A FAST CODE FOR SIMULATION OF THE LONGITUDINAL SPACE CHARGE EFFECT IN ISOCHRONOUS CYCLOTRONS*

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Abstract

The paper describes a fast code for simulation of the longitudinal space charge effect in isochronous cyclotrons. It discusses assumptions made in the code and their validity, equations of motion, and a fast field solver, which was developed specifically for the code. The results of simulations are compared to the Sector Model, developed by W. Joho [1]. With the purpose of illustration the program is used to estimate the maximum beam current in a single-turn extraction mode in the K1200 Cyclotron, NSCL.

1 INTRODUCTION

A design of an intermediate or high beam current isochronous cyclotron with single-turn extraction has to include consideration of the longitudinal space charge effect. As was shown by several authors [1], [2] this effect increases the energy spread within a turn and tends to destroy turn separation.

Particle-in-Cell (PIC) codes are powerful tools for computer study of the longitudinal space charge effect. The main drawback of PIC codes is that they are time-consuming. Calculation of the maximum achievable beam current requires optimization of several cyclotron and beam parameters and can be very time-consuming if a PIC program is used.

This paper describes the code "LSC" which aims specifically at fast estimation of the longitudinal space charge effect in isochronous cyclotrons. Section 2 describes the equations of motion and the field solver used in the code. The code is compared to the sector model, which was developed by W.Joho [1], in Section 3. Section 4 describes a procedure of computation of the maximum beam current that destroys turn separation at extraction.

2 DESCRIPTION OF THE CODE

2.1 Equations of motion

The code simulates the energy-phase motion of beam particles. It ignores the betatron motion and assumes operation in a regime where transverse focusing is strong. The code calculates only the longitudinal component of the field and ignores the radial one. Because the code neglects both the radial component of the space charge force and the betatron motion, it cannot simulate the space charge induced vortex motion within the beam. Therefore, the code accurately simulates the energy spread within a bunch for "low" and "intermediate" beam intensities, when the bunch tilt due to the space charge is small, and overestimates the energy spread for high beam intensity.

The equations of motion used in the code were proposed by M.Gordon and are

$$\frac{\mathrm{d}E}{\mathrm{d}n} = qV_1\cos(\phi) - qV_3\cos(3(\phi-\phi_3)) + 2\pi Rq\mathcal{E}_\theta^{sc} \quad (1)$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}n} = 2\pi h((1+\epsilon)\Delta+\epsilon) - q\left(\frac{\mathrm{d}V_1}{\mathrm{d}E}\right)\sin(\phi) + \\ +\frac{q}{3}\left(\frac{\mathrm{d}V_3}{\mathrm{d}E}\right)\sin(3(\phi-\phi_3)) \quad (2)$$

where E is the energy of a particle and ϕ is its RF phase. n, turn number, is the independent variable, q is the charge state of the particle, V_1 and V_3 are the peak voltage gains per turn produced respectively by the main and the flat-topping RF systems, ϕ_3 is the phase shift between V_1 and V_3, h is the RF harmonic number. ϵ defines the RF frequency as $\omega_{RF} = \omega_{RF0}(1+\epsilon)$, where ω_{RF0} is the RF frequency corresponding to an ideally isochronous field. Δ is the rotation frequency error, expressed as $\omega = \omega_{RF0}/(h(1+\Delta))$. R is the average radius of the particle trajectory. Values of R and Δ are pre-calculated for a measured or calculated cyclotron magnetic field and as a function of energy stored in a table, which is part of the input. Values of V_1 and V_3 as a function of E are also stored in the same table. The classical, explicit, 4^{th} order Runge-Kutta method is used to integrate the equations. The θ-component of the space charge field, $\mathcal{E}_\theta^{sc}$, is calculated by the field solver described in Section 2.2.

2.2 Calculation of the space charge field

The code assumes that the accelerated beam is confined between two horizontal, perfectly conducting planes. In calculating the space charge field for a given R, LSC ignores the radial turn structure of the beam and replaces the fast-oscillating microscopic charge density by its average value at R. Besides, it locally ignores the wedged shape of the beam. Thus, the code adopts a two dimensional Cartesian model in which $y = -R\phi/h$ and $\rho = \rho(y,z)$. The azimuthal space charge field $\mathcal{E}_\theta(R,\phi,z)$ is replaced by $\mathcal{E}_y(y,z)$. The model was first developed by M.Gordon in [2].

The longitudinal distribution can be trapezoidal, rectangular, or triangular. Although the size of the longitudinal distribution changes with radius, its shape stays unchanged. The vertical distribution function is rectangular of a constant size. The code neglects both the curvature of the trajectory and the centrifugal acceleration and assumes that the rest frame of the beam is inertial.

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First, the space charge field is calculated in the rest frame of the beam. The beam is "sliced" into thin, regularly spaced, vertical, charged sheets. The median plane field of a single sheet with unit surface charge density is

$$\mathcal{E}_0(y) = \begin{cases} 4\arctan\dfrac{\sin(\frac{kz_0}{2})}{\sinh(ky)} & : \quad y \neq 0 \\ 0 & : \quad y = 0 \end{cases} \qquad (3)$$

where z_0 is the vertical size of the beam, k is π/gap, and gap is the vertical aperture.

