RECENT STUDIES OF FFAGS IN THE USA*

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Abstract

Muon acceleration is one of the most difficult stages to develop for a Neutrino Factory or Muon Collider. The large transverse and longitudinal admittances which must be designed into the system and the rapidity with which acceleration must take place because of muon decay preclude the use of conventional synchrotron design. The current approaches employ fixed-field architectures for muon acceleration: one being a recirculating linac (RLA) with separate fixed-field arcs for each acceleration turn, and another a fixed-field alternating gradient (FFAG) accelerator. Although both scenarios are being actively pursued, this paper emphasizes the FFAG option.

1 INTRODUCTION

Because of potentially heavy losses from decay, acceleration must occur rapidly for any application requiring a high-energy, intense muon source; e.g. a Neutrino Factory or a Muon Collider. Linear acceleration is the most efficient, but above a GeV it becomes prohibitively expensive. Conventional synchrotons cannot be used because the rapid rate and cycle time required preclude ramping even normal conducting magnets[1]. The current baseline approach employs recirculating linacs with separate, fixed-field arcs for each acceleration turn. However, the RLAs with their separate return arcs have not only proved costly, they also represent a bottleneck in acceptance, and, as a result, set the present neutrino intensity limit for the U.S. Neutrino Factory designs[2]. (As an example, the combined longitudinal and transverse acceptance of the RLA is a factor of 4-5 below that of the Neutrino Factory storage ring[2].)

Given the technical complexity, intensity limitations, and expense of ultra-rapid cycling synchrotrons and recirculating linacs, the idea of using fixed-field, single-path accelerators has been revisited in recent work. The arcs of such machines, composed of large-bore superconducting magnets, can be designed to accommodate the large energy range in acceleration. Lattices have been developed which can contain an energy change of a factor of four[3]. These recent studies of FFAG accelerators represent an effort to reduce cost and promote an acceptance which is better matched to the performance of the ionisation cooling system and the storage ring.

2 FFAGS FOR MUON ACCELERATION

2.1 General

An overriding consideration for a Neutrino Factory is to design an acceleration system which has an exceptionally large acceptance, both transversely and longitudinally, thereby reducing as much as possible the degree of beam cooling required. Hence the naturally-large longitudinal acceptance of the FFAG makes it an attractive option to explore and potentially address the acceptance issues associated with the RLA designs. Further, revisiting FFAG lattices in light of present superconducting technology and magnet design has advanced their reach into the multi-GeV regime, making a chain of FFAG accelerators a potential candidate for a complete acceleration scenario—a scenario applicable to either the Neutrino Factory or Muon Collider.

2.2 Types of FFAG Lattices

A circular accelerator system can be designed with fields that remain constant for the duration of the acceleration cycle using an alternating gradient focussing lattice. The closed orbits are not fixed as in a ramped machine, but rather move across the magnet aperture during the cycle. There are three basic types of alternating gradient structures used in FFAG lattice design. These include:

- Traditional scaling FFAG
- Triplet-based scaling FFAG
- Nonscaling FFAG

The traditional scaling FFAG is comprised of combined-function short FODO cells with edge focussing and B fields which scale with momentum. The consequence of scaling the magnetic fields is that the orbit properties are maintained constant as a function of momentum and the optics are also independent. Such FFAG rings were first designed and studied at MURA[4]. The triplet-based FFAG is a recent innovation based on the scaled-field concept, but formed from a triplet quadrupole structure rather than a FODO one. (It was developed for the KEK Proof of Principle, or POP, machine[5].) Its primary advantage over the previous

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structure is incorporation of a significantly longer straight section in each cell facilitating injection, extraction, rf insertion, etc.

The nonscaling FFAG is a concept unique to muon acceleration where acceleration occurs so rapidly the beam experiences only a few turns in the accelerator. For such rapid acceleration, one does not have to avoid resonances or control lattice parameters as a function of momentum. Instead, one has the freedom to choose parameters optimal for muon acceleration such as minimizing circumference and requiring a large transverse dynamic aperture.

