AN ALTERNATIVE 1D MODEL FOR CSR WITH CHAMBER SHIELDING

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Abstract

The longitudinal CSR impedance is proportional to the radiation power spectrum of a point charge. The code CSRZ, which solves the parabolic equations in the frequency domain, is used to calculate the synchrotron radiation (SR) fields and impedance with rectangular chamber shielding. The most attractive merit of the method for CSR modeling lies in taking into account the realistic chamber shielding. The driven source in the code is rigid and has point-charge distribution in the longitudinal direction. It is found that the calculated impedance shows significant similarity to the spectra of measured infrared signals in electron rings. An alternative 1D model of utilizing the calculated SR impedance for simulation of CSR effects is proposed.

INTRODUCTION

Since the concept of the impedance was introduced to describe CSR effects [1, 2], there have been tremendous efforts in calculating the CSR wake fields and impedance analytically. In parallel to developments of analytical theories, various codes have also been developed to calculate the CSR wake fields and impedance. For a single bending magnet with shielding of beam chamber, up to now only numerical methods are available [3, 4, 5, 6].

In electron storage rings, the bunch length is usually in the order of a few millimeters, or down to sub-millimeters for dedicated THz radiation sources. Measurements in the CSR signals have shown fine structures in their spectra [7, 8, 9, 10]. It is obvious that the energetic spectrum due to synchrotron radiation (SR) of a moving point charge is linearly proportional to the longitudinal impedance

$$\left.\frac{dU(k)}{dk}\right|_{\rm s} = \frac{e^2 c}{\pi}\mathrm{Re}.Z_{\parallel SR}(k). \tag{1}$$

Here $Z_{\parallel SR}(k)$ is general, and represents impedance of a single or a series of bends. It is a function of the orbit curvature, magnet length, and chamber dimensions. For a bunched beam, the energetic spectrum is written as

$$\left.\frac{dU(k)}{dk}\right|_{\rm b} = \left[N + N(N-1)\left|\tilde{\lambda}(k)\right|^2\right]\left.\frac{dU(k)}{dk}\right|_{\rm s}, \tag{2}$$

where N is the bunch population. For a Gaussian bunch with line charge distribution, the beam spectrum is $\tilde{\lambda}(k) = \mathrm{Exp}[-k^2\sigma_z^2/2]$. The relation between predicted radiation spectrum and measured signal spectrum

$$\left.\frac{dU(k)}{dk}\right|_{\rm m} = T(k)\left.\frac{dU(k)}{dk}\right|_{\rm b}. \tag{3}$$

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Here $T(k)$ is a transfer function characterizing the response of the detection system (such as optical efficiency, and low-pass filter).

There are two kinds of studies on CSR detections: 1) Assume that $dU(k)/dk|_s$ and $T(k)$ are known, measuring $dU(k)/dk|_b$ for a bunch give the information of the bunchs profiles. It directs to developing new techniques for beam diagnostics [11]; 2) Assume that $dU(k)/dk|_s$, $T(k)$, and bunch profile are known, one can design and optimize a THz light source.

FEATURES OF CSRZ CODE

The code CSRZ inherits the main features of the method described in Refs. [3, 12]. The code solves a set of parabolic equations simplified from Maxwell's equations, i.e.

$$\frac{\partial \vec{E}_\perp}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp - \frac{1}{\epsilon_0}\nabla_\perp \rho_0 + \frac{2k^2 x}{R(s)}\vec{E}_\perp\right], \tag{4}$$

where $\vec{E}_\perp$ is the transverse electric field, and ϵ_0 is the vacuum permittivity. The curvature of the beam orbit defined by $R(s)$ is assumed to be an arbitrary functions of s. This assumption indicates the most significant feature of CSRZ. The vacuum chamber having an uniform rectangular cross-section adopts the same curvature of the beam trajectory. Freeing $R(s)$ allows CSRZ to investigate the CSR interference between consecutive bending magnets, even coherent wiggler or undulator radiation. With paraxial approximation, the longitudinal electric field is approximated by

$$E_s = \frac{i}{k}\left(\nabla_\perp \cdot \vec{E}_\perp - \mu_0 c J_s\right), \tag{5}$$

where μ_0 is the vacuum permeability, c is the speed of light in vacuum, and $J_s = \rho_0 c$ is the current density.

