

# MAGNETIC FIELD PARAMETRIZATION FOR EFFICIENT SPIN TRACKING WITH POLE\*

J. F. Schmidt<sup>†</sup>, W. Hillert, ELSA, Bonn, Germany

## Abstract

The new spin dynamics simulation suite `POLE` is designed to perform systematic studies of beam depolarization in circular accelerators with short storage times or fast energy ramps. It is based on spin tracking using a Runge-Kutta algorithm with adaptive step width. `POLE` can approximate the magnetic fields of the accelerator with a Fourier series to reduce computing time. Therefore, the magnetic field distribution is simplified with frequency filters by a C++ library before the spin tracking. The versatile library deals with import and export of lattices and particle trajectories from `MAD-X` and `ELEGANT`. The derived magnetic field distributions can be interpolated, Fourier transformed and accessed easily by applications. This contribution discusses advantages and disadvantages of the frequency filtering concept.

## INTRODUCTION

The new spin dynamics simulation suite `POLE` is designed to perform systematic studies of beam depolarization in circular accelerators with short storage times or fast energy ramps. This includes the crossing of depolarizing resonances and the impact of synchrotron radiation on short time scales (up to seconds). The intention is to develop open source tools that are also usable for other facilities.

Spin motion in an accelerator is described by the Thomas-BMT equation [1]. So, its numerical integration is an obvious approach for spin tracking. Neglecting electric fields perpendicular to the beam axis, the Thomas-BMT equation for an electron spin  $\vec{S}(t)$  can be written as

$$\frac{d}{dt}\vec{S} \approx c \cdot \vec{S} \times \left[ (1 + \gamma a)\vec{B}_\perp + (1 + a)\vec{B}_\parallel \right] \quad (1)$$

with the gyromagnetic anomaly  $a = (g_s - 2)/2$ , the Lorentz factor  $\gamma(t)$  and the energy normalized magnetic field

$$\vec{B}(t) := \frac{e}{p(t)} \cdot \vec{B}(t) \quad (2)$$

parametrized by the time  $t$ . All the information about the accelerator lattice, beam optics settings and also beam dynamics are contained in the magnetic field  $\vec{B}(t)$  experienced by the particles. Hence, the major task for efficient spin tracking is to implement a suitable magnetic field parametrization and an according integration algorithm.

The basis for `POLE` is a C++ accelerator lattice library developed at ELSA. It allows to work with lattice information and particle tracking results from `MAD-X` and `ELEGANT` within any C++ code. Building on this, magnetic field

parametrizations can be tested and compared. The first approach realized for `POLE` was a frequency filtering of the magnetic fields to balance spin tracking accuracy against computing time [2]. Advantages and disadvantages of this approach are discussed in this contribution.

## C++ LATTICE LIBRARY

The C++ accelerator lattice library developed at ELSA basically provides two data structures. One represents a lattice consisting of individual elements of different types (e.g. dipole, cavity, quadrupole or kicker) stored by their position in the accelerator. The second structure implements arbitrary physical quantities, which are defined as a function of the position in the accelerator. They can have integer or floating point data type, including two or three dimensional quantities. Common examples are Twiss parameters or the closed orbit. A schematic of the library is shown in Figure 1. Lattice information can be accessed at an arbitrary position. Interpolation and Fourier transformation of the physical quantities are included using the GNU scientific library.

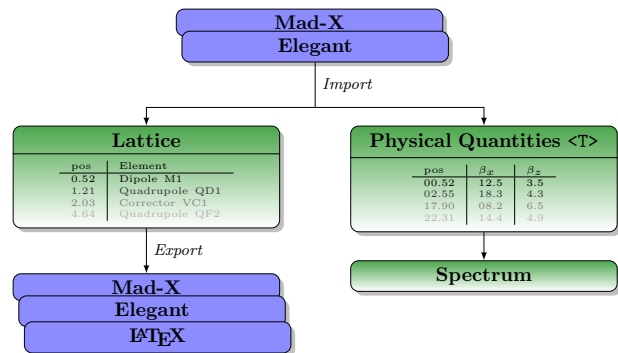


Figure 1: Schematic of the C++ accelerator lattice library.

