

THE LOW- α LATTICE AND BUNCH LENGTH LIMITS AT BESSY-VSR*

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Abstract

An upgrade of the BESSY II synchrotron light source to a Variable pulse length Storage Ring BESSY-VSR has been recently proposed [1], by introducing strongly focusing superconducting cavities in order to manipulate the longitudinal phase space. This will bring up a voltage beating pattern and allows to store simultaneously long and short electron bunches. In the regular user optics rms bunch lengths of ≈ 15 ps and down to 1.5 ps are expected for high current operation. Bunches as short as 300 fs can be provided by using a low- α optics. This paper will focus on the low- α optics for shortest bunches and discuss intrinsic limits of the zero current bunch length, given by different single particle effects.

INTRODUCTION

The bunch lengths given in the abstract are for high current operation, including the potential well effect evoking a 50% bunch lengthening. In the following we will focus only on the zero current bunch length and its limits from single particle effects. For electron storage rings, the equilibrium bunch length σ_0 in zero current limit is defined by the equilibrium energy spread δ_0 , which is reached when the average quantum excitation rate is equal to radiation damping rate:

$$\sigma_0 = \frac{\alpha \delta_0}{2\pi f_s} = \delta_0 \sqrt{\frac{E_0}{f_0} \frac{\alpha}{eU'}} \quad (1)$$

For fixed energy E_0 and revolution frequency f_0 two adjustable parameters allow the manipulation of the zero current bunch length of the storage ring:

1. The momentum compaction factor α defined by the magnetic optics

$$\Delta L/L_0 = \alpha \Delta p/p_0 = \alpha \delta \quad (2)$$

2. The rf-voltage gradient U' given by the voltage amplitude U and its frequency f_{rf}

$$\frac{dU}{dt} = U' = 2\pi U f_{\text{rf}} \quad (3)$$

Both in turn define the longitudinal oscillation, expressed by the synchrotron frequency f_s via $f_s^2 = (\alpha f_0 eU')/(4\pi^2 E_0)$.

Currently, BESSY II provides synchrotron radiation in two different modes, the standard user mode and short pulses in the low- α mode. By introducing a voltage beating pattern [1] and increasing the longitudinal gradient for BESSY-VSR by a factor of 80 it is planned to store long and short

bunches simultaneously. In this paper we will focus exclusively on the shortest bunches in the low- α mode of BESSY-VSR. They will be reduced by a factor of ≈ 9 , down to approximately 250 fs zero current bunch length. Table 1 summarizes the lattice and beam parameters of BESSY II and VSR.

Table 1: Emittance ϵ , momentum compaction factor α and zero current bunch length σ_0 . For BESSY-VSR only bunch lengths of short bunches is shown.

	standard	low- α
emittance ϵ	5 nm rad	40 nm rad
mom.comp. α	$7.3 \cdot 10^{-4}$	$3.5 \cdot 10^{-5}$
σ_0^{BII}	10 ps	2 ps
σ_0^{VSR}	1.1 ps	0.246 ps

Different single particle effects may provide limitations for the achievable minimum bunch length. If these effects are independent of each other their contribution to the total width will be add up quadratically. In order to push for shortest bunches in a storage ring with the BESSY-VSR concept, relevant limiting single particle effects have been evaluated and will be discussed here.

ZERO CURRENT BUNCH LENGTH LIMITING EFFECTS

The zero current bunch length given in Equation (1) is derived from single particle dynamics, as well as the discussed limiting effects.

Intra beam scattering of electrons within a bunch is a multi particle effect, which might also introduce a lengthening. However, because of only few mA beam current in the low- α optics, we expect no lengthening effect.

In this paper we will focus on the following two effects

- Longitudinal quantum radiation excitation,
- Horizontal longitudinal coupling,

as discussed in [2–4].

Longitudinal Quantum Radiation Excitation

The primary idea of this limit [2] is based on the fact that the path length of a radiating electron in a storage ring depends on the place where this photon emission took place. According to Equation (2) the path length for one revolution depends on energy deviation δ and the momentum compaction α , which is defined for the global ring. For a radiating particle changing its energy, the partial momentum compaction $\bar{\alpha}$ becomes important and can be calculated for

