THE COD INVESTIGATION OF THE EXPERIMENTAL RING CSRe

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Abstract

A new multipurpose experimental ring (CSRe) is being constructed in Lanzhou. It is a heavy ion storage ring used for internal-target experiments or high precision spectroscopy with electron-cooling. In order to obtain large acceptance the closed orbit distortion (COD) of storage ring should be considered. The global COD caused by misalignments and field errors of all magnets in the CSRe will be discussed.

1 INTRODUCTION

The experimental ring CSRe shown in Figure 1 is one storage ring of the HIRFL-CSR complex[1]. HIRFL-CSR, a new accelerator project at the Heavy Ion Research Facility in Lanzhou (HIRFL), consists of a main ring (CSRm) and an experimental ring (CSRe). The heavy ion beams from the HIRFL will be accumulated, cooled and accelerated in the CSRm, and then extracted to produce radioactive ion beams (RIB) or highly charged heavy ions. The secondary beams can be accepted and stored by the CSRe for internal-target experiments or high precision spectroscopy with e-cooling.

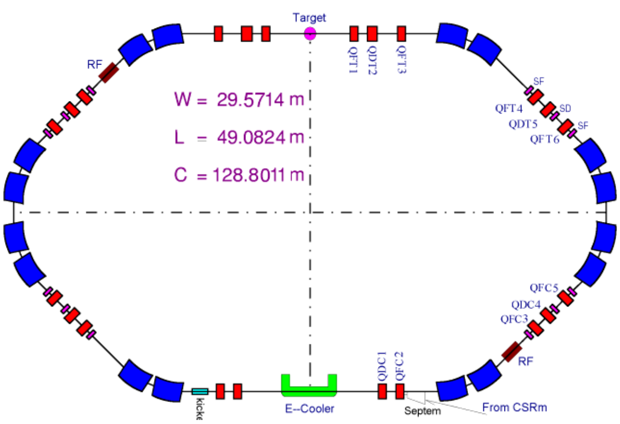

Figure 1: The layout of the CSRe

2 LATTICE

CSRe is not only an experimental ring with internal target but also a high-resolution spectrometer. In order to meet different experiments needs three lattice modes will be adopt. First one is the internal-target mode with small β-function in the target point and large transverse acceptance. The second one is the high-resolution mode used for mass spectrometer with large momentum acceptance. The third one is the isochronous mode for the mass measurements of short lifetime nuclei with Time-of-Flight method.

In order to investigate the closed orbit distortion (COD) of CSRe, only the internal-target lattice mode is discussed. The layout of the CSRe is shown in Figure 1. It is a racetrack shape and consists of two quasi-symmetric parts. One is the internal target part and another is the e-cooler part. Each part is a symmetric system and consists of two identical arc sections. Each arc section consists of four dipoles, two triplets or one triplet and one doublet. 11 independent variables for quadruple are used in the CSRe. Table 1 is the lattice parameters of the CSRe for the internal-target mode, and Figure 2 is the distribution of β-functions and dispersion for this mode.

Table 1: Lattice parameters of the internal-target mode

Circumference (m)	128.80
$B\rho_{Max.}$ (Tm)	8.4
Transition gamma	γ_{tr} = 2.457
Betatron tune values	Qx / Qy = 2.53 / 2.57
Natural chromaticity	Q_x' / Q_y' = -3.70 / -3.55
Max. β-Amplitude	β_x/β_y =25.7/8.7 m (Dipole) β_x/β_y =43.0/20.4 m (Quadruple)
Max. Dispersion	$D_{max}(x)$=7.9 m (Dipole, β_x=14m) $D_{max}(x)$ = 9.4 m (Quadruple)
Injection section	β_x= 30.8 m , D_x = 0 m (Septum) β_x= 31.4 m, D_x = 0 m (Quadruple.)
E-cooler section	β_x/β_y = 12.9/16.5 m , D_x = 0

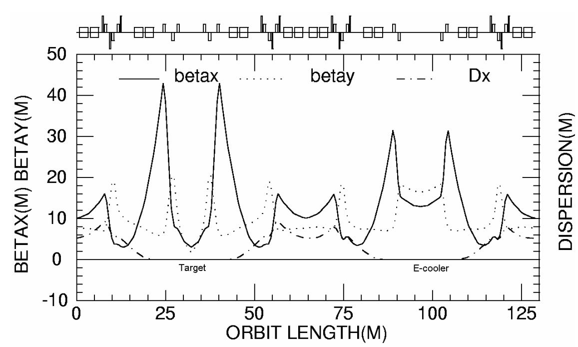

Figure 2 The distribution of the β-functions and the dispersion for the internal-target mode.