2.3 Example of a 16-64 GeV Scaling FFAG

Constant orbit properties imply parallel orbits as a function of momentum. FFAGs obtain constant orbital properties by increasing the magnitude of the B field radially outward. The B field is “scaled” to the desired momentum and position. In a radial-sector FFAG, inward-bending magnets are sector magnets indicating horizontal focussing. In order for the optics to also scale appropriately with momentum, the direction of the magnetic field must alternate from magnet to magnet. With both a reversed bend field and edges constructed parallel to the preceding sector magnet, vertical focussing (horizontal defocusing) is added without disturbing the constant offset, or radial stagger of orbits of different momentum. (The magnet edges lie on radii from the ring center.) Hence, the closed orbit for each momentum alternates from an inward to an outward curvature, forming a scalloped shape. Clearly, in order to close the ring, the reverse-bend D magnets must be shorter than the normal-bend F ones. Usually the reverse bends are made as short as possible within the limits of the accompanying transverse optics with the result that optical parameters are dissimilar in the horizontal and vertical planes. For example, the vertical tune is much lower than the horizontal tune.

A useful quantity is the field index, n, which is kept constant in a scaling FFAG:

$$n = -(R/B)(dB/dR)$$

where R is the radius, and B the magnitude of the field at that radius. The derivative of the B field with respect to radius determines the linearity of the machine. If dB/dR is fairly constant over the aperture of the magnet, then the machine exhibits a large transverse dynamic aperture in addition to the large longitudinal acceptance (as in the POP machine[5]). However, in order to expand the momentum reach of the machine and conserve magnet aperture at the same time, the field has to rise sharply and nonlinearly with radius, usually at the expense of dynamic aperture. The following is an example of a scaling, radial-sector FFAG with a large field index designed to collapse a factor of 4 range in momentum (16-64 GeV) into a modest SC magnet aperture. Table 1 gives parameters of the ring and Figure 1 the optics of a single sector, including a long, dispersion-suppressed, 14 m straight section.

Table 1. Parameters of a 16-64 GeV scaling, radial-sector FFAG.

General	
FFAG type	Radial sector
Energy Range	16-64 GeV
Circumference	1356 m
Arc Cells	
Central Energy	40 GeV
Rigidity	133.4 T-m
Magnetic Field @ 40 GeV	4.86 T
Gradient	125 T/m
Field Index (n)	707
Radius of Curvature	27.5 m
“F’ Length	1.79 m
“D” Length	1.13 m
Cell Tunes, υ_x/υ_y	0.416/0.084
Radial Displacements :	
16 GeV	-3.6 cm
64 GeV	1.8 cm

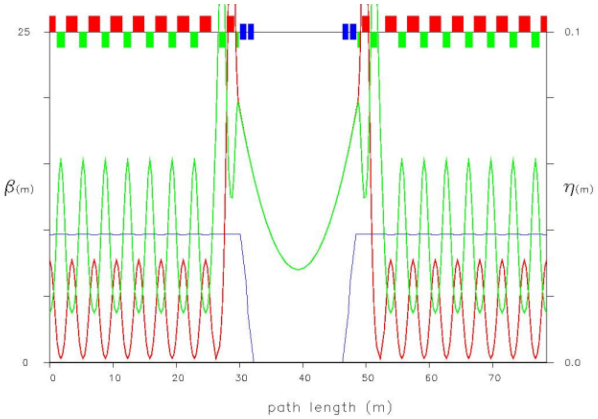

Figure 1. A sector of the 16-64 GeV scaling FFAG showing the 14 m dispersion-suppressed drift.

Even with careful design, the performance limitations of the scaling FFAG generally eliminate it as a candidate for high-energy muon acceleration when a large range in energy is involved. The primary obstacle comes from the large transverse muon beam size, which must be transported along with the large change in energy. Keeping the orbit and optical properties consistent require the B field to scale with momentum, but achieving a large transverse dynamic aperture requires that B’ must be nearly constant. For a large energy range, this implies the horizontal spread of orbits becomes large. For example, at 160 T/m, the 16 and 64 GeV orbits would be separated by 0.75 m; apertures which are incompatible or far too expensive for SC magnets. Curtailing the magnet aperture means the field must rise sharply with radius through addition of higher-order field terms. (The most significant terms are generally sextupole and octupole[6].) Degradation of dynamic aperture follows.

