THE HIGHEST ENERGY OF PROTON LIMITED BY β OSCILLATION IN HIRFL_SSC

Liu Wei, Yifang Wang, Shangyun Yang
Institute of Modern Physics, Chinese Academy of Sciences

Abstract

For meeting the requirements of the experiment of cancer therapy and single particle effect, HIRFL'S main machine SSC (HIRFL-SSC) will accelerate higher energy proton. For deciding the highest energy of proton that can be accelerated by HIRFL-SSC, the β oscillation of proton beam in the cyclotron has been calculated with both 'hard edge' and 'soft edge' magnetic field .The procedure of calculation and result is presented.

1 MAGNET PERIODICAL STRUCTURE OF HIRFL-SSC

HIRFL-SSC is an isochronous separated sector cyclotron. Its magnet geometric arrangement is that with four identical straight-line sector magnet structure ($N = 4$), every sector magnet has 26-degree half angle

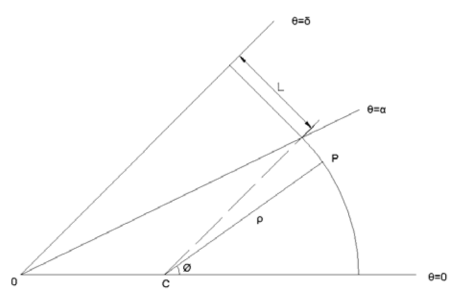

Fig.1 sketch map of half structure period

($\alpha = 26$) and every structure period has 45-degree half angle to the machine center point ($\delta = 45$). The magnet coefficient is $f = \alpha / \delta = 0.5778$. The sketch map of half structure period is shown in Fig.1.

In Fig.1, $\theta = 0$ is the sector central line of the magnet; $\theta = \alpha$ is the edge of the magnet; $\theta = \delta$ is the central line of magnet valley. The average injection radius is 1 meter; the average extraction radius is 3.207 meters.

2 VARIATION OF ß OSCILLATION FREQUENCY WITH PARTICLE ENERGY

2.1 Under Hard-edge Approximation Magnetic field

The hardedge approximation is an ideal simplification to practical magnetic field distribution. In real situation, the magnetic field is decreasing slowly from the inner magnetic area to the outside magnetic area at the magnet edge. Under the hardedge approximation magnetic field is constant along GORDON orbit inside the magnetic area. (The magnetic field is different along different GORDON orbit inside the magnetic area to satisfy isochronous requirement.) And outside the magnetic area, the magnetic field decreases to zero immediately. The GORDON orbit is drawn in Fig.1. The transverse oscillation equations of particle inside the magnetic area are:

$$\frac{d^2 x}{d\phi^2} + (1 - k(\phi))x = 0 \qquad (1)$$

$$\frac{d^2 z}{d\phi^2} + k(\phi) z = 0 \qquad (2)$$

Where

$$k(\phi) = -\frac{(\beta\gamma)^2}{1 + b\cos(\phi)} \ , \ \beta = \frac{v}{c} \ , \ \gamma = \frac{1}{\sqrt{1-\beta^2}} \ ,$$

$b = \dfrac{\sin(\delta - \alpha)}{\sin(\alpha)}$. ϕ is angular coordinate to curvature center of particle orbit in magnetic field.

In magnetic area, define $p_x = \gamma(\frac{dx}{d\phi})$, $p_x = \gamma(\frac{dz}{d\phi})$.

Under hardedge approximation, p_x , p_z change not continuously at magnet edge. There are transformations from magnet edge to magnet valley center:

$$p_x(s_1) = p_x(s_0) + \gamma t x(s_0) \tag{3}$$

$$x(s_1) = (1 + ta\delta)x(s_0) + (a\delta/\gamma)p_x(s_0) \tag{4}$$

$$p_z(s_1) = p_z(s_0) - \gamma t z(s_0) \tag{5}$$

$$z(s_1) = (1 - ta\delta)z(s_0) + (a\delta/\gamma)p_z(s_0) \tag{6}$$

Where $t = \tan(\delta - \alpha)$, $a = b\dfrac{\sin(\delta)}{\delta}$. s_0 is particle position at the edge of magnet and s_1 is particle position at the center of magnet valley.