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a storage ring by

$$\bar{\alpha} = \bar{\alpha}(s_i) = \frac{1}{L_0} \int_{s_i}^{L_0} \frac{D(s)}{\rho(s)} ds, \quad (4)$$

where L_0 is the ring circumference and s_i the point of photon emission. That means, when photon emission takes place the energy changes and the path lengthening depends on the new energy and the partial momentum compaction $\bar{\alpha}$ until a further photon is emitted. The variance of $\bar{\alpha}$, named I_α defines this bunch length limit. So the stochastic fluctuation where the radiation process takes place is a direct measure of the variance of the momentum compaction leading to this so called quantum radiation excitation bunch length limit σ_{re} . As a result of this radiation excitation, the bunch length cannot be reduced below [2]

$$\sigma_{re} = \frac{\delta_0 \sqrt{I_\alpha}}{f_0} \quad \text{with} \quad (5)$$

$$I_\alpha = \langle [\bar{\alpha}(s_i) - \langle \bar{\alpha} \rangle]^2 \rangle. \quad (6)$$

This intrinsic limit depends on the revolution frequency f_0 , the natural energy spread δ_0 and the variance of the momentum compaction factor I_α . For “standard” DBA lattices, where dispersion and its derivative is zero outside of the DBAs, the partial momentum compaction can be well approximated by $\bar{\alpha} = \alpha \cdot (L_0 - s_i)/L_0$, leading to a variance of $I_\alpha = \alpha^2/12$. This simplification can not be used for low- α lattices because of the non vanishing dispersion function. Table 2 shows the numbers for the momentum compaction variance I_α and the associated bunch length limits at BESSY-VSR when using a natural energy spread of $\delta_0 = 8 \cdot 10^{-4}$ and a revolution frequency of $f_0 = 1.25$ MHz.

Table 2: Variance of the Momentum Compaction I_α and the Associated Radiation Excitation Bunch Length Limit σ_{re}

	standard	low- α
I_α	$4.5 \cdot 10^{-8}$	$1.3 \cdot 10^{-8}$
σ_{re}	140 fs	70 fs

For BESSY-VSR the radiation excitation bunch length limit was calculated to be $\sigma_{re} = 140$ fs for the standard user mode and $\sigma_{re} = 70$ fs for the low- α mode.

Transversal Longitudinal Coupling

The zero current bunch length for ultra short bunches becomes additionally dependent on coupling effects between the horizontal and the longitudinal plane (and similar for the vertical plane), which can be distinguished in 1st order and higher order effects with respect to x, x' .

In [4], a 2nd order nonlinear effects was discussed. It was pointed out that the path lengthening for one revolution ΔL_ξ is dependent on the betatron oscillation amplitude:

$$\Delta L_\xi = -2\pi \xi_x J_x. \quad (7)$$

It depends on the horizontal action $J_x = \epsilon/2$ of betatron motion and can be tuned by the chromaticity ξ_x . Using the

emittance from Table 1 and the usual BESSY II chromaticity of $\xi_x \approx 3$ a bunch length limit of < 2 fs is expected and in comparison with the zero current bunch length of BESSY-VSR negligible.

A 1st order, linear path lengthening effect is generated when a particle with horizontal displacement is passing a dispersion producing section, e.g., dipole magnets [3]. Particles with different amplitudes and phases pass the dipole on different trajectories resulting in a longitudinal displacement (a rotation in $x - z$ plane) and another bunch length limit σ_H given by [3]

$$\sigma_H = \sqrt{\epsilon H}/c \quad \text{with} \quad (8)$$

$$H = H(s) = \gamma D^2 + 2\alpha D D' + \beta D'^2. \quad (9)$$

H is the chromatic invariant function, which depends in turn on the optical Twiss functions γ, α, β and on the Dispersion and its derivative D, D' . The coupling strength and consequently this bunch length limit is a local property depending on the emittance ϵ and the local H function. In case of an isochronous ring, i.e., a low- α optics, the H function is typically non vanishing, like the dispersion, but it can be tuned to zero in few selected places by an appropriate optics. The H function changes inside dipoles; it stays constant in sections in between dipoles.

Figure 1 shows Twiss functions, the dispersion and the chromatic H function for the adapted BESSY-VSR low- α lattice. Currently the BESSY II optics consists of 16 equal DBA cells, completely symmetric. In simulations this symmetry was broken in one straight section (Observation Point: OP0) for BESSY-VSR, where the sc cavities will be installed to avoid additional coupling effects between the horizontal and longitudinal plane.