In the above example, the stable dynamic aperture of the arc cell alone was found to be only 450π mm-mr (normalized) at 16 GeV, which is inadequate to transport even a muon beam cooled to the extent needed for a collider. (This represents only ±3σ of the projected transverse emittance for a muon collider[1].)

In summary, the radial-sector FFAG achieves a number of successes: notably a large energy acceptance, a small closed orbit radial spread, and consistent optics at all the energies. However, the disadvantages are the large circumference due to the use of reverse bends and the constriction of dynamic aperture due to the non-linear magnetic field profile. For these last reasons, scaling FFAGs are not presently being considered for high-energy muon acceleration in the U.S. design studies.

2.4 *Example of a 4-16 GeV Nonscaling FFAG*

As mentioned earlier, muon acceleration occurs so rapidly that resonances are not a consideration. The rf systems are assumed to deliver on the order of 0.5-2 GeV per turn. In this case, the beam can be accelerated through an integer, or other resonance-driving "global" tunes if the tune is only valid for a fraction of a turn. With a fast acceleration cycle, the lattice's optical parameters are released from scaling with momentum as well. One is then allowed the freedom to choose machine parameters which are optimal for muon beam acceleration; i.e. minimizing the circumference to limit intensity loss from decay and maximizing the transverse dynamic aperture to accept a less-cool beam. This approach has been termed a non-scaling FFAG accelerator.

First, the reverse bends required by the criterion to maintain constant optics can be eliminated yielding approximately a 20% decrease in total circumference. A more important step is to choose the magnet configuration in the FODO cell to provide the maximum net bend per cell for a given peak excursion of the closed orbit during acceleration. This is accomplished by favourably positioning the dipole bend field over the defocusing quadrupole element. The cells of the non-scaling ring then contain a horizontally focusing quadrupole followed by a vertically focusing, combined-function bending magnet. The allowed bend is further increased by the choice of focussing strength and cell length: the lower or injection momentum experiences a cell phase advance approaching π while the upper momentum approaches zero, depending sensitively on the choice of gradient and relative radial position (magnet aperture). The non-scaling approach yields the smallest design circumference of any lattice and can approach a factor of 2 less than that of a scaling lattice (1.3-1.7 in the present example).

In designing a non-scaling lattice, the optimal lengths for the magnets are obtained analytically by assuming thin-lens kicks and imposing geometric closure on both off-momentum orbits and transversely-displaced orbits (orbits with the same momentum, but with an amplitude). In order to solve the set of coupled equations, the maximum off-axis orbit excursion in the F quadrupole must be chosen along with the F quadrupole's aperture and poletip field. To insure stability in both planes, the D quadrupole strength is set equal to the F quadrupole strength. In the table below, the largest orbit excursion was chosen to be 14 cm and a gradient to be less than 60 T-m (giving a poletip field of about 8T). In practice, adjustments must be made to the approximate lengths given by the thin-lens solutions to produce the desired orbit excursions in the full simulation. The lattice components, parameters, and functions of the non-scaling FFAG cell are given in Table 2 and plotted in Figure 2.

Table 2. Parameters of a 16-64 GeV Non-scaling FFAG.

General	
FFAG type	nonscaling
Energy Range	16-64 GeV
Central Energy	54 GeV
Circumference	900-1100 m
Half-cell straight length	1.3-1.8 m
Rigidity	180 T-m
Maximum Poletip field	8 T
Arc Cell	
Number	200
Length	5.5 m
Half-cell straight length	1.8 m
Bend/cell	0.0157 rad
Quadrupole Gradient	57.1 T/m
"F' length	0.76 m
"F" strength	0.33 m^{-1}
"D" length	1.14 m
"D" strength	0.32 m^{-1}
Cell Tunes:	
@16 Gev	0.45 (162°)
@64 GeV	0.08 (29°)
βmax:	
@16 GeV	33 m
@54 GeV	13 m
@64 GeV	15 m
Maximum Displacements:	
16 GeV	-11 cm
64 GeV	14 cm

Relaxing the requirement for consistency in closed orbits at different energies means that the orbits no longer remain parallel. The peak of the closed orbit excursion always occurs at the center of the F quadrupole. Orbit excursions at 16 GeV are plotted in Figure 3 (top). The corresponding orbit excursion at 64 GeV is almost an inversion of the 16 GeV curve as can be seen in Figure 3 (bottom).