3 THE COD THEORY OF STORAGE RING

3.1 Derivation of the general COD formula

Considering the magnetic field errors, the Hill equation[2] of the particle motion is the following form,

$$\frac{d^2Y}{ds^2}+K(s)Y=\frac{1}{\rho}\frac{\Delta B}{B}=F(S) \tag{1}$$

where Y is the horizontal or vertical displacement (x or y) from the ideal equilibrium orbit, K(s) represents the focusing function $K_x(s)$ or $K_y(s)$. K(s) satisfies the periodicity relation K(s+L) = K(s+C) = K(s), L and C are the periodicity length and circumference of the equilibrium orbit respectively. F(s)= ΔB/Bρ is a measure of the deviation of the field on the ideal orbit from its ideal value ρ(s) is the radius of curvature of the equilibrium orbit at s. Here Bρ is the magnetic rigidity of the particle, and ΔB =Bx for vertical oscillations and ΔB = By-B_0 for horizontal oscillation, both measured on the ideal orbit. The solutions of the homogeneous equation of Eq. (1) are

$$y(s)=\beta^{\frac{1}{2}}(s)e^{\pm iQ\phi(s)} \tag{2}$$

where

$$\phi(s)=\int\frac{ds}{Q\beta(s)} \tag{3}$$

and

$$Q=\int_s^{s+C}\frac{ds}{\beta(s)}=\frac{N\mu}{2\pi},\quad \mu=\int_0^L\frac{ds}{\beta(s)} \tag{4}$$

Introduce a new variable

$$V=\beta^{-1/2}Y \tag{5}$$

Eq. (1) becomes to

$$\frac{d^2V}{d\phi^2}+Q^2V=Q^2f(\phi) \tag{6}$$

where

$$f(\phi)=\beta^{\frac{3}{2}}F(s)=\beta^{\frac{3}{2}}\frac{\Delta B}{B\rho} \tag{7}$$

By using the above transform, the forced oscillations of the Hill equation (1) can be reduced to the forced oscillations of a harmonic oscillator. The periodic solution of Eq. (6) is

$$V(\phi)=\frac{Q}{2\sin\pi Q}\int_\phi^{\phi+2\pi}f(\varphi)\cos Q(\pi+\phi-\varphi)d\varphi \tag{8}$$

From Eq.(8), the square of the amplitude of $V(\varphi)$ is

$$V^2(\phi)=\frac{Q^2}{4\sin^2\pi Q}\int_\phi^{\phi+2\pi}\int_\phi^{\phi+2\pi}f(\varphi_1)f(\varphi_2)\cos Q(\varphi_1-\varphi_2)d\varphi_1d\varphi_2 \tag{9}$$

and its expectation value is

$$\left\langle V^2(\phi)\right\rangle=\frac{Q^2}{4\sin^2\pi Q}\int_\phi^{\phi+2\pi}\int_\varphi^{\phi+2\pi}\left\langle f(\varphi_1)f(\varphi_2)\right\rangle\cos Q(\varphi_1-\varphi_2)d\varphi_1d\varphi_2 \tag{10}$$

In practice the perturbing function F(s) in detail can't be know, but some of its statistical characteristics are known. The distribution of ΔB/B in all magnets follows the statistical rule, and those errors were actually caused by misalignments or manufacture of magnets.

The errors in different magnets are uncorrelated, if φ_1 and φ_2 do not locate in the same magnet, then

$$\left\langle f(\varphi_1)f(\varphi_2)\right\rangle=0 \tag{11}$$

For the magnet numbered i, its length is L_i, and L_i is small compared to the wave length(β_i) of betatron oscillation. When φ_1 and φ_2 are in the same magnet,

$$\cos Q(\varphi_1-\varphi_2)=0, \tag{12}$$

and

$$\Delta\phi_i(s)=\int_s^{s+L_i}\frac{ds}{Q\beta(s)}=\frac{L_i}{Q\beta_i} \tag{13}$$

then

$$\left\langle V^2(\phi)\right\rangle\approx\frac{1}{4\sin^2\pi Q}\sum_i\beta_i\left(\frac{\Delta B_i}{B\rho}\right)^2L_i^2 \tag{14}$$

From Eq. (5) and Eq. (14), we can get the mean square amplitude of the closed orbit distortion (COD) in the magnet numbered j,

$$\left\langle Y_{COD}{}^2\big|_j\right\rangle=\frac{\beta_j}{4\sin^2\pi Q}\sum_{i=1}^M\beta_i\left(\frac{\Delta B_i}{B\rho}\right)^2L_i^2 \tag{15}$$

