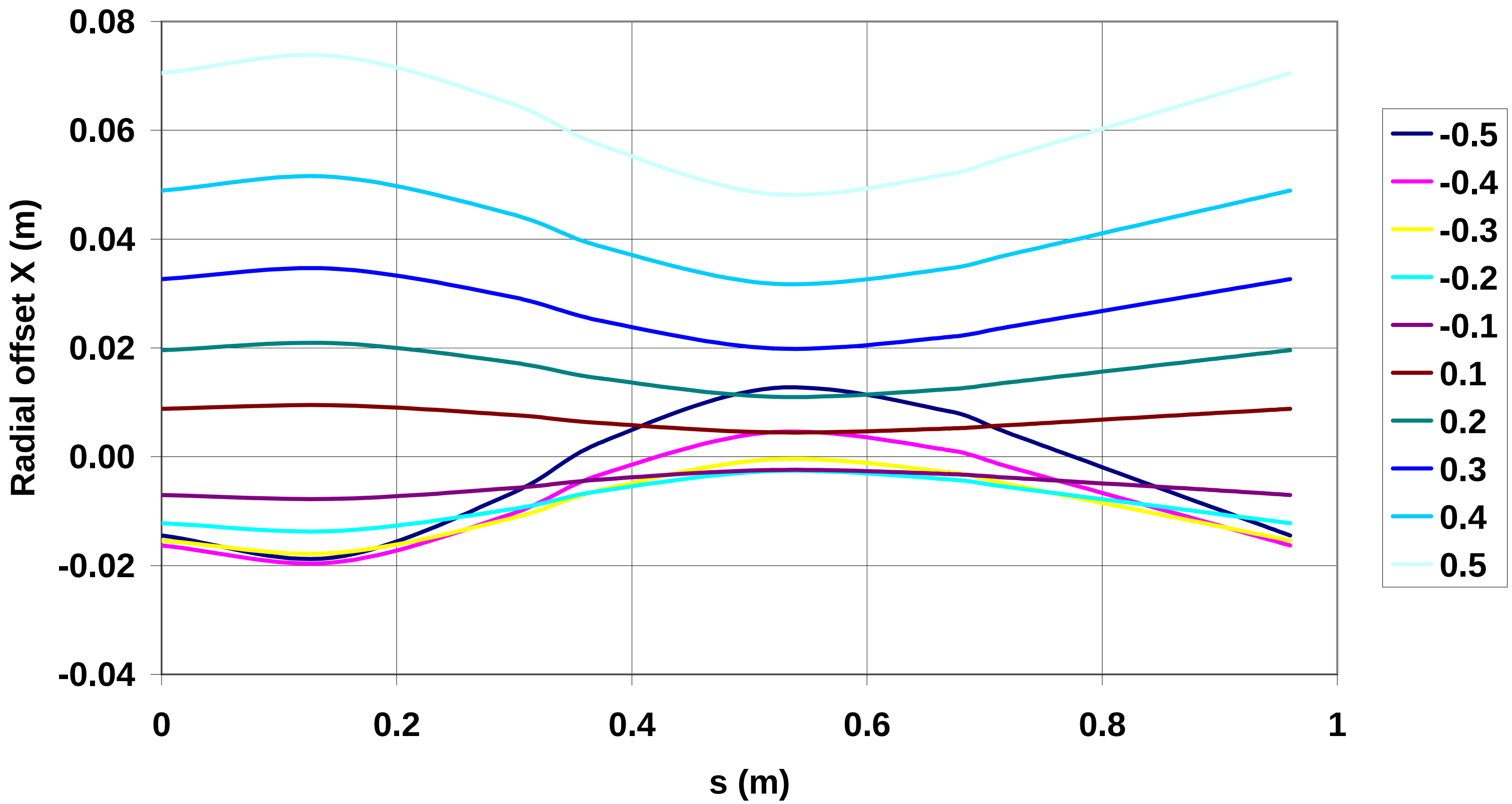
- All 3 rings have 48 cells with doublets of combined-function dipoles
- All F magnets bend away from the ring centre
- Path length varies like $(\Delta p/p)^2$ near reference momentum
  $\Rightarrow$ mimimal radial spread of off-momentum orbits
  $\Rightarrow$ minimal radial aperture
- New Ratio 3:4:5 of circumferences instead of 4:5:6 lowers RF frequencies in all 3 rings
- Ring 1 unchanged
- Longer magnets and cells and lower magnetic fields in Rings 2 and 3
- Long straight section unchanged
- Most examples from Ring 2

