EQUIPARTITIONING DESIGN METHOD OF A QUASI-PERIODICAL FOCUSING-ACCELERATING COUPLED SYSTEM

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Abstract

High-current proton linac sets a very severe requirement on the control of beam loss and emittance growth. Beam physics theory demonstrates that driven by the beam coherent instability; beam emittance grows due to the coupling of the strong space charge effect when an intense beam has different temperatures between different degrees of freedom. Therefore it is necessary to design an intense beam accelerator in line with the equipartitioning principle. However, this design becomes too tedious to reach full equipartitioning because a real proton linac is, in fact, a quasi-periodic coupling system. Taking the advantage of the code TRACE3-D[1], we modified it and added equipartitioning design function to it. It becomes a useful tool to readily realize equipartitioning design, with the assistance of PARMILA code[2]. This paper will introduce the modification to TRACE3-D code, and then illustrate its application to the design of a DTL linac section, which shows the importance of equipartitioning design in the intense beam linac.

1 INTRODUCTION

Beam loss is a major concern in designing an intense beam proton linac, because it may result in a radioactivity on the machine strong enough to exclude the manual access to the machine for the necessary maintenance during its long-term operation. It has been realized that the beam halo formation and emittance growth are important reasons for the beam loss. Beam physics study[3-4] reveals that the beam matching and equipartitioning design can effectively control the beam halo formation and emittance growth.

A proton linac uses different accelerating structure at different energy sections. It usually consists of RFQ, DTL, CCL and/or superconducting linac. Due to the increase of proton energy, the structures form quasi-periodic accelerating-focusing systems. PARMTEQ and PARMILA codes are commonly used for the designs of RFQ and the other accelerators, and TRACE3-D is frequently applied for the beam matching design. However, these codes were unable to achieve equipartitioning design. It is a tedious work to reach an equipartitioning design in a quasi-periodic system when one directly applies the equipartition theory without code assistance. RFQ design code PARMTEQ has been modified for the equipartitioning design and applied the RFQ design of Frankfurt University. We modified TRACE3-D code in order for it to be capable of designing an equipartitioned linac. The modified code can readily achieve an equipartitioned linac design under the assistance of PARMILA code. In this paper, we will briefly introduce our modification to TRACE3-D code and use it for an example DTL linac to show its application. A major effort was put on keeping the beam matched while equipartitioned in the modification. Before describing the modification, the equipartition theory applied in the code will be outlined, as the basis of the code modification.

2 BASIS OF THE CODE MODIFICATOIN

Beam theory has demonstrated that beam should satisfy the equipartition condition in order to avoid the beam emittance growth and beam halo formation due to the coupling in different freedoms through strong space charge effect in an intense beam. For a matched beam in a uniform focusing channel, the condition is given in the following form:

$$\gamma_0 \frac{\varepsilon_{nx}}{\varepsilon_{nz}} \frac{z_m}{a} = 1 , \qquad (1)$$

or, in another form, which we used in the code:

$$\frac{\varepsilon_{nx}}{\varepsilon_{nz}} \frac{k_x}{k_z} = 1 . \qquad (2)$$

In the formula, ε_x and ε_z are the beam normalized emittances in transversal and longitudinal directions, a and z_m the beam RMS size in the two directions, k_x and k_z the wave numbers with space charge.

Even though the formula is derived for uniform focusing channel, it is also applicable for a matched periodic focusing channel, if the wave numbers in equation (2) are recognized, under smooth approximation, as the ones of a periodic system. However, the real proton linac usually is not a real periodic system due to the acceleration. As the result, the wave numbers change along the linac. The variation of longitudinal wave number is usually first fixed by linac longitudinal dynamics. Then the focusing strength of the magnets is adjusted to achieve the equipartition condition. The zero-current wave number in longitudinal direction can be expressed in terms of the following form:

$$k_{z0} = \left[-\frac{q}{mc^2}\frac{2\pi}{\lambda}\frac{1}{\beta_0^3\gamma_0^3}\frac{1}{L}\sum_{i=1}^{n} E_{0i}L_iT_i\sin\phi_i \right]^{\frac{1}{2}} \qquad (3)$$

with q being the charge of the particle, m the mass of the particle, c the speed of the light in vacuum, λ the RF wavelength, β and γ the relativistic factors, L the total cell length in one period, E_{0i}, L_i, T_i and φ_i the accelerating gradient, cell length, transit time factor and synchronous phase of the i-th cell in one period, n the total number of the cells in one period. The transversal phase advance of zero-current in one period σ_{x0} can be found by the trace of the transfer matrix M:

$$\cos\sigma_{x0} = \frac{Tr(M)}{2} \quad . \qquad (4)$$

The wave number may become a complex function of the parameters of a periodic system, and hence the equipartitioning design may turn out to be very tedious work. FODO channel of a DTL is a simple periodic system. The transversal transfer matrix of one period is:

$$M = D_4Q_FD_3G_2D_2Q_DD_1G_1 \qquad (5)$$

in which,

$$D_i = \begin{bmatrix} 1 & (L_i - l_Q)/2 \\ 0 & 1 \end{bmatrix}, \quad G_i = \begin{bmatrix} 1 & 0 \\ k_i & 1 \end{bmatrix},$$

$$Q_F = \begin{bmatrix} \cos\theta_F & \frac{1}{\sqrt{\kappa_F}}\sin\theta_F \\ -\sqrt{\kappa_F}\sin\theta_F & \cos\theta_F \end{bmatrix},$$

$$Q_D = \begin{bmatrix} \cosh\theta_D & \frac{1}{\sqrt{\kappa_D}}\sinh\theta_D \\ \sqrt{\kappa_D}\sinh\theta_D & \cosh\theta_D \end{bmatrix},$$

and $\theta_Q = \sqrt{\kappa_Q}\,l_Q$, $\kappa_Q = \frac{q}{mc}\frac{B_Q}{\beta_0\gamma_0}$, B_Q is the magnetic gradient of quadrupoles, l_Q is the length of the quadrupole. Q stands for F or D, corresponding to focusing and defocusing magnets. The RF defocusing effect is represented by ki, with $k_i = \frac{-\pi q E_{0i} L_i T_i \sin\phi_i}{mc^2\lambda\beta_0^3\gamma_0^3}$. To realise equapartition design, one must select a properite B_Q value to satisfy the contion of eq.(2).

3 MODIFICATION OF TRACE3-D CODE

It is obvious from the last section that the equipartition design is a rather complicated procedure for a quasi-periodic system, even in a simple case like a FODO channel of a DTL linac. So a code is necessary to design an equipartitioned linac composed of hundreds of periods. We noticed that TRACE3-D code is suitable to design a matched linac and expanding its capability to equipartitioning design is possible with the aid of PARMILA code. The longitudinal dynamic design of the PARMILA code outputs an input file for TRACE3-D, which gives out the quasi-periodic system and hence is a good start point. This output file can be further used for equipartitioning design by adjusting the magnetic gradient of focusing magnets in the modified TRACE3-D code. And more ever, TRACE3-D code itself includes the emittance growth due to the linear part of space charge effect and to RF field.

As the command K for periodic system in TRACE3-D was originally ineffective, it is adopted to enter the subroutine of equipartitioning design in the modified code. One can make choice of either equipartitioning design or a design of constant zero-current phase advance, which is also a kind of frequently used design method.

Because matching is the basis of the equipartitioning design, before equipartitioning design, one needs to design the input beam to be matched with the first period of the linac by making use of the matching design function of TRACE3-D. The modified code will revise the focusing field generated by PARMILA to satisfy the equipartition conditions, period by period. It is essential to maintain the beam always to be matched with focusing channel along the linac while changing the field of the magnets. So repeat may be necessary in some case. When a satisfied result is reached, the code will output a list of the focusing field for each magnet in the linac. It can then be put into PARMILA to replace the originally designed field.

4 APPLICATION EXAMPLE

To show the application of the modified code in equipartition design, an example is presented in this section. Table 1 lists the major parameters of a section of a DTL linac used in the example.

Table1 Major Parameters of the DTL Linac

Frequency f	324 MHz
Inject Energy W_0	3 MeV
Output Energy W	13 MeV
Accelerating Gradient E_0	2.5 MV/m
Synchronous Phase φ_0	-30^0
Number of Cells	51
Beam Current I	60 mA
Lattice	FODO
Nor. Emittance (RMS) ε_{nx}	0.0931 mm-mrad
ε_{ny}	0.0925 mm-mrad
ε_{nz}	0.135 Deg-MeV

The equipartitioning factor $\frac{\varepsilon_{nz}}{\varepsilon_{nx}}\frac{k_z}{k_x}$ is plotted in Figure 1 for the cases with a constant zero-current phase advance of 50 degree and with an equipartitioning condition. It is observed that the longitudinal temperature of the non-equipartitioned beam is three times higher than that of the transversal temperature at the entrance of the linac. For the equipartitioned beam the factor keeps unit almost all along the linac.

