STUDY OF ION-INDUCED INSTABILITIES IN BTCF

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Abstract

This paper gives the study of the ion-induced instabilities in the electron ring of Beijing $\tau - charm$ factory (BTCF). First according to the linear theory of ion trapping, the appropriate ion cleaning gap to avoid the multi-turn ions is calculated. Then a program is written to simulate the fast beam-ion instability (FBII) to estimate the one-turn ions' effects. The growth time of the FBII in the electron ring of the BTCF given by the tracking coincides with the theoretical prediction.

1 INTRODUCTION

Among the new generation colliders, Beijing $\tau - charm$ factory's ring length is much shorter than that of B factories and with less bunches in the ring, but it has its own problems. For example, the last turn's ions couldn't be completely eliminated by a not very short cleaning gap, and the one-turn ions still can induce the coupled bunch instability. This paper is aimed to find out the appropriate multi-turn ions cleaning gap, and estimate the coupled bunch instability's growth time due to the one-turn ions with the ion cleaning gap. The BTCF high luminosity mode lattice is adopted in the following, its main parameters are given in Table 1.

Table 1: Some parameters of BTCF high luminosity mode

Circumference C (m)	385.421
Beam Energy E (GeV)	2.0
Revolution Frequency(Mhz)	0.7778
Harmonic Number h	612
Momentum Compaction α_p	0.0144
Working Points Q_x / Q_y	11.23/11.27
$\tau_x / \tau_y / \tau_e$ (ms)	34.18/32.86/14.43
Natural Emittance ε_{x0} (nm)	148
β_x^* / β_y^* (m)	0.66/0.01
Enery Spread σ_e / E	5.84×10^{-4}

2 MULTI-TURN IONS' CLEANING

According to the linear theory of ion trapping, when the bunches are equidistantly distributed along the bunch train and all bunches are equally populated, if an ion is trapped in the bunch train, the ion will perform semi-resonant kind oscillation near the beam orbit, its motion equation is

$$\frac{d^2 z}{ds^2} + \Omega_{x,y}^2 z = 0, \tag{1}$$

$$\Omega_{x,y} = \left[\frac{2\lambda_e r_p c^2}{A\sigma_{e,x,y}(\sigma_{e,x} + \sigma_{e,y})} \right]^{1/2}, \tag{2}$$

where z denoting the ion's transverse displacement x or y with respect to the axis of the beam line , $\Omega_{x,y}$ the single ion's oscillation angular frequency, λ_e the average electron line density along the ring, r_p the classic proton radius, c the light velocity, A the mass number of the ion, $\sigma_{e,x}$ and $\sigma_{e,y}$ the beam size where the ion is trapped.

Assuming only one bunch train along the ring, and the ion cleaning gap is ΔT long with the revolution time T_0, so at one location along the ring, the multi-turn ion couldn't be stable only if the following condition is met.

$$\left| \cos\left[\Omega_{x,y}(T_0 - \Delta T)\right] - \frac{\Omega_{x,y}\Delta T}{2} \sin\left[\Omega_{x,y}(T_0 - \Delta T)\right] \right| > 1 \tag{3}$$

According to this criterion, the ion trapping ratio, which is defined here as the total length of multi-turn ion stable regions along the ring divided by the whole ring length, is calculated for different ion cleaning gap length.

In the preliminary design of BTCF, the following filling pattern in the electron ring of BTCF is adopted: one out of six rf buckets to be filled with electrons and the interval of adjacent bunch centers is 6 rf bucket length. So the maximum filled bunch number is 612/6, that is 102 for BTCF. In order to eliminate multi-turn ions, in fact only 86 bunches filled, leaving continuous 96 buckets unfilled as the ion cleaning gap. The ion cleaning gap take up 15.68% of the whole ring. The design bunch current is 6.7*mA*, the total electron beam current is 576*mA*. For simplicity, this filling pattern is called the design filling pattern in the following.

With eq.(3) and the BTCF high luminosity lattice parameters, Fig. 1 gives the trapping ratio of CO^+ vs. different cleaning gap length, and Fig. 2 gives the CO^+ trapping ratio vs. the bunch current under the design filling pattern. It can be seen that for the design filling pattern, with the cleaning gap of 15.68% of the whole ring, the trapping ratio is of about 10%, about 90%

multi-turn ions along the ring are unstable. So multi-turn ions' cleaning gap given by the preliminary design is appropriate. And from Fig .2, the trapping ratio doesn't change too much when the bunch current decreases from 6.5*mA* to 4.5*mA*.