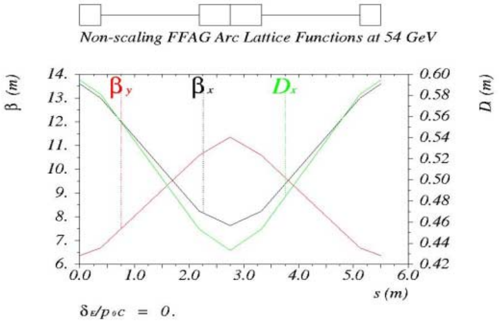

Figure 2. Lattice functions at 54 GeV for a 16-64 GeV non-scaling FFAG.

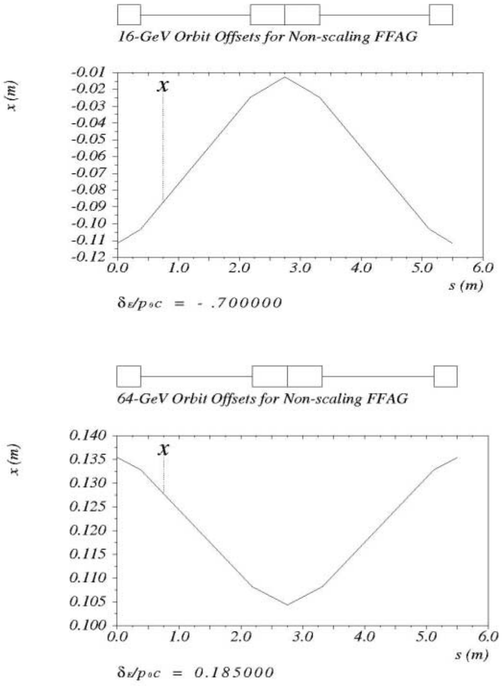

Figure 3. 16-GeV (top) and 64-GeV (bottom) orbit amplitudes for a 16-64 GeV non-scaling FFAG.

The need for a large transverse dynamic aperture is automatically satisfied in this design because only linear elements are used. Because of the short cell structure, variations in the maximum beta functions are not significant enough to instigate a serious beta wave during acceleration due to a transverse optics mismatch between different energies. (The phase advance of injection should probably be reduced somewhat, to about 150°, which brings the 33 m peak in the beta function down to under 20 m in the current design.)

2.4 Pathlength Dependencies in FFAGs (rf phase-slip)

A main drawback to FFAGs in both the scaling and the non-scaling versions is the large changes in pathlength as a function of energy. The pathlength dependence is clearly linear with momentum for radially-staggered, parallel orbits as in the scaling case, but it is parabolic in non-scaling FFAGs. This comes about when the transverse excursion of off-momentum orbits (see Figure 4) is larger than the contribution from the longitudinal pathlength change. Nominally, for small momentum deviations from the central momentum, the fact that the lower momentum stays to the inside of the central orbit and the high momentum to the outside means a smaller total pathlength for low momentum and a larger one for higher momentum. The scaling FFAG follows this norm except the pathlength variations are large because of the large momentum acceptance. For large transverse apertures, and correspondingly large excursions across the magnet apertures, as is the case in the non-scaling FFAG, the transverse path changes overtake these longitudinal variations. Because of their transverse offset, both high and low momentum have total pathlengths larger than the central orbit, giving a parabolic shape to the circumference change as a function of momentum.

