SIMULATING SPIN DYNAMICS AND DEPOLARIZATION USING POLE*

J. F. Schmidt†, O. Boldt, F. Frommberger, W. Hillert, ELSA, Bonn, Germany

Abstract

The spin dynamics in circular accelerators with fast energy ramps, or short storage times of up to some seconds, can be investigated with spin tracking appropriately. Additionally, the spin motion of lepton beams is affected significantly by synchrotron radiation. Hence, spin dynamics simulations require spin tracking with a large number of particles to compute the beam polarization and thus take considerably long computing times. Therefore, high efficiency is crucial to perform systematic polarization studies.

The new simulation tool POLE provides the ability to balance accuracy against computing time. To that end, adjustable approximations of magnetic fields and synchrotron radiation are implemented. POLE is accessible for a wide range of lepton storage rings because it uses the common MAD-X lattice files and the corresponding particle tracking results.

MOTIVATION

The spin dynamics in circular accelerators is determined by the periodic sequence of the magnets. If the spins precess in phase with any component of the magnetic field distribution, a depolarizing resonance is excited. For electron beams synchrotron radiation causes incoherent spin motion and thus each crossing of a resonance leads to depolarization. Therefore, the investigation and correction of depolarizing resonances is a major challenge, when dealing with polarized electrons in accelerator rings. Hence, systematic simulations request easy access to lattice and beam optics modifications and high computing efficiency. An appropriate spin dynamics simulation tool for fast energy ramps and storage rings with short storage times is not available. For this reason, POLE is developed.

Its results can be tested experimentally at the in-house ELSA stretcher ring accelerating polarized electrons to typically 2.4 GeV during a fast 4 GeV/s energy ramp and applying sophisticated methods for the correction of depolarizing resonances [1].

THE BASIC CONCEPT OF POLE

The Thomas-BMT equation [3] describes the spin motion of a relativistic particle in electromagnetic fields. Solving it numerically is a common approach for the computation of spin dynamics in accelerators and is also used in the case of POLE. For this purpose we applied a Runge-Kutta algorithm with an adaptive step size. Neglecting electric fields, the Thomas-BMT equation can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{S} \approx c \cdot \vec{S} \times \left[(1+a\gamma)\vec{B'}_{\perp} + (1+a)\vec{B'}_{\parallel}\right] \qquad (1)$$

with the time dependent spin vector $\vec{S}(t)$, the gyromagnetic anomaly $a = (g_s - 2)/2$ and the energy normalized time dependent magnetic field $\vec{B'}(t)$. Here, the energy dependence of the magnetic field is separated from $\vec{B'}$ and given by the Lorentz factor $\gamma(t)$.

An important consequence of synchrotron radiation are depolarization effects caused by incoherent spin motion. Their simulation requires the computation of many spin vectors, which are finally averaged to derive the polarization vector. Some spins can be computed parallel on multicore processors, but the most significant reduction of computing time is achieved by confining the magnetic fields $B'(t)$. We have shown in [4] that this can be done by a spectral analysis of the field distribution. It allows for filtering the magnetic fields in the frequency domain and yields a smoother shape. Thus the step size during the integration of equation (1) is enlarged and can significantly exceed the length of a magnet.

MAGNETIC FIELD APPROXIMATIONS

The magnetic fields in the accelerator are decisive for the spin motion. They determine the strengths of depolarizing resonances. POLE computes the field distribution based on a common MAD-X lattice [2]. For this purpose, the MAD-X "*Twiss Module*" provides the magnet positions and strengths as well as the closed orbit. Of these, a field distribution $\vec{B'}(t)$ is derived for one revolution on the closed orbit. A Fourier transformation for each axis yields magnetic field spectra, consisting of the harmonics of the revolution frequency ω_{rev}. Its amplitudes and phases are used to approximate $B'(t)$ as a Fourier series:

$$B'(t) \approx \sum_{i=0}^{i_{\max}} A_i \cos(\omega_i t + \phi_i) \quad \text{with } \omega_i = i \cdot \omega_{\mathrm{rev}} \ . \qquad (2)$$

This approximation is then applied to equation (1) for each axis. That way, the frequency components of the field distribution can be filtered by selecting any ω_i for equation (2).