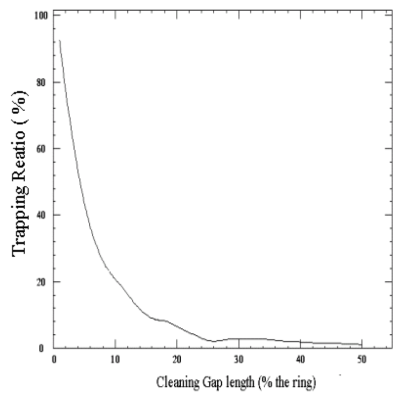

Figure 1: CO^+ trapping ratio vs. gap

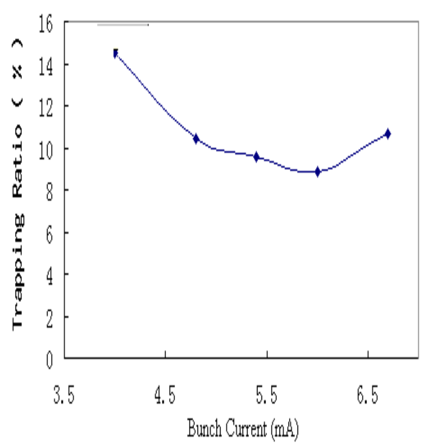

Figure 2: CO^+ trapping ratio vs. bunch current

3 THEORETICAL PREDICTION OF FBII GROWTH TIME

The linear theory of fast beam-ion instability assumes the linear interaction between the bunch and the ion cloud, the ion cloud's transverse size is $1/\sqrt{2}$ times of the bunch size, its coherent angular frequency is

$$\bar{\tilde{\omega}}_i = c\left[\frac{4N_b r_p}{3AL_{sep}\bar{\sigma}_{e,y}(\bar{\sigma}_{e,x}+\bar{\sigma}_{e,y})}\right]^{1/2}, \quad (4)$$

The coupled motion in the bunch train like $y \sim \exp(\sqrt{t/\tau_c})$, the characteristic time τ_c is given by [1]

$$\frac{1}{\tau_c} = \frac{4d_{gas}\sigma_{ion}\beta_y N_B^{3/2} n_B^2 r_e r_p^{1/2} L_{sep}^{1/2} c}{3\sqrt{3}\gamma\left[\bar{\sigma}_{e,y}(\bar{\sigma}_{e,x}+\bar{\sigma}_{e,y})\right]^{3/2} A^{1/2}} \quad (5)$$

where $\bar{\sigma}_{e,x}$ and $\bar{\sigma}_{e,y}$ is the average beam size along the ring, d_{gas} the residual gas' particle density, σ_{ion} the ionization collision section, n_B the bunch number in the bunch train, N_B the electron numbers in one bunch, L_{sep} the interval between adjacent bunches, r_e the classical electron radius.

If taking into account of the coherent frequency spread, the linear theory gives the coupled bunch motion in the bunch train like $y \sim \exp(t/\tau_e)$, the growth time is given by [1]

$$\frac{1}{\tau_e} = \frac{1}{\tau_c}\frac{c}{2\sqrt{2}l_{train}(\Delta\tilde{\omega}_i)_{rms}}, \quad (6)$$

where $(\Delta\tilde{\omega}_i)_{rms}$ is the RMS spread of the ion coherent angular frequency, the bunch train length is $l_{train} = n_B L_{sep}$.

According to eq.(5) and eq.(6), assuming 10% relative coherent frequency spread along the ring of BTCF, for $1.0nTorr$ CO gas pressure, the growth rates $\tau^{-1}{}_c$, $\tau^{-1}{}_e$ versus the bunch number are plotted in Fig. 3. It can be seen that the fastest growth rate given by linear theory is about $0.2ms$. After considering the ion coherent frequency spread, the fastest growth time is about $1.5ms$.

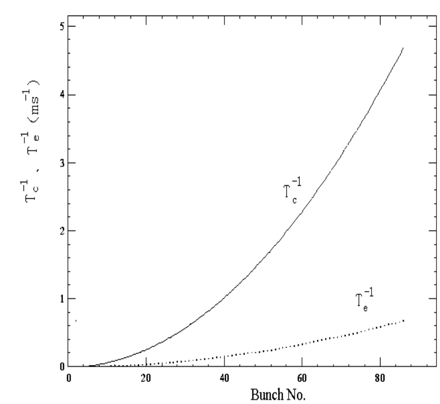

Figure 3: $\tau^{-1}{}_c, \tau^{-1}{}_e$ vs. bunch No.

4 SIMULATION OF FBII IN BTCF

A simulation program is written for the fast beam-ion instability based on the weak-strong model [2], in which the bunch is represented by rigid three dimensional Gaussian distribution, while the ions are represented by ion macro-particles. In the program the ion macro-particles are produced at ten almost equidistantly distributed locations along the ring. Under the design filling pattern, the macro-particles ion created in the last turn are intentionally cleaned in program. When a bunch passing by an ion, its velocity change is given by

$$\Delta v_y + i\Delta v_x = -\frac{n_e r_p c}{A}\cdot\sqrt{\frac{2\pi}{(\sigma_{e,x}^2-\sigma_{e,y}^2)}}\cdot f(x,y), \quad (7)$$

where n_e is the electron particle numbers in one bunch, *x* and *y* are ion's displacements from the bunch center, *f(x,y)* is a dimensionless function dependent on the bunch sizes,