3.2 The most important errors in storage ring

For storage ring, there are many errors by manufacture and misalignment, but only three kinds of errors are most serious. One is the dipole field error ΔB_y caused by manufacture, this error will result in a horizontal COD,

$$\left\langle x_{c1}{}^2\right\rangle=\left\langle x_{COD}{}^2\big|_j\right\rangle=\frac{\beta_{xj}}{4\sin^2\pi Q}\sum_{i=1}^M\beta_{xDi}\left(\frac{\Delta B_{y_i}}{B\rho}\right)^2L_{Di}{}^2 \tag{16}$$

The second one is the rotation ($\Delta\psi_D$) of dipole around the beam direction. This rotation will cause one horizontal dipole field error ΔB_x, namely,

$$\Delta B_x=B_0\sin\Delta\psi_D\approx B_0\Delta\psi_D \tag{17}$$

Then, $\Delta\psi_D$ will caused a vertical COD

$$\left\langle y_{c1}{}^2\right\rangle=\left\langle y_{COD}{}^2\big|_j\right\rangle=\frac{\beta_{yj}}{4\sin^2\pi Q}\sum_{i=1}^M\beta_{yDi}\left(\frac{B_0\Delta\psi_{Di}}{B\rho}\right)^2L_{Di}{}^2 \tag{18}$$

The third one is the transverse displacement ΔY_q (Δx_q or Δy_q) of quadruple. The displacement ΔY_q will cause one dipole field error ΔB, namely,

$$\Delta B_x=B\rho K_q\Delta y_q\ \text{ or }\ \Delta B_y=B\rho K_q\Delta x_q \tag{19}$$

Thus the COD caused by ΔY_q will be

$$\left\langle x_{c2}{}^2\right\rangle=\left\langle x_{COD}{}^2\big|_j\right\rangle=\frac{\beta_{xj}}{4\sin^2\pi Q}\sum_{i=1}^M\beta_{xQ_i}\left(K_{qi}\Delta x_{qi}\right)^2L_{Q_i}{}^2 \tag{20}$$

and

$$\left\langle y_{c2}{}^2\right\rangle=\left\langle y_{COD}{}^2\big|_j\right\rangle=\frac{\beta_{yj}}{4\sin^2\pi Q}\sum_{i=1}^M\beta_{yQ_i}\left(K_{qi}\Delta y_{qi}\right)^2L_{Q_i}{}^2 \tag{21}$$

According to Eq.(16), Eq.(18), Eq.(20) and Eq.(21), the mean expectation value of COD for a ring can be estimated as the following,

the horizontal COD

$$x_c\approx\sqrt{\left\langle x_{c1}{}^2\right\rangle+\left\langle x_{c2}{}^2\right\rangle} \tag{22}$$

the vertical COD

$$y_c\approx\sqrt{\left\langle y_{c1}{}^2\right\rangle+\left\langle y_{c2}{}^2\right\rangle} \tag{23}$$

4 COD ESTIMATION OF THE CSRE

By using the above formula (16), (18), (20) and (21), the COD in magnets of the experimental ring CSRe can be estimated. Here we only discuss the expectation value of COD in the quadruple and the dipole with the max. β-function or max. dispersion. In CSRe the errors required by project will be less than ΔB /B = $\pm 5\times 10^{-4}$, and $\Delta x_q = \Delta y_q$ =0.15mm, $\Delta\psi_B$ = 0.5 mrad.

For CSRe

- the number of dipole N_D = 16,
- the arc length of dipole L_D=2.3562m,
- the bending radius of dipole ρ_D = 6.0m,
- the number of quadruple N_Q = 22,
- the average length of quadruple $\tilde{L}_Q$ = 0.65m,
- the average gradient of quadruple $\tilde{K}_Q$ = 0.5 T/m.

According to the internal-target lattice mode of the CSRe,

* the tune value of the machine Q_x/Q_y = 2.53/2.57,
* the max. β_x and β_y values in dipole and quadruple are list in table 1,
* the average β_x in all dipoles $\tilde{\beta}_{x\,D}$ = 13m,
* the average β_x in all quadruples $\tilde{\beta}_{x\,Q}$ = 15m,
* the average β_y in all dipoles $\tilde{\beta}_{y\,D}$ = 7m,
* the average β_y in all quadruples $\tilde{\beta}_{y\,Q}$ = 12m.