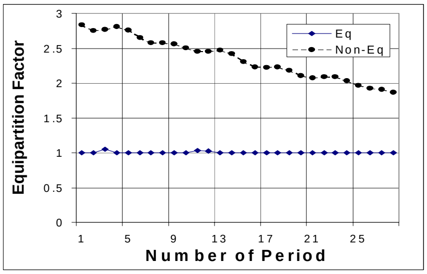
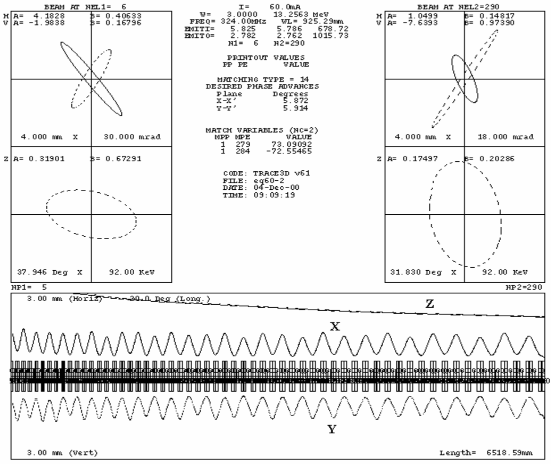


Figure 1: Equipartition factor along the linac for equipartitioned and non-equipartitioned beams (a constant zero-current phase advance of 50 degree in transversal direction).



```
         I=     60.0mA
    W=    3.0000    13.2563 MeV
 FREQ= 324.00MHz    WL= 925.29mm
EMITI=    5.825     5.786   678.72
EMITO=    2.782     2.762  1015.73
        N1=   6    N2=290

       PRINTOUT VALUES
      PP PE        VALUE

     MATCHING TYPE = 14
    DESIRED PHASE ADVANCES
     Plane        Degrees
     X-X'          5.872
     Y-Y'          5.914

    MATCH VARIABLES (NC=2)
     MPP MPE         VALUE
      1  279      73.09092
      1  284     -72.55465

       CODE: TRACE3D v61
       FILE: eq60-2
       DATE: 04-Dec-00
       TIME: 09:09:19
```

Figure2: TRACE3-D output for the equipartitioned beam in the DTL. Up-left plot is the input beam phase spaces; up-right plot is the output beam phase spaces; bottom plot is the beam envelope in three directions.

TRACE3-D output the graph of the beam phase spaces and envelopes for the equipartitioned beam, as shown in Figure 2. The beam envelopes in the bottom plot exhibit a good matching of the beam with the focusing lattice.

To compare the emittance growth of the equipartitioned and non-equipartitioned beams, the magnetic field from TRACE3-D runs was copied into PARMILA input file. PARMILA runs with 10,000 macro-particles show the emittance variation in Figure 3. It is noticed that the transversal emittance of the equipartitioned beam increases only a little, several times less than that of the non-equipartitioned beam. On the other hand, in longitudinal direction, emittance growth for equipartitioned beam is more than that of the non-equipartitioned beam. The results reflect the fact that the equipartitioning design prevents the temperature transfer from the longitudinal direction to the transversal direction.

The example in this section shows that the modified TRACE3-D code can be readily used for the design of equipartitioned beam in a quasi-periodic system with acceleration.

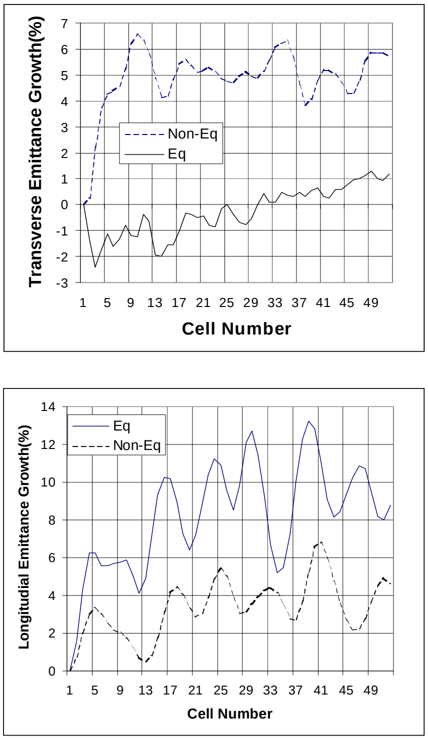


Figure3: Beam emittance growth for equipartitioned and non-equipartitioned beam in transversal (up) and longitudinal (low) directions.

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