VARIOUS METHODS TO MEASURE THE BETATRON TUNE OF THE SYNCHROTRON

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Abstract

Generally in the synchrotron, the frequency of transverse oscillation of the bunched beam for each single turn (usually called betatron tune or just ”tune”) is one of fundamental controllable knobs to avoid the instability of the accelerator. In this report, it is not mentioned about the effect to the beam instability but it is focused to compare various methods to measure the betatron tune by using turn-by-turn transverse beam position. For the presentation, it is used the data of J-PARC (Japan Proton Accelerator Research Complex) Main Ring.

SINUSOIDAL FITTING BY SINGLE BPM

Here we treat linear betatron motion [1], the periodic position measured by the single BPM can be represented by the following equation,

$$x_n = A \cdot \sin(2\pi\nu_x \cdot n + B) + C \tag{1}$$

$$A = \sqrt{x_0^2 + (\alpha_x x_0 + \beta_x x_0')^2}$$

where n is turn number the bunch revolved in the ring, x_n is transverse beam position of turn n, A is amplitude which depend on initial position and momentum (x_0, x_0'), α_x, β_x are Twiss parameters at the location of the BPM, ν_x is the decimal part of the betatron tune, B is initial phase and C is the offset which is originated from closed orbit distortion(COD) and BPM error. Figure 2-(a) shows turn-by-turn horizontal positions measured in the injection phase of the J-PARC MR. For each 5 turns the data is fitted by Eq.(2) with least square method. Figure 2-(b) shows the tune variation obtained from the fitting. Advantage of this method is it is model independent. Disadvantage is that we have to set adequate initial parameters to get the successful result.

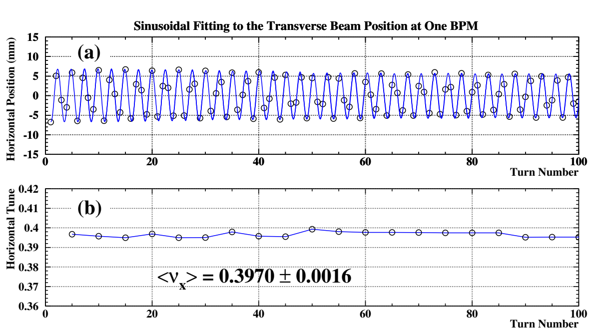

Figure 1: (a) Sinusoidal fitting to the beam position for each 5 turns. (b) Tune variation obtained from the fitting.

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FFT BY SINGLE BPM

FFT of Turn-by-Turn Position

When we perform discrete Fourier transformation(DFT) to the turn-by-turn positions, we get the power spectrum of the tune in which there are two peaks corresponding to ν_x and $1 - \nu_x$ solutions. For the sake of the efficient calculation, it is used fast Fourier transformation (FFT) with well known Cooley-Tukey algorithm [2]. When calculating FFT, the zero padding method is used to set the data size to 2^n. For example, in the Fig. 3, FFT spectrum $(0 < \nu_x < 0.5)$ of the 230 turns of beam positions(using the same data of Fig.2) are shown. In the Fig. 3-(a) the

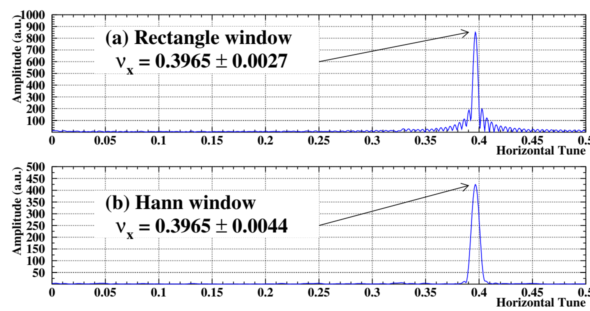

Figure 2: Spectrum by FFT of single BPM’s turn-by-turn positions. The measured ν_x is shown in the graph. The error value is estimated from HWHM of the peak. (a) FFT with rectangle window. (b) FFT with hann window.

rectangle window is applied, and in the Fig. 3-(b) hann window is applied. The error of tune is estimated by the half width at half maximum(HWHM) of the peak. Advantage of this method is it is model independent and furthermore we don’t need any adequate initial parameters. Thanks to this feature we can perform totally automatic calculation of the tune. Disadvantage is that the S/N of the FFT peak is dependent on the resolution of the turn-by-turn positions. For example of J-PARC MR, it is required at least 0.2mm of betatron amplitude to distinguish the FFT peak of the betatron motion from the noise floor.