Figure 1 shows the effect of such a filter on the crossing of the integer resonance $a\gamma = 3$ in the ELSA stretcher ring. In this example, the computed vertical degree of polarization after crossing the resonance is plotted as a function of the maximum frequency $\omega_{\max} = i_{\max} \cdot \omega_{\mathrm{rev}}$ of the field distribution. The computed polarization converges with increasing $\omega_{\max}$, so this approximation of the field distribution is reasonable.

* Work supported by BMBF

† schmidt@physik.uni-bonn.de

ISBN 978-3-95450-122-9

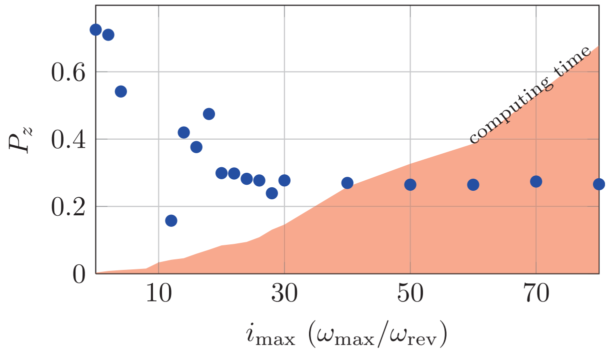

Figure 1: Simulated vertical polarization after crossing of integer resonance $a\gamma = 3$ as a function of the maximum considered frequency of the magnetic field spectrum

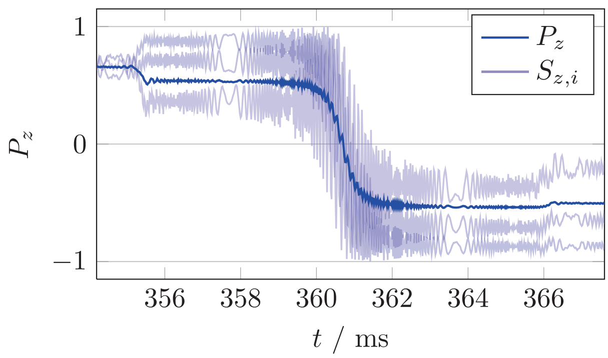

Figure 2: Simulated vertical polarization with synchrotron side-bands during crossing of integer resonance $a\gamma = 6$

Integer resonances can be modeled by this field spectrum, generated from a one revolution field distribution. Intrinsic resonances are caused by horizontal fields in phase with the tune. Thus, they are affected by the single particle trajectories and the relevant frequencies are especially not revolution harmonics. To implement intrinsic resonances, a representative particle on the emittance ellipse is tracked with the MAD-X "*ptc_track Module*". Tracking e.g. thousand turns results in a frequency resolution of $\omega_{\text{rev}}/1000$, so that the trajectory is closed in a good approximation if the tune is set with three decimal places. Computing the corresponding spectra with POLE still takes less than 30 s including the MAD-X execution. Many frequencies in-between the revolution harmonics have negligible contributions. Hence, the large number of frequencies can be reduced by filters effectively.

RESONANCE CROSSING AND SYNCHROTRON RADIATION

As described above, the particle's energy is separated from the magnetic fields B' in equation (1). Therefore, an energy ramp can be applied in the spin dynamics simulation with POLE by an arbitrary function as the Lorentz factor $\gamma(t) = \gamma_{\text{ramp}}(t)$. We use additional terms in $\gamma(t)$ to approximate the longitudinal motion of the individual particles. This approach is debatable for the simulation of crossing of intrinsic resonances, since they are determined by the single particle trajectories and hence, the coupling of longitudinal motion with the transversal trajectories might not be negligible. For integer resonances (and aside from resonances), we achieve promising results with simple models for $\gamma(t)$. Several models were tested on the expected influence on spin motion, in order to get a computing time as short as possible. They implement synchrotron oscillations as well as synchrotron radiation.