The total field of the beam is a convolution between the field described by Eq.3 and a distribution of the charged sheets. In calculating the convolution the code takes advantage of Fast Fourier Transformations and the convolution theorem (for more details see [3]). The calculated longitudinal field is then transformed into the laboratory frame according to the Lorenz transformation for the electromagnetic field.

3 COMPARISON TO THE SECTOR MODEL

Both the code and the Sector Model (SM) are based on similar assumptions. Therefore, the magnitude of the energy spread simulated by LSC and that predicted by SM have to be similar. This was used to validate the code. The energy spread induced by the space charge was calculated in three cyclotrons: NSCL K1200 Cyclotron, PSI Injector II, and PSI Main Ring Cyclotron. Table 1 summarizes the result.

Table 1: Energy spread simulated by LSC and calculated from SM as $\Delta E^{sc} = 2800\, q I_{peak}\, n^2/\beta_{max}$ where n is the number of turns.

	E_{max} (MeV)	I (mA)	n	ΔE^{sc} (MeV) LSC/SM
K1200	200	0.01	819	1.3/1.6
PSI INJ.II	72	1	99	2.6/2.5
PSI MRC	590	1	225	6.3/5.8

Cyclotrons K1200, PSI Injector II, and PSI Main Ring strongly vary in such parameters as the maximum energy, the beam current, the number of turns, etc. However, the difference between the energy spread simulated by LSC and that predicted by SM does not exceed 20% for all three cases. This indicates that the code works correctly within adopted approximations.

It is worth noting that both LSC and SM overestimate the energy spread in the PSI cyclotrons. As was mentioned in the introduction the vortex motion limits the growth of the energy spread within a turn. PIC simulations [4] show that a 1 mA bunch in PSI Injector II deforms into a galaxy-like shape in 15-20 turns.

4 COMPUTATION OF THE MAXIMUM BEAM CURRENT

To minimize the energy spread within a final turn the code optimizes selected beam and cyclotron parameters. Five parameters can be varied: ϵ, V_3, ϕ_3, the initial central phase (ϕ_{ci}), and the initial phase width of the beam ($\delta\phi_i$). The downhill simplex method is used for optimization [5].

The maximum achievable beam current, which destroys turn separation, is calculated from the equation

$$\Delta E_{min}(I) = A \cdot \delta E_{extr} \qquad (4)$$

where $\Delta E_{min}(I)$ is the minimum energy spread as a function of beam current, A is a parameter ($0 \leq A \leq 1$), which defines a criterion for turn separation, and δE_{extr} is the energy gain at extraction.

For illustration the method was applied to the NSCL K1200 Cyclotron. $^{16}O^{8+}$, 200 MeV/u field was used in the example, V_1 was 450 kV (actual dee voltage was 150 kV). Three parameters, ϵ, $\delta\phi_i$, and ϕ_{ci} were optimized. Fig.1 shows the minimum energy spread vs. beam current. The dashed line $\Delta E = 225$ keV corresponds to the energy spread at which turn separation vanishes(A=1). As follows from the figure, the maximum beam current in the single turn extraction mode cannot exceed 4.3 μA. Fig.2 shows the last two turns of the beam corresponding to the maximum current. At this current, separation between the turns just vanishes. Note, the whole procedure of computation of the maximum current took less than thirty minutes. (600 MHz single-processor ALPHA was used).

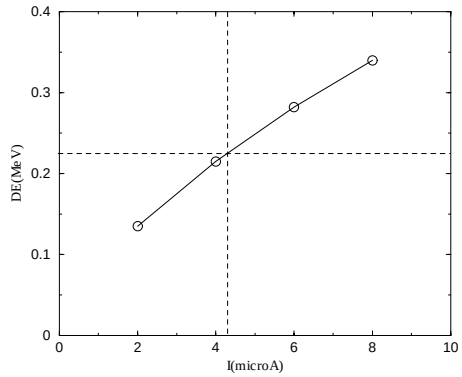

Figure 1: Minimum energy spread vs. beam current for the K1200 $^{16}O^8$, 200 MeV field. V_1 was 450 kV (actual dee voltage was 150 kV). The dashed line $\Delta E = 225$ keV corresponds to the energy spread which destroys turn separation.

5 CONCLUSION

The code LSC, which simulates the longitudinal space charge effect in isochronous cyclotrons, was developed and tested. The code is fast and user friendly. It does not require use of high-performance, multi-processor computer

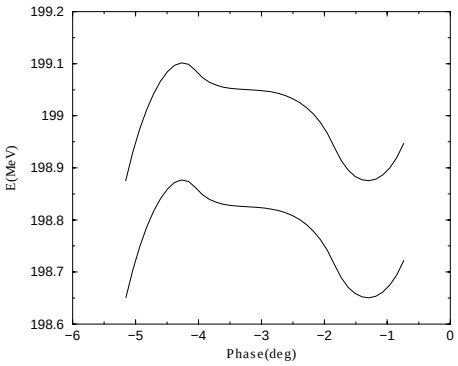

Figure 2: Energy vs. RF phase for the last two turns of the beam corresponding to the maximum current. Separation between the turns just vanished

facilities and can be successfully run on virtually any modern computer. As follows from Section 4, the whole procedure of computation of the maximum current in a cyclotron with the single-turn extraction takes approximately a half an hour if 600 MHz single-processor ALPHA is used.

6 ACKNOWLEDGMENT

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7 CODE DISTRIBUTION

The code can be downloaded from the web page http://www.nscl.msu.edu/~pozdeev.

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