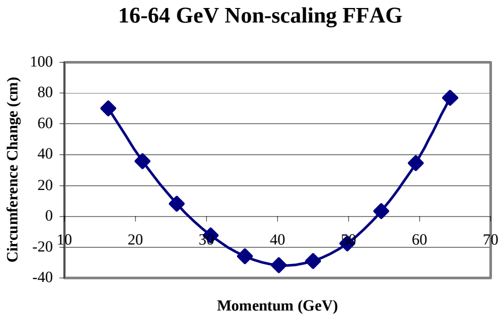

Figure 4. Circumference dependence on momentum

The circumference change is problematic because as the highly relativistic beam changes energy, its time of arrival is incorrect relative to the rf phase—it walks significantly away from the synchronous phase. For example, if the rf system is 200 MHz, then a pathlength change of 50 cm from the central orbit means the bunch arrives 1/6 of an rf wavelength out of phase and will not accelerate properly or even remain bunched. (The corresponding momentum compaction is given in Figure 5 as a function of momentum.) Further, this phase slip cannot be accommodated by the rf system in the microsecond circulation time characteristic of these machines. (If the acceleration cycle were slower, as in past or existing FFAGs, the rf phase can be made to track the change in arrival time.)

Recently workshops have focussed on the phasing problems of the FFAGs and a number of solutions are being advanced[7]. These are:

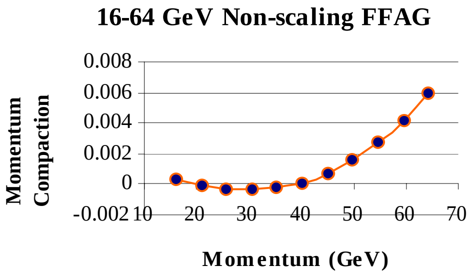

Figure 5. Momentum Compaction dependence on momentum

- Chicanes

Chicanes can correct pathlength differences which increase with momentum. They have been successfully implemented in the scaling FFAG in this paper[8]. However, chicanes cannot resolve a parabolic pathlength dependency on momentum and are therefore ineffective for nonscaling FFAGs.

- Broad-band rf

Broadband rf with a low Q value can be formed and phased quickly to adhere to the longitudinal dynamics of the bunches as they evolve during acceleration: the shape of the rf accelerating voltage can be tailored or shifted in the timeframe of a single turn. It is more natural to adjust the rf waveform to track the beam than to force the lattice to control the time of arrival of accelerating bunches, a procedure which is inherently nonlinear. The disadvantage, however, to broadband rf is its low achievable acceleration voltage of 1 MeV or less.

- Low frequency rf

If the rf wavelength is long enough then the relative phase shifts of beam bunches relative to the rf waveform in both the scaling and non-scaling FFAGs will not represent a significant mismatch or loss of beam from the bucket. Rf frequencies of about 25 MHz or lower will keep the bunches intact, but at the price of increased rf voltage and power.

- Multiple rf frequencies

Multiple rf frequencies can be mixed to produce an appropriate waveform to match the beam traversal times. Since the Δt, or phase-slip per turn is approximately constant on either side of the parabola for a non-scaling FFAG, two close frequencies could be used to produce a fairly accurate phase slip. However, rhis phase slip may not be a problem because for a kilometre circumference and a rf frequency of 300MHz, the change in frequency needed from 1 km to a 1.0007 km is less than a MHz. The bottom of the parabola does represent transition and a transition jump (accompanied by a 180° change in phase) must take place in order to match the entire cycle of a non-scaling FFAG.

3 SUMMARY

Scaling FFAGs for the most part have not been found to be applicable to muon acceleration in the multi-GeV regime. This is due to their poor transverse dynamic aperture resulting from strong nonlinear field profiles. Although superconducting magnets with large horizontal apertures are required for efficient acceleration in a non-scaling FFAG, this approach does provide the necessary transverse and longitudinal acceptance match to high-energy muon beams. Initially it was felt that the circumference change, or phase-slip, posed a serious problem, but since then numerous solutions have been proposed. In conclusion, it looks promising to build a chain of muon accelerators from FFAGs and replace the costly and restrictive RLAs, which so far have been the baseline accelerator for the feasibility studies of a Neutrino Factory in the U.S [2,9].

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