To initial conditions:

$$\begin{pmatrix} y_1(0) \\ p_{y1}(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and

$$\begin{pmatrix} y_2(0) \\ p_{y2}(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(y can present either x direction or z direction), for different energy using Ronge-Kutta method to solve equation (1) and (2) from $\phi = 0$ to $\phi = \delta$, the values at the edge of magnet

$$\begin{pmatrix} y_1(s_0) \\ p_{y1}(s_0) \end{pmatrix}$$

and

$$\begin{pmatrix} y_2(s_0) \\ p_{y2}(s_0) \end{pmatrix}$$

can be calculated. And then for x and z direction with transformations ((3), (4)) and ((5), (6)), the values at the center of magnet valley

$$\begin{pmatrix} y_1(s_1) \\ p_{y1}(s_1) \end{pmatrix}$$

and

$$\begin{pmatrix} y_2(s_1) \\ p_{y2}(s_1) \end{pmatrix}$$

are obtained. According to magnet symmetry,, $\sin^2(\nu_y\delta) = -y_2(s_1)p_{y1}(s_1)$ can be proved, where ν_y represents the focusing frequency along x and z direction respectively.

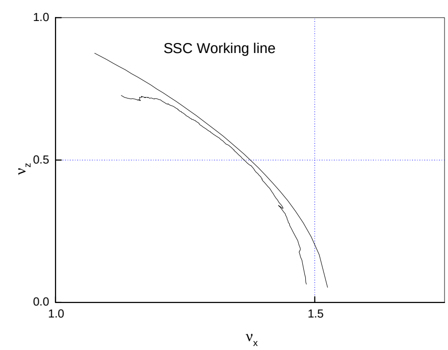

Fig.2 Transverse oscillation frequency under hard edge approximation and the measured magnetic field

Under hard edge approximation, the change of ν_z to ν_x with varied energy is shown as the right curve in Fig.2. The curve is from upper to lower with increasing of particle energy, from $\gamma = 1$ to $\gamma = 1.34$. At the position $\gamma = 1.34$, there is $\nu_z = 0$. At this time z direction becomes defocusing. From hard edge approximation analysis, the accelerated proton maximum energy is 316MeV under such an accelerator structure.

2.2 Under Measured Magnetic Field

To approach real situation, we calculate the variation of ß oscillation frequency with particle energy change according to practical measured data of magnetic field. First of all, theoretical isochronous field of accelerated particle is established on magnetic sector axial line by K_b, K_r method. In accordance with practical measured data of magnetic field distribution, the calculation is extended to two dimensions. Magnetic field mesh is $\Delta r = 10mm$ from $r = 840mm$ to $r = 3420mm$, $\Delta\theta = 0.5^\circ$ from 0° to 90°. By using angle θ to machine center as variation, the transverse oscillation equations are:

$$\frac{d^2x}{d\theta^2} + (1-n)\frac{r^2}{\rho^2}x = 0 \qquad (7)$$

$$\frac{d^2z}{d\theta^2} + n\frac{r^2}{\rho^2}z = 0 \qquad (8)$$

Where $n = -\frac{\rho}{B\sqrt{r^2 + \dot{r}^2}}(r\frac{\partial B}{\partial r} - \frac{\dot{r}}{r}\frac{\partial B}{\partial\theta})$, r is particle radial coordinate to machine center, $\dot{r} = dr/d\theta$, ρ is particle curvature radius on equilibrium orbit, B is magnetic field at particle's position. They are calculated by using particle equilibrium orbit and two-dimension isochronous magnetic field on equilibrium orbit.

Define $p_y = \frac{dy}{d\theta}$. To initial conditions:

$$\begin{pmatrix} y_1(0) \\ p_{y1}(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and

$$\begin{pmatrix} y_2(0) \\ p_{y2}(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

using Ronge-Kutta method to solve equations (7) and (8) from $\theta = 0$ to $\theta = \pi/2$, the values over one period of magnetic structure

$$\begin{pmatrix} y_1(\pi/2) \\ p_{y1}(\pi/2) \end{pmatrix}$$

and

$$\begin{pmatrix} y_2(\pi/2) \\ p_{y2}(\pi/2) \end{pmatrix}$$

can be obtained, and also the Twiss matrix $\begin{pmatrix} y_1(\pi/2) & y_2(\pi/2 \\ p_{y1}(\pi/2) & p_{y2}(\pi/2) \end{pmatrix}$.

From $\mu_y = \arccos(\frac{y_1(\pi/2) + p_{y2}(\pi/2)}{2})$ we can calculate $\nu_y = \frac{N\mu_y}{2\pi}$.

The calculation result of isochronous field of proton beam with extraction energy 270MeV is shown as left curve in Fig.2. The curve is from upper to lower with the increasing of particle energy, from 18.6MeV to 270MeV. To even higher particle energy $\nu_z = 0$ at the extract position, there is defocusing at z direction. Therefore, according to analysis of practical measured isochronous field, the accelerated proton has maximum energy 270MeV approximately.

REFERENCES

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