Both data structures can be imported from the established simulation tools `MAD-X` and `ELEGANT`. Lattice definitions for these widespread tools exist for many accelerators and can be made available in any C++ program with about two lines of code. Of course, a lattice can also be constructed and modified within C++. Additionally, the library provides a lattice export, which also facilitates the automatic lattice conversion between `MAD-X` and `ELEGANT`. Automated drawings of an accelerator can be made with a `LaTeX` export using the `tikz-palattice` package [3]. A shared library version will be published as open source software soon.

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2015). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI.

\* Work supported by BMBF  
<sup>†</sup> schmidt@physik.uni-bonn.de

## FREQUENCY FILTERING OF MAGNETIC FIELDS

Starting point for the development of POLE was the simulation of crossing integer depolarizing resonances in the ELSA stretcher ring to study the dedicated correction scheme [4]. Integer resonances occur at integer spin tune  $\gamma a = n \in \mathbb{N}$  and are driven by revolution harmonic horizontal field distributions, which are mainly composed of magnet misalignments and quadrupole fields due to vertical closed orbit distortions. In particular, single particle trajectories are not involved, because of the non-integer betatron tunes. Therefore, only a single turn field map for a particle on the closed orbit is needed for spin tracking. POLE calculates this field map parametrized by the time (up to the revolution time) using a lattice and a closed orbit from MAD-X or ELEGANT. A Fourier transformation for each axis  $\chi \in \{x, z, s\}$  yields the amplitudes  $A_i^\chi$  and phases  $\phi_i^\chi$  for the revolution harmonics  $f_i = i \cdot f_{\text{rev}}$ . They can be used to approximate the magnetic field map as a Fourier series:

$$\vec{B} \approx \sum_{i=0}^{i_{\text{max}}} \vec{A}_i \cos(2\pi f_i t + \phi_i) . \quad (3)$$

The precision of this approximation can be adjusted by  $f_{\text{max}}$ . This low pass is visualized in Figure 2 for vertical ( $z$ ) and horizontal ( $x$ ) fields in the ELSA stretcher ring. The higher  $f_{\text{max}}$  is, the higher is the spatial resolution of the field map whereas the integrated field is independent of  $f_{\text{max}}$ .

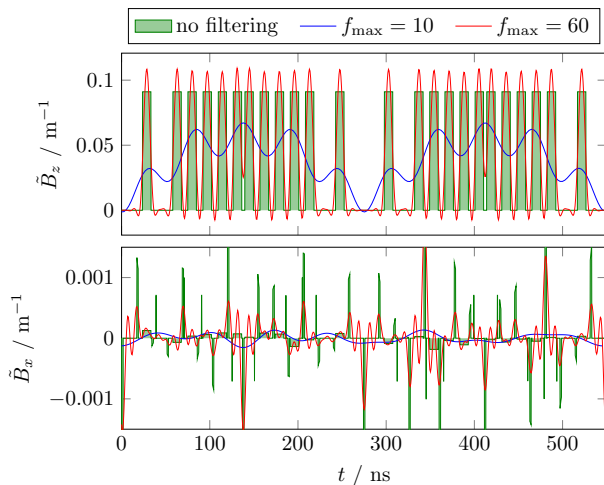


Figure 2: Magnetic field approximation by Fourier series up to different maximum frequencies (equation 3) for one turn in the ELSA stretcher ring.