Then the formulae (16), (18), (20) and (21) can be simplified as following,

$$\left\langle x_{c1}{}^2\right\rangle \approx \frac{\beta_{xj}}{4\sin^2\pi Q}\left(\frac{\Delta B_y}{B}\frac{L_D}{\rho_D}\right)^2 N_D\tilde{\beta}_{xD} \qquad \left\langle y_{c1}{}^2\right\rangle \approx \frac{\beta_{yj}}{4\sin^2\pi Q}\left(\frac{L_D}{\rho_D}\Delta\psi_D\right)^2 N_D\tilde{\beta}_{yD}$$

$$\left\langle x_{c2}{}^2\right\rangle \approx \frac{\beta_{xj}}{4\sin^2\pi Q}\left(\tilde{K}_q\Delta x_q\tilde{L}_Q\right)^2 N_Q\tilde{\beta}_{xQ} \qquad \left\langle y_{c2}{}^2\right\rangle \approx \frac{\beta_{yj}}{4\sin^2\pi Q}\left(\tilde{K}_q\Delta y_q\tilde{L}_Q\right)^2 N_Q\tilde{\beta}_{yQ} \tag{24}$$

According to the above four formulae and (22), (23), we can obtain the estimated COD of the mean expectation value for the CSRe as shown in table 2.

Table 2: Estimated COD of the mean expectation value

	Horizontal				Vertical			
	β_{x_j} mm	x_{c1} mm	x_{c2} mm	x_c mm	β_{y_j} mm	y_{c1} mm	y_{c2} mm	y_c mm
Dipole	14	5.3	2	6	8.7	3.1	1.2	4
Quadruple	43	9.4	3	10	20	5	2	6

5 SIMULATION RESULTS OF COD

For the alignment of magnet, 6 errors should be taken in to account. Namely 3 position errors(Δx , Δy , Δz) and 3 angle errors($\Delta\varphi$, $\Delta\theta$, $\Delta\psi$). As shown in Fig. 3,

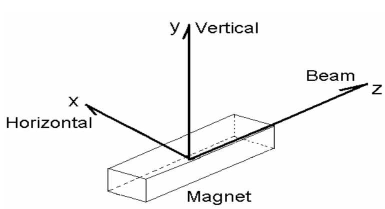

Figure 3 The local reference coordinate system.

φ is rotation angle around the x-axis, θ is rotation angle around the y-axis, and ψ is rotation angle around the z-axis. All rotations obey the right-hand screw rule with the axes. For CSRe project the required misalignments are listed in table 3.

Table 3: The magnet misalignments of the CSRe

Error	**B**	**Q**
ΔX (mm)	0.5	**0.15**
ΔY (mm)	0.5	**0.15**
ΔZ (mm)	2.0	0.5
Δφ (mrad)	0.5	0.5
Δθ (mrad)	0.5	0.5
Δψ (mrad)	**0.5**	0.5

Assuming that the distribution of every error is a $\pm 2\sigma$ Gauss distribution, and the random seed number is 40. Then we can do the COD simulation of the CSRe by means of the lattice calculations with errors. The simulation results are shown in Fig. 4.

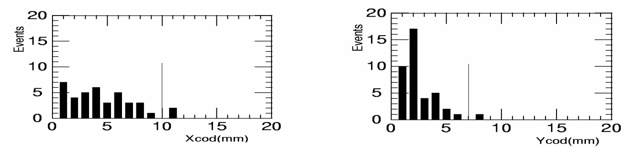

Figure 4 (a) COD distribution in the dipole with the max. $\beta_{x,\,y}$.

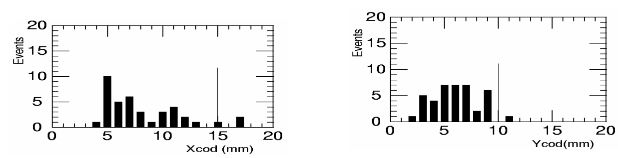

Figure 4 (b) COD distribution in the quadruple with the max. $\beta_{x,\,y}$.

6 CONCLUSION

According to the estimation and simulation, we can get the result that the simulated max. COD is nearly 3/2 times of the estimated expectation value. Thus defining the aperture of vacuum chamber, the following COD should be considered,

x_{COD}(D)= 10mm, y_{COD}(D)= 7mm,

x_{COD}(Q)= 15mm, y_{COD}(Q)= 10mm.

For any two points in storage ring the relation of COD and β-function can be obtained from Eq.(15),

$$COD_1/COD_2 = (\beta_1/\beta_2)^{1/2} \tag{25}$$

Example, in the entry of the e-cooler section, the β_y = 18m, comparing to the quadruple with max. β-function, the y_{COD} value of the entry can be calculated from Eq. (25), namely y_{COD} = 10mm.

REFERENCE

[1] J.W. Xia, et al., 'HIRFL STATUS AND HIRFL-CSR PROJECT IN LANZHOU', Proceedings of the first Asian Particle Accelerator Conference, KEK, Japan, 1998.

[2] E.D. Courant and H.S. Snyder, 'theory of the Alternating-Gradient Synchrotron', ANNALS OF PHYSICS: 3, 1-48 ,1958