Orbit Functions $\sqrt{\beta_x}$, $\sqrt{\beta_y}$, D_x in Ring 2



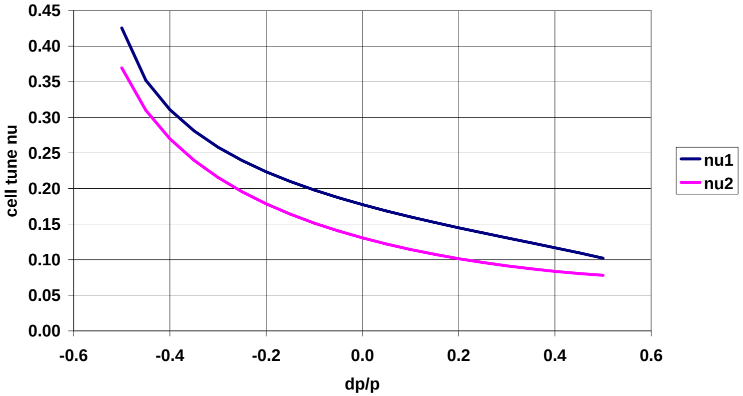
- Two combined-function magnets and one RF cavity shown schematically
- Note $\beta_x, \beta_y < 2$ m and $D_x < 0.1$ m

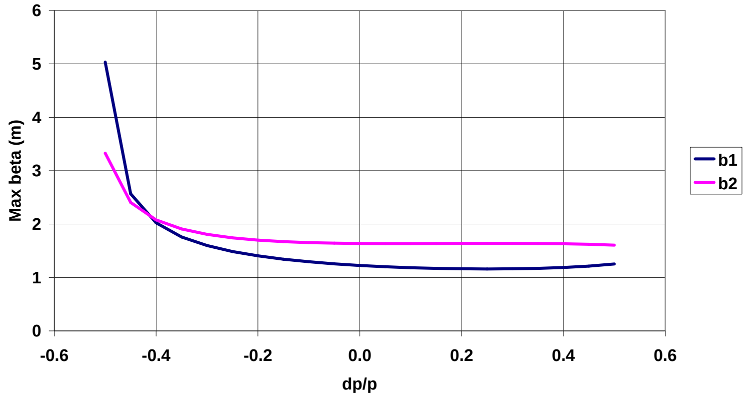
Horizontal orbit offset X in m along a cell of Ring 2



- Momentum range $-0.5 \leq \delta p/p \leq +0.5$
- Colour coded $\delta p/p$

Cell Tunes and β-functions vs. $\delta p/p$ in Ring 2





- Keep cell tunes away from 0.5 for $\delta p/p \rightarrow -0.5$ and away from 0 for $\delta p/p \rightarrow +0.5$
- Avoid steep increases of β-functions for $\delta p/p \rightarrow \pm 0.5$
- Playing with shape of dipoles and values of ν_1 and ν_2 at $\delta p/p = 0$ changes behaviour
- Observed effects of small changes in cell tunes ν_1 and ν_2 at $\delta p/p = 0$, magnet shape and edges demonstrate need for careful optimisation

