EFFECTIVE EMITTANCE GROWTH IN BEAM WITH GAUSSIAN DENSITY PROFILE

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Abstract

Quasistationary sheet beam propagation was studied in case of particle nonlinear transverse oscillations. Equations for the beam envelope and rms emittance were obtained. The effective emittance is proved to increase with time. Beam parameters required for stationary equilibrium state existence are found.

INTRODUCTION

Effective emittance growth or filumentation appeared in channels due to different reasons makes the problem of matching more difficult. One of the filumentation reasons is nonlinearity of own beam electromagnetic fields. It is very important to find scaling laws of this phenomena.

Real beam does not have uniform charge density. In most practically important cases the beam density may be presented as Gaussian. If own beam electromagnetic fields prevalence above beam emittance, the effective emittance growth may be observed, resulting in particle losses despite the initial beam rms size matched with channel [1]. In papers [1,2] the nonuniform beam behaviour was investigated, but solutions obtained were not self-consistent. In paper [1] radius equation of 2nd order was obtained, but rms emittance time-dependence was calculated approximately. In paper [2] rms emittance dependence from time was not taken into account, because authors supposed beam cross-section oscillated in self-similar manner.

In this report solutions for sheet beam with nonuniform charge density are presented. Density distribution was aproximated as parabolic with density maximum on beam axis. Such distribution shape is sufficiently close to Gaussian and allows to consider the problem analitically and to obtain equations for beam envelope and beam rms emittance in self-consistent manner.

MODEL CALCULATIONS

Let consider quasistationary intensive sheet beam. If beam lifetime is significantly more than the time of transition processes in beam, one can describe the beam behaviour by means of smooth function R(z), where R(z) – beam tranverse size, z – longitudinal coordinate. In case of uniform charge density in beam cross-section we can use K-V-invariant [3] for beam description:

$$\mathrm{I} = (R'x - Rx')^2 + \frac{\varepsilon_0 x^2}{R^2} \qquad (1)$$

where x' is derivative of x with respect to z, R' – derivative of R with respect to z, ε_0 - beam rms emittance squared, x – transverse coordinate.

Let consider the beam with nonuniform density n(x,z):

$$\mathrm{n(x,z)} = \mathrm{a}_0(\mathrm{z}) - a_2(z)x^2 \qquad (2)$$

So particle transverse motion is described by equation:

$$\mathrm{x}'' = -\alpha_1(\mathrm{z}) + \alpha_3(z)x^3 \qquad (3)$$

Here $\alpha_1(\mathrm{z}) = \mathrm{k}\,\mathrm{a}_0(\mathrm{z})$, $\alpha_3(\mathrm{z}) = \mathrm{k}\mathrm{a}_2(\mathrm{z})/3$, $\mathrm{k} = 4\pi \mathrm{e}^2/\mathrm{mc}^2$.

We can built the integral for equation (3) with help of relation

$$\mathrm{x}'(\mathrm{x,z,I}) = \sum_{k=0}^{\infty} a_k(z,I)x^k \pm \Big(\sum_{k=0}^{\infty} b_k(z,I)x^k\Big)^{1/2} \qquad (4)$$

Let substitute (4) in (3) and neglect all summands with 5th power and higher. Then let introduce kinetic distribution function

$$\mathrm{f(I)} = 2\mathrm{n}_0\sigma(1-I).$$

Here n_0 – normalization constant, time-independent, σ – Heaviside function. We can use the distribution function differed from Maxwellian because charged particle beam is not thermodynamically equilibrium system. So one can obtain for the beam charge density:

$$\mathrm{n} = (\mathrm{n}_0/\mathrm{u})(1 - \varepsilon_0{}^2 x^2 / 2u^2)\sigma(R - |x|), \qquad (5)$$

where

$$\mathrm{R} = u\sqrt{2/\varepsilon_0}\,\Big(1 + \sqrt{1 + (\varepsilon_0' u^2)' u^2 / 3\varepsilon_0{}^2}\Big)^{-1/2}, \qquad (6)$$

function u is solution of equation

$$u'' = -\alpha_1(\mathrm{z})\mathrm{u} + \varepsilon_0(z)/u^3. \qquad (7)$$

Taking into account the whole current conversation, one can obtain for dimensionless beam radius and effective emittance next system [4]:

$$\begin{cases} \alpha'' + 1 = \beta / \alpha^3 \\ (\beta' \alpha^2)' = 12(1-\beta)/\alpha^2 \end{cases} \tag{8}$$

Here α and β are dimensionless radius and rms emittance respectively, $\alpha = u(l_0/l_1)^{2/3}$, $\beta = \varepsilon_0 l_0{}^2, l_1 = c/\omega_p$, $l_0 = J/2evn_0 L$, J is the whole beam current, L - the width of beam, ω_p is plasma frequency, corresponding to density value n_0, v is beam velocity.