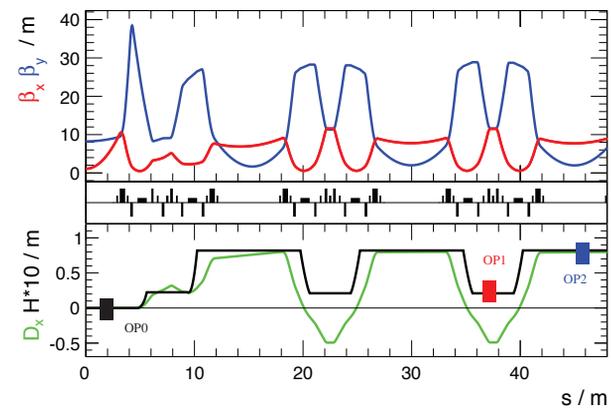


Figure 1: Optical functions of the adapted low- α lattice for BESSY-VSR. The H function multiplied by 10 is shown in black and is a measure of the bunch length limit. Observation points, used in elegant simulations are named OP0, OP1 and OP2.

Using this setting the H function is 21 cm in the DBAs (OP1) between the dipoles and nearly four times larger 80 cm in the straight sections (OP2) where the IDs are placed. Ac-

According to Equation (8) the bunch length limit was calculated using the emittance of 40 nm. Adding up this limitation quadratically with the zero current bunch length of ≈ 250 fs the effective bunch lengths σ_{eff} shown in Table 3 are expected.

Table 3: Limit from horizontal coupling and the effective bunch length are listed for the three observation points.

	limit σ_H	effective bunch length σ_{eff}
OP0	–	246 fs
OP1	100 fs	265 fs
OP2	195 fs	314 fs

The effective “zero current bunch length” increases by 8% in the DBAs to 265 fs and by 28% in the straights to 314 fs. Elegant [5] was used to verify these numbers with long term tracking for 100 longitudinal damping periods. Figure 2 shows the resulting bunch length for all three observation points, which is in excellent agreement with the analytical calculation. In case the bunch length σ_0 could be tuned to zero, the residual bunch length would be 200 fs because of this coupling and the radiation excitation effect. By reducing the emittance for the low- α lattice this limit may be further reduced.

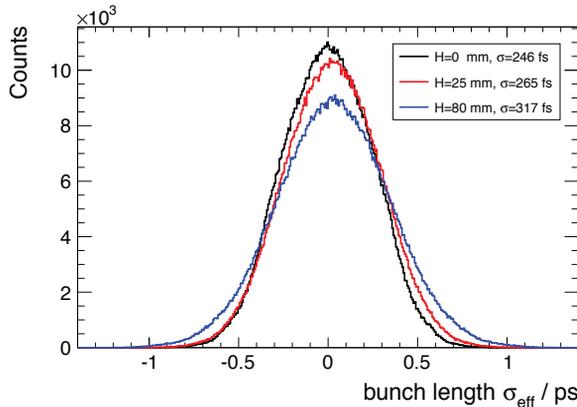


Figure 2: Results of the effective bunch length for all three observation points produced by long term tracking with elegant.

The bunch length limit inside the dipoles as given by the linear coupling is not constant but depends on the observation point, because of the changing H function. If the optical functions at the dipole entrance are chosen appropriately, the value of σ_H could become zero, but only at one point. If it is required to have a short bunch conserved along a distinguished length $(-L, L)$ inside the dipole, e.g. a observation section from a user beam line, a minimization approach for σ_H is required. An expression like $\sigma_{H,\text{min}}^2 \sim \int (H ds)_{\text{min}}$ needs to be evaluated, where the integration is performed over the interval $(-L, L)$. This approach is very similar to the minimization of the natural emittance contribution of

dipoles and extendedly describe in [6]. From this note we take the resulting minimum average value of H , determined over the length $(-L, L)$ inside the dipole. It is given by

$$H_{\text{min}} = \frac{1}{(12\sqrt{15})} \frac{L}{\rho^2}, \quad (10)$$

assuming symmetrically centered dispersion and β -function with minimum values of

$$D = \frac{\rho}{24} \left(\frac{L}{\rho} \right)^2 \quad (11)$$

$$\beta_{\text{min}} = \frac{L}{60}. \quad (12)$$

The resulting β function would be extremely small, and in case of storage rings there is limited tuning range to achieve these optimized functions.

CONCLUSIONS AND OUTLOOK

This paper presents limits for ultra short bunches in BESSY-VSR. As a starting point for simulations and calculations the well proven low- α lattice currently used at BESSY II was chosen. With the BESSY-VSR sc-cavity installation and this optics the bunches will be shortened down to 250 fs. In principle the momentum compaction α can be further reduced and the optics can be optimized for lowest zero current bunch length. However, evaluating bunch length limits showed that such optimization will not be successful, because the bunch length limit is given by horizontal longitudinal coupling and radiation excitation, increasing the zero current bunch length by nearly 30% in straight sections up to ≈ 315 fs.

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