Apertures and Fields of F and D Magnets

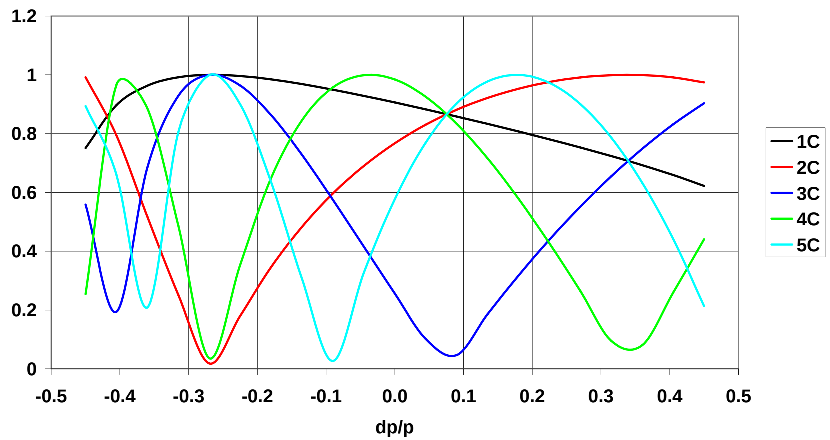
- Inner and outer horizontal aperture radius from radial offsets and betatron amplitudes
- Variation of B across horizontal aperture in rows labelled B at inner/outer apert radius
- Fields in F magnets change sign inside aperture, those in D magnets do not

Magnet	F			D		
Ring	1	2	3	1	2	3
Inner hor apert radius	-27	-32	-35	-10	-12	-13
Outer hor apert radius	28	86	81	19	67	63
B at inner apert radius	-0.69	-1.11	-2.08	0.83	1.51	2.89
B at outer apert radius	0.17	0.58	0.70	0.44	0.38	1.10
Vert half apert	8	10	7	15	13	12

- Conventional iron-dominated magnets in Rings 1 and 2, excited by resistive room-temperature coils, or by coils of high-temperature superconductor
- Maximum B in Ring 3 much reduced, but still beyond iron-dominated magnets

Extraction at Variable Energy from Ring 3

- How many kickers are needed to extract at variable energy?
- Optimum tune advance $Q_x \approx (2n+1)/4$ with integer $n \geq 0$ between kicker and septum
- Process table of cell tunes q_x vs. $\delta p/p$ in apr26j_sum.xls



- Get guidance from graph of $|\sin 2\pi k q_x|$ against $\delta p/p$ for $k = 1 \ldots 5$ cells between kicker and septum
- Good values of k have $|\sin 2\pi k q_x| \approx 1$
- Cover whole $\delta p/p$ range with $k = 1$ or 2
- Extraction at any energy reduced to extraction at maximum energy by using one of two kicker magnets at efficiency $\geq \sqrt{3}/2$
- Graphics package doesn't know about zeroes of $|\sin 2\pi Q_x|$

Injection and Extraction

- Extraction in two stages
  - Full-aperture fast kicker deflects extracted beam by $2\sigma'\sqrt{5}$ such that upright beam ellipses just touch, while skew ellipses with $\gamma > 1/\beta$ are separated
  - Septum magnet at optimum phase deflects extracted beam, such that it misses components downstream, and sends it into a transfer line
- Energy of extracted beam varied by changing number of turns
- Injection uses components in reverse order, septum magnet two cells from kicker magnet, and close to optimum phase

Ring	1	2	3
Kick angle (mrad)	11.5	7.5	4.4
Rise time (ns)	120	120	120
Aperture width (mm)	52	102	94
Aperture height (mm)	28	23	20
Kicker length (m)	0.2	0.2	0.2
Kicker field (T)	0.047	0.091	0.14

Crossing the Error-Driven Resonance $Q = n$

- Baartman gave a tolerance at FFAG workshop in Oct 2004 for resonance at $Q = n$

$$B_n/\overline{B} = 2Q\sqrt{Q_\tau}\Delta A/C$$

with average magnetic field $\overline{B}$, n-th Fourier component B_n of the vertical magnetic field, tune Q, tune change per turn Q_τ, and circumference C

	Ring	C (m)	Q	Turns	$10^3 Q_\tau$	$B_n/\overline{B}/\Delta A$ (1/m)
$\mu^\pm$		400	20.3	9	284	0.16
H^+	1	34.56	10.44	1500	8.5	0.0514
H^+	2	46.08	8.52	3000	5.2	0.0266
C^{6+}	2	46.08	8.52	1500	1.03	0.0376
C^{6+}	3	57.6	8.66	5000	2.5	0.0150