In (8) rms emittance time-dependence was obtained in self-consistent manner, because f(I) built automatically satisfies to Vlasov equation, and relation for density (5), i.e. zero moment of distribution function, has a parabolic dependence from x.

System (8) has stationary equilibrium solution:

$$\alpha = \beta = 1 .$$

This solution corresponds to beam radius value

$$R = l_0 (c/\omega_p l_0)^{2/3},$$

and effective emittance value

$$\varepsilon_0 = \eta / l_0^{\ 2} .$$

Here η is normalizing constant.

To investigate equilibrium solution stability one have to build Lyapunov function. It was found that near the equilibrium stationary solution the equations of first approximation are not applicable, and according to Raus-Gurvitz criterium the equilibrium point is singular point of higher order.

The system (8) was solved by means of Runge-Kutta-Feldberg method of 4th order. Analysis of the results obtained shows that:

1. if β=1, then phase curve $\alpha'(\alpha)$ is closed and beam envelope performs harmonic oscillations with an amplitude depending from initial beam radius value.
2. if beam radius has initial value equaled to equilibrium stationary value, and rms emittance differs from equilibrium value, effective emittance growth is observed. The amplitude of the radius oscillation grows too.
3. if initial values of radius and rms emittance are significant different from equilibrium ones, the significant growth of the effective emittance is observed. The particle oscillations have two characteristic frequencies (Fig.1).

CONCLUSIONS

Nonlinear dynamics of sheet beam was studied in colissionless approximation because the time between binary collisions is more than the pulse duration. Transverse charge density nonuniformity lead to essentially nonlinear particle transverse oscillations, they can be described by Duffing equation with zero right part. Depending on nonlinearity power the growth of effective emittance can be observed at a time corresponding to about a quarter of the plasma wavelength. The beam parameters exist corresponding to the case when the effective emittance and the beam transverse size does not grow. The results obtained are valid under the condition $l_0 > c/\omega_p$, i.e. when minimum characteristic system size is more than maximum beam plasma wavelength.

In the problem studied above the envelope equation of 4th order was obtained. The coefficient of first radius derivative is not equal to zero differing from the case of envelope equation for uniform density beam. In the absence of energy dissipation in the channel, this fact means that the reason for the growth of the beam effective emittance with a nonuniform density profile is the transition of part of potential energy into kinetic energy of transverse motion of the particles.

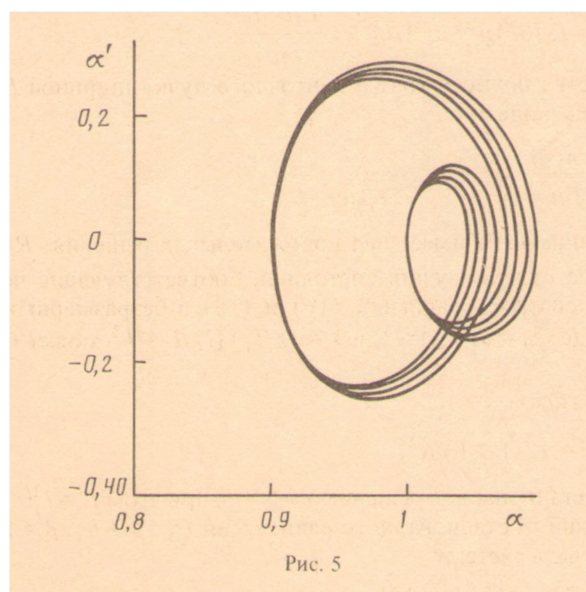

Figure 1: Phase curve for initial values $_0\alpha \neq 1$ and $_0\beta \neq 1$.

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