FFT of Δ-signal of BPM

One of the most popular method to measure the tune is using FFT power spectrum of the BPM’s Δ-signal. Where Δ-signal means the waveform data of the difference of the voltage of a pair of pickups. Figure 4-(a) shows the example of the power spectrum (FFT with hann window) of Δ-signal (using the same data of Fig.2). Figure 4-(b) is the enlarged view around 20th harmonics of revolution frequency(f_{rev} = 185.76kHz) and it’s sideband peaks.

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The tune can be calculated by the following formula,

$$\nu_x = \frac{\Delta f}{f_{rev}} \qquad (2)$$

where Δf is the difference of frequency between the peak of n-th harmonics of f_{rev} and it's sideband peak which is originated from betatron motion. To avoid the fake peak of the noise in the lower frequency, relatively higher n is selected. Advantage of this method is it it model in-

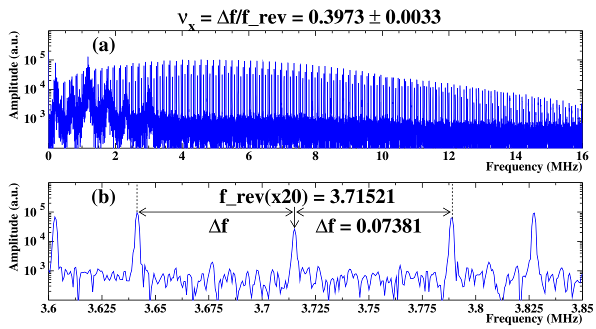

Figure 3: (a) Spectrum by FFT of Δ-signal of a pair of BPM's pickups. (b) Enlarged view around 20th harmonics of f_{rev} and it's sideband peaks. The obtained tune is shown at top of the graph, where error is estimated from HWHM of the sideband peak.

dependent and valid for small betatron amplitude. If we use dedicated equipment (Spectrum Analyzer) which can take many frames of FFT spectrum, we can measure longer range of tune shift. This is useful to investigate the instability caused by the resonance area of the tune. Disadvantage is that we need revolution frequency and expected value of the tune to find the sideband peak successfully.

PHASE ADVANCE BY SINGLE BPM

Using the transfer matrix between BPM(s_1) and BPM(s_2) the transverse position (x) and momentum ($x' = dx/ds$) can be described by Eq.(3)

$$\begin{bmatrix} x(s_2) \\ x'(s_2) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} x(s_1) \\ x'(s_1) \end{bmatrix} \qquad (3)$$

where s is the length along the closed orbit from an initial point, m_{ij} is the component of the transfer matrix between s_1 and s_2. The momentum x' can be determined as following,

$$x'(s_1) = -\frac{m_{11}}{m_{12}} x(s_1) + \frac{1}{m_{12}} x(s_2) \qquad (4)$$

$$x'(s_2) = \frac{m_{12}m_{21} - m_{11}m_{22}}{m_{12}} x(s_1) + \frac{m_{22}}{m_{12}} x(s_2) \qquad (5)$$

Twiss parameters (α_x, β_x) can be calculated using measured (x, x') as following [3],

$$\alpha_x = \frac{-<x \cdot x'>}{\sqrt{<x \cdot x><x' \cdot x'> - <x \cdot x'>^2}} \qquad (6)$$

$$\beta_x = \frac{<x \cdot x>}{\sqrt{<x \cdot x><x' \cdot x'> - <x \cdot x'>^2}} \qquad (7)$$

where $<x \cdot x'>$ is variance defined as bellow,

$$\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_n, \quad \bar{x'} = \frac{1}{N}\sum_{n=1}^{N} x'_n \qquad (8)$$

$$<x \cdot x'> = \frac{1}{N}\sum_{n=1}^{N} (\bar{x} - x_n)(\bar{x'} - x'_n) \qquad (9)$$