The Thomas-BMT equation is integrated using a Runge-Kutta algorithm with adaptive step width. Figure 3 shows the step widths as a function of the position in the ELSA stretcher ring accumulated over many turns. Without frequency filtering the step width is very short, especially at the magnet edges. They had to be smoothed to achieve a continuous function integrable with the Runge-Kutta method. For this purpose, smooth edge fields have been implemented in

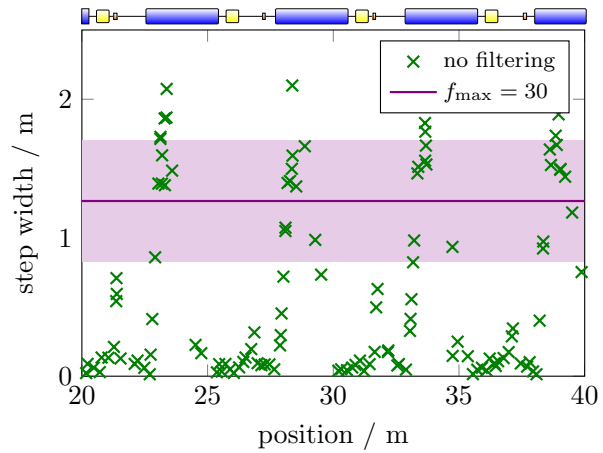


Figure 3: Step widths of the Runge-Kutta algorithm as a function of the position in the ELSA stretcher ring accumulated over many turns.

the lattice library. The step width increases only within the bending magnets due to their constant field or within drift spaces. Applying the frequency filter, the individual magnets are not resolved any more, the average step width increases and its spread decreases. This reduces the computing time.

To generalize the spin tracking full particle dynamics have to be included in calculating the field map. Therefore, single particle trajectories can also be imported from MAD-X or ELEGANT. They include field contributions related to the betatron tunes, which do not occur with revolution harmonic frequencies. Thus, a multi turn field map is needed to calculate the spectrum. E.g. thousand turns will result in a frequency resolution of  $f_{\text{rev}}/1000$ . Most contributions are negligible and can be filtered out by additionally cutting all entries below an amplitude  $\vec{A}_{\text{min}}$ . In this way, the number of summands of the Fourier series does not increase dramatically. However, each particle needs its own field map and the Fourier transformations require considerable computing time. In the following, only crossing integer resonances based on a particle on the closed orbit is discussed.

### Convergence

Figure 4 shows exemplary spin tracking results plotted against the maximum frequency  $f_{\text{max}}$  of the field approximation.  $P_f$  is the vertical polarization after crossing a single integer resonance in the ELSA stretcher ring. One might suppose that each resonance is excited exclusively by the field distribution with the frequency  $\gamma a \cdot f_{\text{rev}}$ . This is not the case, because of the different reference frame of spin precession (only within the magnets) and the field map (continuous with position or time).

For the resonance  $\gamma a = 3$  the result converges already with only a few contributing frequencies. In this case the step width can even exceed the length of a magnet and the computing time is short. Certainly, the convergence depends on the resonance, because each resonance is driven by another field contribution. Additionally, it can also depend

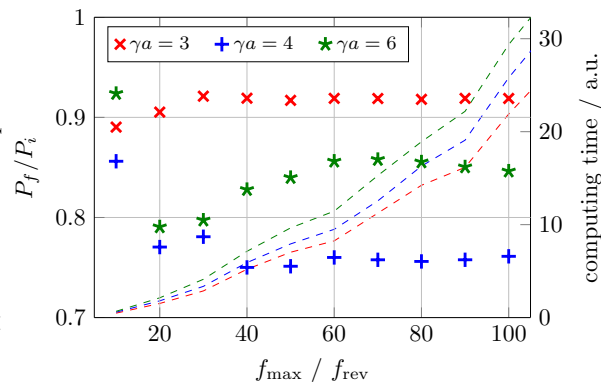


Figure 4: Convergence of vertical polarization after crossing an integer resonance in the ELSA stretcher ring, simulated with different maximum frequencies  $f_{\max}$  of the field approximation. Computing time as dashed lines.

on the individual field distribution. In fact, a reasonable compromise between accuracy and performance has to be found separately for each case. Using higher frequencies as a precaution is not advisable, because the computing time increases faster than linear with  $f_{\max}$ .

