BASICS OF SURFACE AND VOLUME FEL'S

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Abstract

Ordinary Free-Electron Lasers [FEL's] can be found in successful operation in the spectral range from millimeters to ultraviolet wavelengths. However the operation of the common FEL's in the extreme ultraviolet and X-ray wavelength regimes faces certain adverse effects. Some of the main obstacles in the way of the realization of X-ray FEL are electron momentum spread and angular divergence. Moreover it is desirable to have more compact FEL's. Surface and Volume FEL's are one of several attractive alternatives. These provide the possibility of more compact devices. And can in principle operate from submillimeter to X-ray. We note that in volume FEL the radiation wavelength is of the same order as the diffraction grating period. The diffraction grating provides the volume distribution feedback. The wave undergoes Bragg diffraction. We study the basic equations of Volume FEL's. Equations and formulas have been recently given by Baryshevsky for vacuum Volume FEL's. In addition to providing an independent check on the basic formulas of vacuum Volume FEL of Baryshevsky we do not ignore the nonperturbative part of the current. The power spectrum of current density and velocity fluctuations are given. The density and velocity fluctuations in entering electron beam cause noise excitation and should be considered. We also briefly comment on the subpicosecond X-ray pulses arising from Volume FEL's and the interaction of such a pulse with a crystal.

Keywords: X-Ray FEL, Short-wavelength Lasers, Volume Free-Electron Laser, Noise.

1 INTRODUCTION

Simply, FEL action arises out of the interaction of dynamics of the electron beam with the electromagnetic [EM] field. The question arises what kind of interaction and what is the specific nature of the wave which leads to the stimulated emission in FEL's? It is known that [1] for the stimulated emission to take place the electron beam must form *coherent bunches* and respond in a *collective manner* to the radiation field. One way to fulfill these requirements is to let the electron beam to transverse an *undulatory magnetic field* [wiggler, for example]. The spatial variation of the wiggler field forms a beat wave [interference pattern] with the electromagnetic wave. Stimulated Emission is the result of the interaction of the electron beam with the beat wave. For FEL action the beat wave must be in synchronism with the electron beam. This is possible since electrons are limited to travel with speed less that of light *in vacuo* and since the beat wave[1] has the same frequency as that of EM wave but its wavenumber is the sum of the wavenumbers of the EM and wiggler fields it has a phase velocity less than the EM wave and consequently the electrons can be synchronized with it.

We now state the well-known fundamental equations which are used in the foundations of Free-Electron Laser [FEL] theory. The charge and current densities enter as sources in the Maxwell's Equations and are respectively written as

$$\mathbf{J}(\mathbf{x},t) \quad = \quad -e\sum_{j=1}^{N_b}\mathbf{v}_{(j)}(t)\delta[\mathbf{x}-\mathbf{x}_{(j)}(t)], \qquad (1)$$

and

$$\rho(\mathbf{x},t) \quad = \quad -e\sum_{i=1}^{N_b}\delta[\mathbf{x}-\mathbf{x}_{(j)}(t)]. \qquad (2)$$

We have enclosed the particle label j in brackets to avoid confusion with index notation used for vectors and other tensors.

The wave equation which arises out of the Maxwell's equations assuming the magnetic permeability μ to be a constant can be written as

$$\nabla\times\nabla\times\mathbf{E}(\mathbf{x},t) = -\mu\frac{\partial}{\partial t}\mathbf{J}(\mathbf{x},t) - \mu\frac{\partial^2}{\partial t^2}\mathbf{D}(\mathbf{x},t). \quad (3)$$

The explicit form of the current must be determined by finding the particle velocity, which in turn appears in Lorentz force equation, viz,

$$\frac{d}{dt}\mathbf{p}_{(j)}(t) \quad = \quad -e[\mathbf{E}+\mathbf{v}_{(j)}(t)\times\mathbf{B}]. \qquad (4)$$