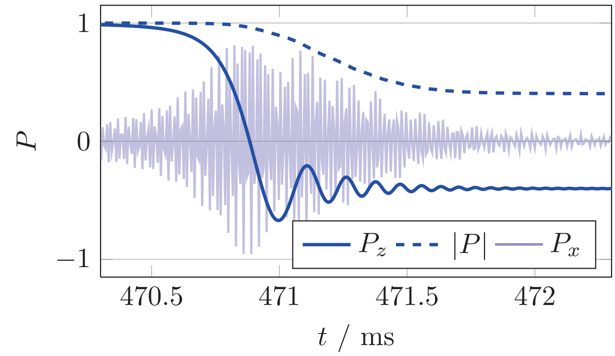

Figure 3: Simulation of spin diffusion during crossing of integer resonance $a\gamma = 7$

The synchrotron oscillation can be modeled by

$$\gamma_i(t) = \gamma_{\text{ramp}}(t) + A_i \cos(\omega_i t + \phi_i) \qquad (3)$$

for particle i, with A_i corresponding to the beam's energy width and ω_i appropriately distributed around its synchrotron frequency. Thus, the individual γ_i reproduce the energy distribution of the beam. Another model including several superposed oscillations for each particle was already studied [4] before developing POLE, but finally lead to very similar results. In Figure 2, a simulation of 100 spins crossing the integer resonance $a\gamma = 6$ with 4 GeV/s in the ELSA stretcher ring is shown. The vertical degree of polarization P_z and the vertical component of three exemplary spins S_i is plotted. P_z is decreased at the resonance energy. Additionally, it is decreased before and after the main resonance, due to the energy oscillation around the reference energy. These are the synchrotron side-bands of the resonance. For this reason, a complete spin-flip does not occur, since P is partially tilted in the horizontal plane and P_z is reduced.

Furthermore, synchrotron radiation can be included by random modulation of the phases $\phi_i(t)$ in equation (3), modeling stochastic energy changes of each particle. As a consequence, there is no fixed phase relation between the precessions of any two spins and therefore, any polarization component perpendicular to the precession axis vanishes with time. This spin diffusion can be observed in Figure 3. It shows a simulation of 1000 spins crossing an isolated resonance at $a\gamma = 7$ with 4 GeV/s. A horizontal polarization P_x occurs because of the increasing opening angle of the precession cone at the resonance. It vanishes after the

ISBN 978-3-95450-122-9

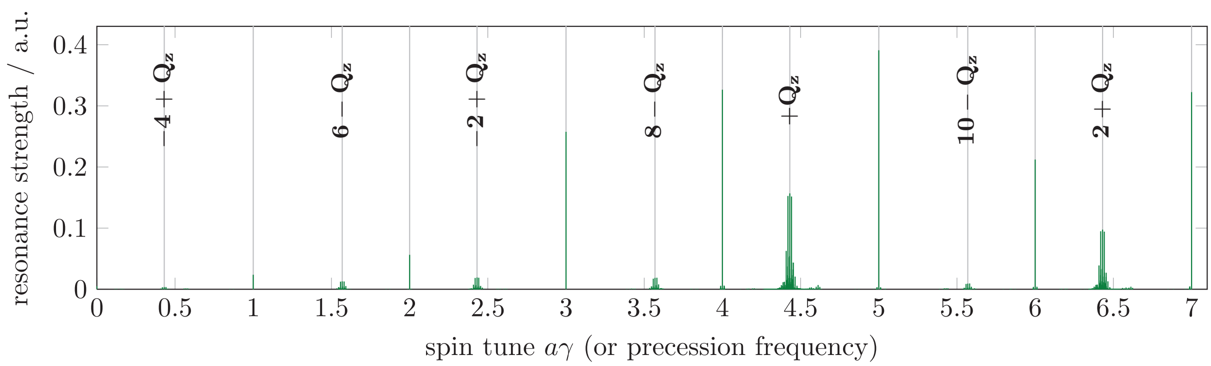

Figure 4: First results for resonance strengths in the ELSA stretcher ring predicted with POLE based on a MAD-X particle tracking for emittance 500 nmrad, vertical tune $Q_z = 4.431$ and periodicity $P = 2$