Using the above Twiss parameters, phase space in physical coordinate is transformed to normalized coordinate as following,

$$\begin{bmatrix} X \\ X' \end{bmatrix} = \begin{bmatrix} 1/\sqrt{\beta_x} & 0 \\ \alpha_x/\sqrt{\beta_x} & \sqrt{\beta_x} \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix} \qquad (10)$$

Figure 5 shows normalized phase space (using the same data of Fig.2). In terms of single BPM, a revolution of normalized phase space can be represented by Eq.(11).

$$\begin{bmatrix} X_{n+1} \\ X'_{n+1} \end{bmatrix} = \begin{bmatrix} \cos\Theta_x & \sin\Theta_x \\ -\sin\Theta_x & \cos\Theta_x \end{bmatrix} \begin{bmatrix} X_n \\ X'_n \end{bmatrix} \qquad (11)$$

where n is turn number and Θ_x is phase advance of one revolution in the normalized coordinate.

When we define the vector $\vec{X}_n = (X_n - X_0, X'_n - X'_0)$, where (X_0, X'_0) is offset originated from COD+BPM error, the decimal part of the tune can be calculated as following,

$$\nu_x = \theta_x/2\pi, \quad \theta_x = \cos^{-1}\frac{\vec{X}_n \cdot \vec{X}_{n+1}}{|\vec{X}_n| \cdot |\vec{X}_{n+1}|} \qquad (12)$$

The relation of Θ_x and θ_x is,

$$\begin{aligned} \Theta_x &= 2\pi \cdot N_x + \theta_x \qquad (13) \\ &= 2\pi(N_x + \nu_x) \qquad (14) \end{aligned}$$

where N_x is integer part of the tune.

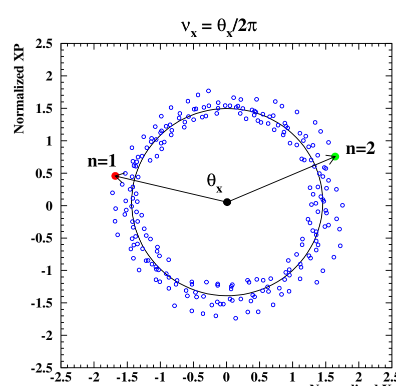

Figure 4: Normalized phase space(230 turns). Red point is 1st turn($\vec{X}_1$) and green point is 2nd turn($\vec{X}_2$). θ_x is the angle between $\vec{X}_1$ and $\vec{X}_2$.

Advantage of this method is we can get the tune for each single turn. Disadvantage is that it is model dependent.

When determining x' it is used the transfer matrix calculated from the expected values of Twiss parameters which is used to preset the magnets. To get the better resolution of θ_x it is required larger betatron amplitude than other methods. As shown in Fig. 6, the error of the obtained tune is larger than the other methods.

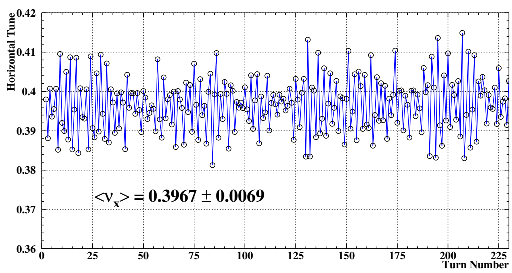

Figure 5: Variation of the tune obtained by the normalized phase advance of single BPM. The average number and error(RMS) is shown in the graph.

FFT BY ALL BPMS IN THE RING

If there are enough number of BPMs to reconstruct the complete orbit along the ring we can count the frequency of the betatron oscillation. The equation of the transverse motion along s-axis in physical coordinate is expressed as following,

$$x(s) = a\sqrt{\beta_x(s)}\cos(\varphi_x(s) + b) + c(s) \quad (15)$$

$$\varphi_x(s) = \int_0^s \frac{ds}{\beta_x(s)} \quad (16)$$

where a, b are constant, $c(s)$ is COD+BPM error and $\varphi_x(s)$ is phase advance along s-axis.