Since the right-hand-side [rhs] of Eq. 4 depends on momentum and on the left-hand-side [lhs] we have velocity that means we must eliminate the momentum in favor of velocity by using the well-known relation $\mathbf{p}_{(j)}(t) = m\gamma\mathbf{v}_{(j)}(t)$. After some simple algebra we eliminate the momentum in Eq. 4 to obtain

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[1] also called pondermotive wave

$$\frac{d}{dt}\mathbf{v}_{(j)}(t) = -\frac{e}{m\gamma}\{\mathbf{E}+\mathbf{v}_{(j)}(t)\times\mathbf{B} - (\mathbf{v}_{(j)}(t)\cdot\mathbf{E})\mathbf{v}_{(j)}(t)\}. \quad (5)$$

It is convenient to go to Fourier coordinates or reciprocal[2] coordinates ω and $\mathbf{k}$ by defining the Fourier Transformation [FT] of any function $f(\mathbf{x},t)$

$$f(\mathbf{k},\omega) = \int_{-\infty}^{\infty} d\omega \int d^3k \exp\left(i\omega t - \mathbf{k}\cdot\mathbf{x}\right) f(\mathbf{x},t). \quad (6)$$

Applying FT to Eq. 3 we obtain

$$-\mathbf{k}\times\mathbf{k}\times\mathbf{E}(\mathbf{k},\omega) = -i\mu\omega\mathbf{J}(\mathbf{k},\omega) + \mu\omega^2\mathbf{D}(\mathbf{k},\omega). \quad (7)$$

Rewriting the above equation in component form we have

$$k^2E_i(\mathbf{k},\omega) - (k_jE^j)k_i = -i\mu\omega J_i(\mathbf{k},\omega) + \mu\omega^2\varepsilon_{ik}E^k(\mathbf{k},\omega), \quad (8)$$

where the electric displacement $\mathbf{D}$ has been related to the elecrtic field $\mathbf{E}$ via the electric permitivitty tensor ε_{ik}. One can easily solve Eq. 8 for the electric field if the permitivitty tensor is diagonal [i.e. the case of diffuse beam where the wave frequency is much greater than electron plasma frequency], we immediately obtain

$$\mathbf{E}_i(\mathbf{k},\omega) = \frac{i\omega}{\varepsilon(\omega^2 - k^2c^2)}\{J_i(\mathbf{k},\omega) - \frac{c^2}{\omega^2}(k_lJ^l)k_i\}. \quad (9)$$

This agrees with the expression in [1] if one changes units [in this case replaces ε by $1/(4\pi)$]. When the permitivitty tensor is not restricted to a particular form one may solve Eq. 8 by using Green function techniques.

By using the above basic equations we can independently show the relations given in [2] for volume FEL. One advantage of our approach is that we use the equations from the very outset in terms of the reciprocal four-vector $(\omega,\mathbf{k})$. For example one can immediately write the expression of power and consequently emissivity by plugging Eq. 9 into the expression of radiated power

$$P = -2(2\pi)^4 \lim_{T\to\infty}\int_0^\infty d\omega \int d^3k \Re[\mathbf{E}(\mathbf{k},\omega)\cdot\mathbf{J}^*(\mathbf{k},\omega)]. \quad (10)$$

Another advantage is that since the VFEL depends on the Bragg's condition, and in analogy with theory of diffraction it is more expedient to formulate the problem in terms of reciprocal space. Moreover the dispersion relation can be written with less effort if one starts with reciprocal space from the outset.

2 SURFACE AND VOLUME FEL'S

Many different types of FEL's have been suggested and some of these are now in successful operation. Almost all the current types of FEL's use the spontaneous radiation mechanism and employ for feedback two parallel mirrors forming a cavity or one-dimensional diffraction grating in which transmitted and diffracted waves propagate along the electron beam direction forming of what is called a one-dimensional distributed feedback [DFB].