The rise of computing time with the number of frequencies is not only caused by the smaller step width, but also by the calculation of the Fourier series with more summands. Actually, about 90% of the run time is used by the cosine function called to evaluate the Fourier series for each integration step (at  $f_{\max} = 30$ ). So this is the only part of the code worthwhile for optimization. The Fourier series could be calculated in advance and interpolated for each integration step. In any case the frequency filtering is only reasonable, if some ten frequencies are sufficient.

The “wavelength”  $\lambda_{\min} := c/f_{\max} = L/i_{\max}$  is a measure for the spatial resolution of the field map approximation by the Fourier series. This indicates that the maximum frequency which is needed for a given spatial resolution scales with the circumference  $L$  of the accelerator. In case of larger accelerators the spin tracking does not converge before analogously higher  $f_{\max}$ . Therefore, this approach seems to be not feasible for accelerators significantly larger than the ELSA stretcher ring ( $L = 164.4$  m).

The main motivation for the frequency filtering approach was an expected decrease of computing time by filtering the field map. If this is valid at least for small accelerators, can only be tested by comparison with other spin tracking methods.

## SPIN TRACKING WITH ROTATION MATRICES

To compare the discussed spin tracking based on frequency filtering with other field parametrizations and tracking algorithms, a much more common approach is currently implemented. Here, the Thomas-BMT equation (Eq. 1) can be solved as

$$\vec{S}(s_0 + l) \approx \mathbf{R}_{3 \times 3}(s_0, l) \vec{S}(s_0), \quad (4)$$

where the spin vector  $\vec{S}$  is tracked from the position  $s_0$  through a magnet with length  $l$  by a three dimensional rotation matrix  $\mathbf{R}_{3 \times 3}$ . The matrix contains the spin precession in the magnet and is constructed from the corresponding magnetic field  $\vec{B}$ . Its direction determines the precession axis and the rotation angle  $\theta$  is calculated from the absolute value according to the Thomas-BMT equation:

$$\theta = \xi^z \cdot \left| \vec{B} \right| \cdot l \quad (5)$$

with  $\xi_x = \xi_z = \gamma a$  and  $\xi_s = a$ . This element-by-element tracking assumes a constant orbit within a magnet. Its precision could be improved by dividing a magnet in multiple slices to approximate a rotation axis changing continuously. Single particle trajectories can be included.

This basic approach parametrizes the field by the consecutive elements instead of the continuous time  $t$ . Hence, regions without spin precession in-between the magnets are skipped. The algorithm is implemented easily using the lattice library. The linear algebra calculations are performed using the open source C++ library Armadillo [5]. First results promise computing times comparable to the frequency filtering method with very few frequencies or even faster and without any dependence on the lattice circumference.

## CONCLUSION

Frequency filtering of the magnetic fields was a promising approach for simulating the crossing of depolarizing resonances especially for integer resonances, but it is not suitable for large accelerator rings without time-consuming dedicated studies for special cuts and filters. If the effort to optimize the filters is worthwhile for smaller machines depends crucially on the performance compared to other methods, which is currently studied. For that purpose, the C++ lattice library allows to compare different field parametrizations and tracking algorithms based on the same lattice and data structure. It will be published as open source software soon.

## REFERENCES

- [1] V. Bargmann and L. Michel and V.L. Telegdi, Phys. Rev. Lett. 2.10 (1959) 435.
- [2] J. F. Schmidt, et al. “Simulating Spin Dynamics and Depolarization using POLE”, IPAC’13, Shanghai, MOPWO005 (2013), <http://www.JACoW.org>
- [3] J. F. Schmidt, tikz-palattice package, The Comprehensive TeX Archive Network, [www.ctan.org/pkg/tikz-palattice](http://www.ctan.org/pkg/tikz-palattice)
- [4] J. F. Schmidt, et al. “Optimizing Polarization with an Improved Integer Resonance Correction Scheme at ELSA”, IPAC’14, Dresden, TUPRO039 (2014), <http://www.JACoW.org>
- [5] C. Sanderson, “Armadillo: An Open Source C++ Linear Algebra Library for Fast Prototyping and Computationally Intensive Experiments”, Technical Report, NICTA (2010)