$$f(x,y) = w\left(\frac{x+iy}{\sqrt{2(\sigma_{e,x}^2 - \sigma_{e,y}^2)}}\right) - \exp\left(-\frac{x^2}{2\sigma_{e,x}^2} - \frac{y^2}{2\sigma_{e,y}^2}\right) w\left(\frac{x\frac{\sigma_{e,y}}{\sigma_{e,x}} + iy\frac{\sigma_{e,x}}{\sigma_{e,y}}}{\sqrt{2(\sigma_{e,x}^2 - \sigma_{e,y}^2)}}\right), \quad (8)$$

w(x+iy) is the complex error function. On the other hand, the angle change of the bunch center due to the ion is

$$\Delta y' + i\Delta x' = \frac{r_e}{\gamma} \cdot \sqrt{\frac{2\pi}{\left(\sigma_{e,x}^2 - \sigma_{e,y}^2\right)}} \cdot f(x,y), \quad (9)$$

according Courant-Snyder invariant

$$J_y = \left[\frac{(1+\alpha_y^{\ 2})}{\beta_y} y_b^{\ 2} + 2\alpha_y y_b y_b' + \beta_y y_b'^2\right], \quad (10)$$

defining the relative quantity $A_y = \sqrt{\beta_y J_y} / \sigma_{y,e}$, which representing the bunch center's oscillation amplitude in the unit of bunch size. In this paper $\log_{10}(A_y)$ is used in plots.

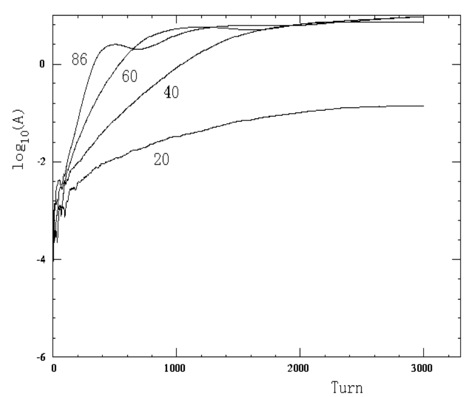

Figure 4: $\log_{10}(A_y)$ vs. tracking turn

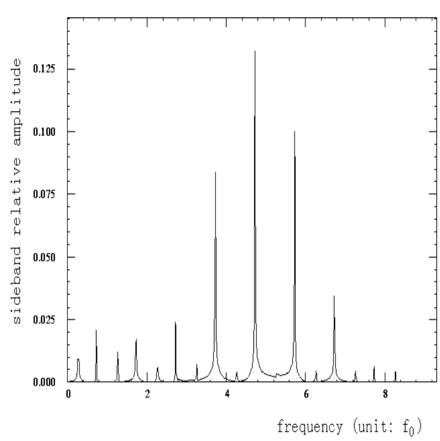

Figure 5: beam spectrum given by tracking

For BTCF high luminosity mode lattice, if residual CO gas' pressure $p_{CO} = 50nTorr$, tracking 3000 turns, Fig. 4 showing the amplitude changes of the 20th, 40th, 60th, 86th bunch's $\log_{10}(A_y)$ changes. Fitting with an exponential function for the 86th bunch, its growth time is given as $0.0304ms$. While the CO gas pressure p_{CO} is $10nTorr$, $5nTorr$, the growth time given by tracking for the 86th bunch is $0.1606ms$, $0.3897ms$ respectively. It can be drawn that the growth time is almost inversely proportional to the residual gas pressure. So for the residual CO gas' pressure $1.0nTorr$, the instability growth time can be estimated about $1.69ms$, which coinciding with the linear theory's prediction $1.5ms$ with 10% ion coherent angular frequency spread.

By doing the FFT of the recording vertical bunch centers' displacements, the beam spectrum is shown as Fig. 5. From eq.(4), with the average beam sizes along the ring, the average ion coherent frequency is given as $\overline{\tilde{\omega}_i} = 2.29 \times 10^7 \, rad/s$, $\overline{\tilde{\omega}_i} / \omega_{rev}$ is about 4.69. For BTCF, the coupled bunch motion represented by the lower side band is unstable. From the beam spectrum shown in Fig. 5 the highest peak of the lower side bands is centered at $4.74\, f_{rev}$, which matches the theory prediction, too.

5. CONCLUSIONS

For the electron ring of the BTCF, the cleaning gap of 15.68% the whole ring length given by the preliminary design can assure about 90% multi-turn ions unstable, one-turn ions trapped in the bunch train can induced a coupled bunch instability with growth time slower than $1.5ms$. The simulation program gives almost the same growth time of fast beam-ion instability comparing with the theoretical prediction.

6. ACKNOWLEDGEMENTS

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[1] Handbook of Accelerator Physics and Engineering, ed. Chao A, Tigner M, World Scientific, 1998
[2] Ohmi K. Phys. Rev. 1997, **E55:**7750