[2] reciprocal to time and space

It is of interest to consider FEL's which are compact and can operate in a wide-spectral range in particular ultraviolet and X-ray. The surface and volume FEL's have been suggested [2] to achieve the requirements of being compact and of operating in the ultraviolet and X-ray regions. Moreover they potentially provide a large tunability from submillimeter to X-ray. The main idea of SFEL and VFEL is that they work on the basis of non one-dimensional distributed feedback system. Now what is non one-dimensional distributed feedback system? Even a one-dimensional diffraction grating can act as non one-dimensional distributed feedback system provided that Bragg diffractive angle does not equal $\pi/2$ i.e. the wave does not move in the backwards direction. The gain of VFEL when the condition of synchronism is satisfied is proportional to $\rho_0^{1/S+1}$ where ρ_0 is the electron beam density and S is the number of diffracted waves[3]. From this relation we can immediately see why VFEL is more effective in terms of radiation process compared to ordinary FEL and in principle could be realized with a more compact device.

By going to the Fourier space we can write the fluctuation spectrum of the particle velocity by using the Lorentz equation Eq. 5

$$\delta\mathbf{v}_{(j)}(\omega) = \frac{ie}{m\gamma\omega}\int\frac{d^3k'}{(2\pi)^3}\exp\left(-i\mathbf{k}'\cdot\mathbf{r}_0\right) \left(\frac{\omega}{\omega-\mathbf{k}'\cdot\mathbf{v}_0}\mathbf{E}(\mathbf{k}',\omega-\mathbf{k}'\cdot\mathbf{v}_0) + \left(\frac{\mathbf{k}'}{\omega-\mathbf{k}'\cdot\mathbf{v}_0} - \mathbf{v}_0/c^2\right) [\mathbf{v}_0\cdot\mathbf{E}(\mathbf{k}',\omega-\mathbf{k}'\cdot\mathbf{v}_0)]\right). \quad (11)$$

This expression is the zeroth-order approximation. We can go beyond it in several ways. The first is to use simple iteration, i.e. calculate the next-order by replacing the velocity by $\mathbf{v}_0 + \delta\mathbf{v}_0$ [where $\delta\mathbf{v}_0$ is found from Eq. 11] in the expression Eq. 11 in this manner we can go beyond the linear approximation used in [2]. This allows us to incorporate the [perturbative] effects of the radiated field on the beam current. We must also check that this procedure converges, as it should, since it is by assumption a perturbative method.

In order to obtain an expression for gain we outline the procedure but not the equations here. We can readily obtain an expression for $\delta\mathbf{J}(\mathbf{k},\omega)$ by plugging Eq. 11 into the FT of the current expression [see definition Eq. 1] i.e. Eq. 1. Once this is done we may find an expression for $\mathbf{E}_i(\mathbf{k},\omega)$.

[3] We recall that for ordinary FEL gain when the condition of synchronism is satisfied is proportional to $\rho_0^{1/3}$

For diffuse beam case i.e. Eq. 9 this is clear. Even in more complicated case the same holds. We may rewrite Eq. 7 so that the source term i.e. explicit current terms are transported to the rhs of this equation. It now remains to expand the E field in terms of eigenmodes. Having done this we can obtain a dispersion relation involving the susceptibility χ, ω and k. Thus one can extract imaginary part of k which is equivalent to gain. By choosing different forms for χ we can model various diffraction gratings for the SFEL and VFEL schemes.

The changes in the power spectrum δP follows from Eq. 10,

$$\begin{aligned} \delta P &= -2(2\pi)^4 \lim_{T\to\infty} \int_0^\infty d\omega \int d^3k \\ & \Re[\mathbf{E}(\mathbf{k},\omega)\cdot \delta\mathbf{J}^*(\mathbf{k},\omega)]. \end{aligned} \tag{12}$$

and can be obtained directly by plugging in the expression for current fluctuations $\delta\mathbf{J}(\mathbf{k},\omega)$