Figure 7-(a) shows the beam orbit reconstructed by 186 BPMs of J-PARC MR (using the same data of Fig.2).

When we define new coordinate,

$$X(s) = \frac{x(s) - c(s)}{\sqrt{\beta_x(s)}} \quad (17)$$

$$\Phi_x(s) = \frac{\varphi_x(s)}{\varphi_x(L)} \quad (18)$$

where L is the length of closed orbit of one revolution. Then Eq.(16) can be represented as following,

$$X(\Phi_x) = a' \cos(2\pi Q_x \Phi_x + b') \quad (19)$$

where a', b' are constant, Q_x is the tune including integer part. Figure 7-(b) shows the Normalized orbit along $\Phi_x(s)$.

FFT algorithm requires regular interval discrete data. Although usually intervals of BPMs in the ring are not equivalent. To make the data regular interval it is interpolated by quadratic function. In the Fig. 7-(b) the points marked purple triangle are the interpolated data.

Finally the data with regular interval points are transformed by FFT with hann window. The result is shown in Fig. 8. We can see the clear peak at Q_x.

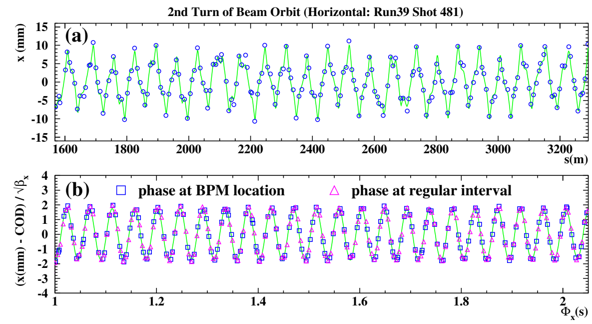

Figure 6: (a) Horizontal orbit by s-axis. Blue circle points are the BPM data. Green line is the expected orbit. (b) Normalized orbit by $\Phi_x(s)$. Blue square points are the BPM data, purple triangle points are interpolated points with regular interval.

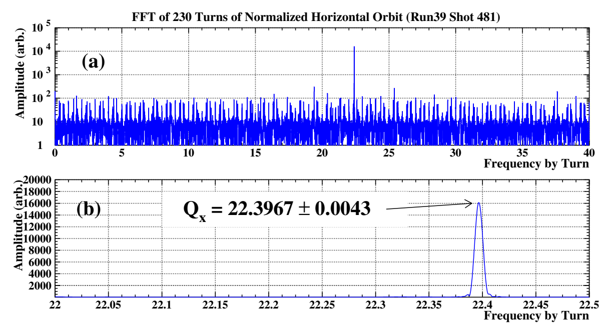

Figure 7: (a) FFT power spectrum of $X(\Phi_x)$. (b) Enlarged view around the Q_x. Obtained tune is shown in the graph. The error is estimated by HWHM of the peak.

Advantage of this method is we can measure the integer part of the tune and there is no ambiguity such as ν_x and $1 - \nu_x$. Disadvantage is that it is model dependent. Although once the boundary condition of $\Phi_x(L) = 1$ is satisfied the accuracy of the expected Twiss parameter is not so required. This feature is the key point of this method. For example the result of $Q_x = 22.3967 \pm 0.0043$ is consistent with the values by model independent methods in spite of presetting the tune $\varphi_x(L)/2\pi = 22.40$ at that time.

REFERENCES

[1] S.Y. Lee, "Accelerator Physics", World Scientific Publishing Co. Pte.Ltd., 1999.

[2] Cooley, James W., and John W. Tukey, "An algorithm for the machine calculation of complex Fourier series", Math. Comput. 19, 297-301 1965.

[3] K. Ohmi et. al., "Optics Measurement at the Interaction Point using Nearby Position Monitors in KEKB", Proceedings of IPAC10, Kyoto, Japan, May 2010.

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