- Relaxed tolerances in $\mu^\pm$ ring, rather tight ones in our rings because of small Q_τ
- Tolerances for half-integral and non-linear resonances behave similarly $\propto \sqrt{Q_\tau}$
- Patented low-tune-variation lattices avoids crossing resonances

Alternatives for RF Systems

- Consider two different RF systems
  - Harmonic number jumping HNJ at constant frequency of about 1300 MHz adjusts energy gain ΔE in RF cavities such that time of flight between neighbouring cavities changes by integral number Δh of RF cycles, where

$$\Delta E = -E_0\beta^2\gamma^3\Delta h/h$$

    with rest energy E_0, harmonic number h and relativistic parameters β and γ
  - Frequency modulated systems vary frequency around 10 MHz such that a turn takes a constant number of RF cycles h, and need relative rate of frequency change with f_i, β_i and γ_i, which at injection is too high for ferrites

$$\frac{1}{f_i}\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{c\Delta E}{CE_0\beta_i\gamma_i^3}$$

- Contribution of radial offset to time of flight neglected
- Presenting more than one RF system implies that we don't have a satisfactory one

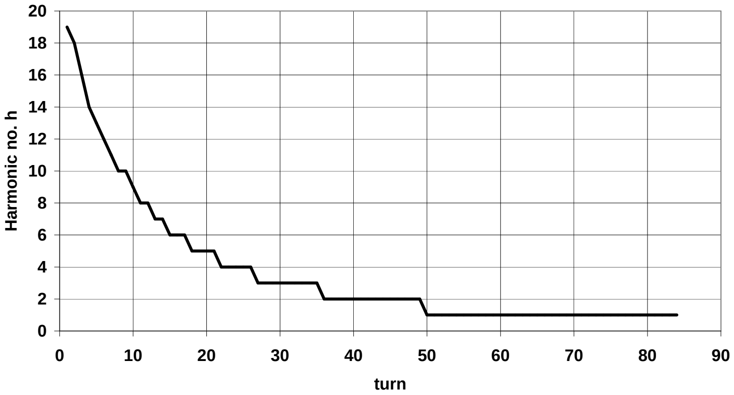
RF Systems Using HNJ

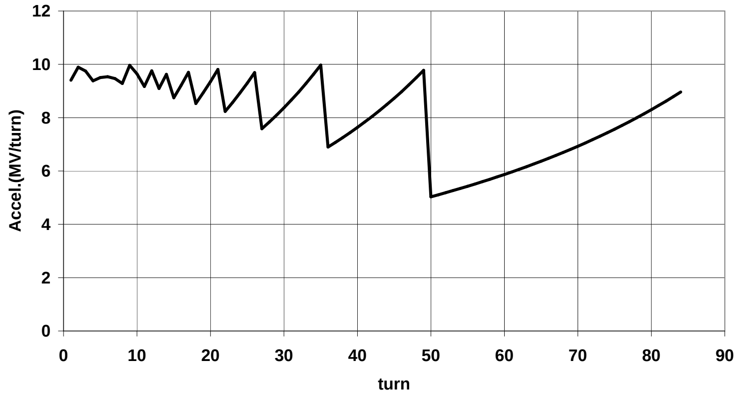
RF system parameters at $f_{\rm RF} \approx 1.3$ GHz, initial and final harmonic numbers h_i and h_f, initial step $|\Delta h|$, number of turns, maximum circumferential voltage V

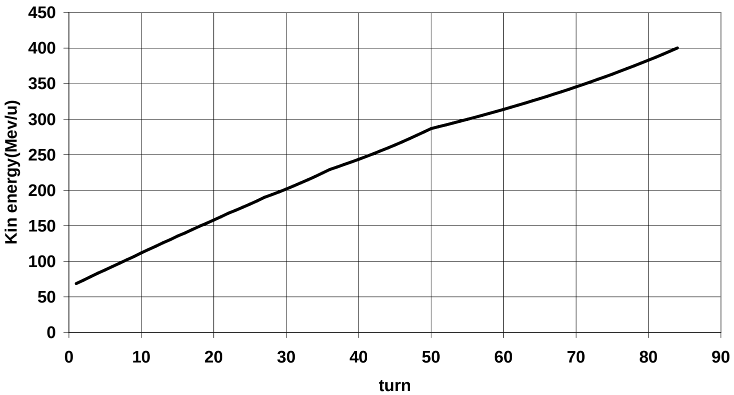
Ring		h_i	h_f	$\|\Delta h\|$	turns	V (MV)
1	H^+	1158	598	8	289	0.11
2	H^+	747	298	25	116	2.4
2	C^{6+}	1448	507	27	253	0.61
3	C^{6+}	619	309	19	81	10.8