$$\begin{aligned} \delta\mathbf{J}(\mathbf{k},\omega) &= -e\sum_j \exp(-i\mathbf{k}\cdot\mathbf{r}_{0j})\{\delta\mathbf{v}_{(j)}(\omega-\mathbf{k}\cdot\mathbf{v}_0) \\ & -i\mathbf{v}_0[\mathbf{k}\cdot\delta\mathbf{r}_j(\omega-\mathbf{k}\cdot\mathbf{v}_0)]\}, \\ \delta\mathbf{r}_j(\omega) &= -(i/\omega)\delta\mathbf{v}_{(j)}(\omega), \end{aligned} \tag{13}$$

into Eq. 12. We note that since the above expression for the current fluctuations is written in terms of velocity fluctuations and that later are related to $\mathbf{E}$ field through expression Eq. 11 the power fluctuation spectrum depends in the usual manner on the EM field, as it should.

The power spectrum of current density and velocity fluctuations associated with noise are clearly important considerations. It is well-known that at least three excitation amplitudes [the current density, the longitudinal velocity and electric field] which all enter with fluctuations. The basic E-field fluctuations arise due to zero-point fluctuations and are usually too small unless one is in the cavity quantum electrodynamics regime, i.e. the fluctuations are restricted to a region which are on the order of wavelength. Shot noise gives rise to density fluctuations and velocity fluctuations arise due to velocity spread.

We restrict ourselves to a simple and transparent approach to this issue in the present note, similar to [4]. As a first approximation we can assume that particles are randomly distributed when they enter the interaction area. The total number of particles in the beam under the most simplest assumptions is

$$n = \frac{NAu}{2\Delta f} \tag{14}$$

where Δf is the spectral width within which the fluctuations are to be evaluated. The beam is assumed to have a cross-sectional area $A = \pi w_b^2$ where w_b is the beam radius. The current density fluctuations for a stream of particles under the assumptions that they have a uniform velocity u and have a completely random distribution leads to the expected shot noise form

$$\delta J^2 = 2e\frac{J}{A}\Delta f \tag{15}$$

Let us next consider the velocity fluctuation noise. The mean-squared velocity fluctuations and mean square velocity deviation are

$$\begin{aligned} \overline{|\delta u|^2}/c^2 &= \frac{2}{3}\frac{\delta f}{f}\frac{1}{\lambda\overline{N}A}\frac{\Delta\gamma}{\gamma^3}, \\ \overline{\delta u^2}/c^2 &= \frac{1}{3}\frac{\Delta\gamma}{\gamma^3}, \\ \overline{|\delta u|^2}/\overline{\delta u^2} &\sim \frac{2}{3}\frac{\delta f}{f}\frac{1}{\lambda\overline{N}A}. \end{aligned} \tag{16}$$

In the last equation in 16 we have taken the ratio of mean-squared velocity fluctuations to mean square velocity deviation which shows that the former are reduced relative to the later by a factor which is directly proprtional to fractional bandwidth divided by number of particles within one wavelength. The key observation [4] is that although the velocity fluctuations appear small, they become density fluctuations by bunching and the resulting fluctuations can become quite large. This leads to the classic result that the gain becomes worse as we go towards smaller wavelength, in fact it the “worseness” factor goes as $1/\lambda^3$. In the case of SFEL and VFEL it is claimed [2] that the frequency of photons emitted even at small angles to the electron velocity does not depend on electron energy but is determined by the Bragg condition. However in a real device it is very difficult if not impossible to satisfy the Bragg condition for all the photons emitted from the beam. Thus a certain fraction of the photons will not satisfy the Bragg condition and will contribute to “noise power” of the beam. It is straightforward to estimate the loss of gain due to the part of the beam which gives rise to the photons that do not satisfy the Bragg condition. Clearly for this part of the beam the analysis takes the straightforward form as outlined briefly above and given in more detail in [4]. Thus the total gain will be a “mixture” of a $\rho_0^{1/4}$ and $\rho_0^{1/3}$ part rather then the pure $\rho_0^{1/4}$ [or $\rho_0^{1/(S+1)}$ in the case of multi-wave dynamical diffraction] dependence as in [2]. We note that in order to determine Imk_z [in notation of [2]] and the gain an expansion is made in [2] near the exact Bragg condition, with $\delta << 1$ [in notation of [2]] representing the deviation from this condition. However we cannot assume this for all the photons emitted from a real beam. Thus the the gain formula contains contributions from both parts one depending on $\rho_0^{1/4}$ and the other on $\rho_0^{1/3}$.