resonance crossing and thus also the absolute value of polarization $|P|$ is reduced. This makes it impossible for an electron beam to restore polarization once tilted out of the vertical direction.

Accordingly, POLE enables the computation of depolarization effects with a computing time in the order of minutes. However, there is a strong influence of the model parameters, such as amplitude and frequency of the phase modulations. Also limitations of the approach for intrinsic resonances have to be studied.

RESONANCE STRENGTHS

Recently, we implemented an additional method for the analysis of depolarizing resonances, which does not need any spin tracking and therefore has especially low computing time. The strengths of depolarizing resonances is calculated from the magnetic field distribution and can be used to compare and weight resonances for certain lattices or parameter sets.

The resonance strength is proportional to the amplitude of the horizontal field component in phase with the spin precession, which can be parametrized by the precession angle θ. This is not equivalent to the time dependent field $B'(t)$, since the precession is interrupted in-between the dipole magnets. Therefore, a resonance strength can not be identified as a single frequency component in the spectra described above, which are computed from a field distribution parametrized by the time t. This is also indicated in Figure 1, where otherwise only one frequency (e.g. $i = a\gamma$) would affect the polarization at all. For this reason, the field distribution $B'_x(t)$ must be transformed from the rest frame of the particle (t) into the rest frame of the spin (θ) to enable the computation of resonance strengths. To that end, the integral kicks (in mrad) for each magnet are calculated and summed up for all magnets corresponding to the same θ. Afterwards, the resulting field distribution $B'_x(\theta)$ is Fourier transformed. Its frequencies are the equivalent of the precession frequencies and thus correspond to the spin tune $a\gamma$ at which the resonance is excited.

Figure 4 shows a first result of the resonance strengths in the ELSA stretcher ring computed with POLE up to $a\gamma = 7$. All first order intrinsic resonances due to the vertical tune occur, as well as all integer resonances, which are caused by random torsion[1] of the dipoles applied in MAD-X.

The resonance strengths enable the comparison of depolarizing effects for different beam optics, closed orbit distortions or individual magnet settings. Their computation with POLE is part of the automatic field calculation based on a MAD-X lattice file described above. Therefore, POLE can also be used, e.g. for the analysis of resonance correction schemes or the design of new accelerators.

CONCLUSION

Recent results of POLE are in good accordance with comprehension. Besides, its development is an ongoing process. One essential aspect will be the comparison of simulation results with polarization measurements that can be performed at ELSA with various energies and ramping speeds, using beam manipulation tools, e.g. tune-jump quadrupoles and fast corrector magnets [5]. POLE offers an easy access to efficiently simulate spin dynamics and thus can also be a valuable tool for other facilities.

REFERENCES

[1] W. Hillert, et al. "Crossing of Depolarizing Resonances in Circular Electron Accelerators," IPAC2011, San Sebastián.

[2] MAD - Methodical Accelerator Design, madx.web.cern.ch

[3] V. Bargmann and L. Michel and V.L. Telegdi, Phys. Rev. Lett. 2.10 (1959) 435.

[4] O. Boldt et al., "Spinor Based Calculations of Depolarizing Effects in Circular Lepton Accelerators," IPAC2011, San Sebastián.

[5] J.-P. Thiry et al., "The New Orbit Correction System at ELSA," MOPME006, these proceedings.

[1] Rotation around beam axis, gaussian distributed around zero rotation with 1 mrad standard deviation.

ISBN 978-3-95450-122-9