- Large circumferential acceleration V, in particular in Ring 3
- Dividing V among equistant cavities doesn't work, grouping few cavities closely might
- ILC/EMMA cavities have 70/40 mm beam ports, smaller than aperture needed
- Fewer turns than assumed in resonance calculation
- HNJ might work for H^+ in Ring 1 and C^{6+} in Ring 2
- A.G. Ruggiero presents HNJ in machines with much larger C and several new ideas

RF System Programming for HNJ in Ring 3







- Use available $V_{\rm RF}$ also at turn ≤ 50 and make $|\Delta h| > 1$
- Get almost linear variation of E
- Higher Q_τ good for resonance crossing
- Don't know how to achieve $V_{\rm RF}$

Frequency Modulated RF Systems

Broad-band transmitters at frequencies between 6.5 and 19 MHz feed low-Q RF cavities filled with modern permeable materials from various suppliers

Ring	Ion	h	f_i(MHz)	f_f(MHz)	N	T (ms)	ΔE(keV/u)
1	H^+	6	6.733	13.05	1500	0.9063	15.34
2	H^+	4	**6.526**	15.97	3000	1.023	73.01
2	C^{6+}	8	6.733	**18.97**	1500	0.9200	40.61
3	C^{6+}	5	9.485	18.60	5000	1.702	66.24

- Low harmonic numbers h related to ratio 3:4:5 of circumferences C
- Fill every 2nd bucket with H$^+$ in Ring 1 and C^{6+} in Ring 2
- Keep acceleration time T below about 1 ms at constant ΔE

$$T = \frac{CN(\beta_f\gamma_f - \beta_i\gamma_i)}{c(\gamma_f - \gamma_i)}$$

  with number of turns N and final relativistic β_f and γ_f

- Fall-back solution with just one bunch could have even lower RF frequencies

RF Power Parameters in FM Systems

- Start from LEIR cavities with $R_L = 660\ \Omega$ and double power $W_L = V_L^2/(2R_L)$
- Assume stable phase angle $\pi/4$ from nearest zero crossing
- Adjust number of cavities N such that $V \leq 6$ kV

Ring	Ion	N	V (kV)	W (kW)
LEIR		1	4	12
1	H^+	4	5.4	22.3
2	H^+	18	5.7	24.9
2	C^{6+}	20	5.7	25.0
3	C^{6+}	32	5.9	26.0

- RF system in Ring 1 is easy
- RF system in Ring 2 fills about 40% of available straight sections, needs 0.5 MW
- RF system in Ring 3 fills about 67% of available straight sections, needs 0.83 MW

Conclusions

- 3 rings for H^{+} and C^{6+} ions with non-scaling, small-aperture lattices
- Changing circumference ratio to 3:4:5 made magnetic fields in Rings 2 and 3 smaller, but not quite small enough for room-temperature iron-dominated magnets
- Feasible full-aperture kicker magnet in each ring injects beam onto closed orbit
- Two feasible full-aperture kicker magnets in each ring extract beam from closed orbit at any energy, and send it to septum magnet
- Demonstrated that integral resonances at $Q = n$ can be crossed, if components are installed with tight but perhaps not impossible tolerances on error-driven errors
- Two RF systems based on HNJ and FM
  – In HNJ systems upper limit on h due to beam port diameter yields excessive ΔE, at least for H^{+}in Ring 2 and C^{6+}in Ring 3
  – In FM systems RF frequencies in range where FM cavities are operated, but large RF power needed, more turns to reduce voltage and power blocked by resonances
- Consider hybrid RF system, starting with HNJ, finishing with FM
- No engineering design and no cost estimates