Yet another consideration which must be taken into account is that of injection of beam into the interaction region. This clearly has practical importance since it is necessary to minimize the fluctuations of the beam about the steady state trajectories in order to minimize a uniform resonant interaction. In ordinary FEL’s this is done by tapering the wigglers [1]. However, for example even with tapered field when electron beam is injected into a combined helical wiggler and axial solenoidal field [ASF], large scale fluctuations accompany injection when ASF is close to gyroresonace and lower the linear gain [1]. Thus in SFEL and VFEL one has to take into account of the photons emitted from electrons which suffer such large scale fluctuations. These photons are not expected to satisfy the Bragg condition.

The fast destruction of synchronism condition between a particle and emitted EM wave is expected to be a key feature for the VFEL scheme with electron passing through a diffraction grating. The influence of multiple scattering is expected to be reduced when the electron beam moves either in the split of grating [vacuum VFEL] or over a surface of a grating [SFEL] at a distance $d \leq \lambda\gamma$. No estimate or analysis of the fast destruction of synchronism has been given in [2].

It is pointed out in [2] that the set of equations given in (8) of [2] is a linear system and consequently one can ignore the non-perturbative part of the current. As is well-known [1] this is not correct. Indeed in the presence of media we get non-linearities for example under simple conditions we can obtain the well-known non-linear Schrodinger equation. Non-linearities can be included in the FEL equations in the standard way [1]. In the presence of non-linearities it is non-trivial to satisfy the Bragg-condition. The simplest non-linear formalism is obtained when one restricts the interaction of electron beam with a wave(s) of single frequency and takes the average of Maxwell's equation over a wave period to remove the fast time oscillations [1]. In this case the numerical procedure is simplified. One of the central equation in this analysis is the non-linear pendulam equation [1]. This leads to a self-consistent nonlinear theory of FEL which describes the interaction through *the linear regime and includes the saturation of growth mechanism* [1]. In ordinary FEL saturation can occur in several ways [1]. It is important to consider the mechanisms for the saturation in SFEL and VFEL.

3 APPLICATIONS

It is of interest to consider the interaction of short-time [sub-picosecond] X-ray pulses with materials [e.g. crystals]. It has been noted by Baryshevsky [3] that short x-ray pulse passing through a crystal results in large time delay of radiation. This time delay of pulse can be observed by extremely short [sub-picosecond] pulse which can be produced in VFEL or FEL operating in the SASE mode. Our comment is restricted to the appilcation of short-time pulses and time delay of radiation to the physics of dynamical stripes. It is expected that the time-scale of phonon assisted dynamical stripes is on the order of picosecond otherwise it on the order of femto-seconds. Thus the relevant time-scale of dynamical stripes varies from pico-seconds to femto-seconds, i.e. sub-picosecond time scale. Moreover the relevant length scale of fundamental stripes also lies in x-ray region. It is natural to suggest that one can use the x-ray pulses from VFEL or FEL operating in the SASE mode to study both spatial and temporal structure of dynamical stripes which occur for example in cuprates, manganites and nickelates. To our knowledge this suggestion has not been made before.

4 CONCLUSIONS

We have looked at the basic equations of surface and volume FEL's. We go beyond the linear approximation by including the [perturbative] effects of the radiated field on the beam current. Moreover we suggest that the noise power in the electron beam, when the exact Bragg condition is not exactly satisfied must be taken into account in the gain and related quantities. We have also indicated for the first time that the x-ray pulses produced from VFEL or FEL operating in the SASE mode to study both spatial and temporal structure of dynamical stripes which occur for example in cuprates, manganites and nickelates.

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