The original motivation of developing the independent code CSRZ was intended to study the multi-bend CSR interference in a storage ring. In CSRZ, the beam is assumed to have a point charge form in the longitudinal direction. Then the longitudinal CSR impedance is calculated by directly integrating E_s over s

$$Z_{\parallel SR}(k) = -\frac{1}{q}\int_0^\infty E_s(x_c, y_c)ds, \tag{6}$$

where (x_c, y_c) denotes the center of the beam in the transverse x–y plane. The code can also monitor the electric fields at specified position. Suppose that the bunch spectrum $\tilde{\lambda}(k)$ is known, then the time-domain fields can be calculated by

$$\vec{\mathcal{E}}(x, y, s; t) = \frac{c}{2\pi}\int_{-\infty}^{\infty}\tilde{\lambda}(k)\vec{E}(x, y, s; k)e^{ikz}dk \tag{7}$$

with $z = s - ct$.

ENERGETIC SPECTRUM FOR A SINGLE BEND

To demonstrate how the chamber shielding modify the SR energetic spectrum, we start from calculating the longitudinal SR impedance of a single bending magnet with constant bending radius in a curved chamber. The parameters are chosen as the same: the bending radius $R = 5$ m, and the horizontal and vertical dimensions of the chamber's cross-section are $a = 6$ cm and $b = 3$ cm, respectively. The magnet length is varied as $L_b = 0.5, 2, 8$ m. The impedance results are shown in Fig. 1. In the same figure, we also plot the result given by the parallel plates model [3]. Since the SR energetic spectrum is simply proportion to the real part of SR impedance, we can easily conclude that the energetic spectrum significantly deviates from the steady-state model. When $L_b = 0.5$ m, which indicates a short magnet, the spectrum is very smooth. When the magnet gets longer, the impedance becomes fluctuating and eventually results in a series of resonant peaks. The resonant peaks are actually correlated to the eigenmodes of the curved chamber [13]. When the curved chamber is long enough, some specific modes which fulfill the phase matching condition can be strongly excited by the beam and become dominant in the radiation fields. For more discussions, the readers are referred to Ref. [14].

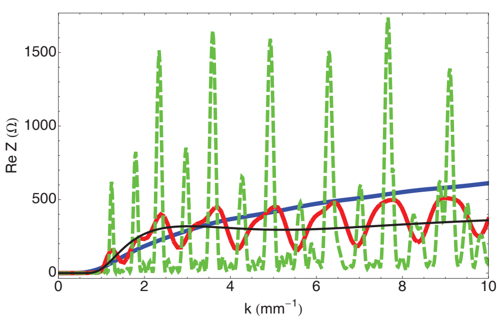

Figure 1: Real part of the CSR impedance for a single bending magnet. The impedances have been normalized by the magnet lengths. Blue solid line: $L_b = 0.5$ m; red solid line: $L_b = 2$ m; green dashed line: $L_b = 8$ m; black solid line: parallel plates model.

The measured SR power spectrum from the U12IR beam line at the NSLS-VUV ring consists of similar resonant peaks [7] as shown in the previous example. It was pointed out in Ref. [10] that these peaks should be attributed to the resonant modes of the curved vacuum chamber. Using the CSRZ code, it is straightforward to obtain the power spectrum by calculating the SR impedance. With parameters of $R = 1.91$ m, $L_b = 1.5$ m, $a = 80$ mm and $b = 42$ mm, the SR impedance is compared with the measure far IR spectrum in Fig. 2. Following Ref. [10], the beam orbit has an offset of 2 mm to the side of outer wall in the calculation. Besides the general agreement in the positions, as also obtained in Ref. [10], we also observe the excellent match in the widths of the peaks. The discrepancy in the amplitude of the spectra at low- and high-frequency parts can be explained by the transfer function $T(k)$ as defined in Eq. (3). In this case, the transfer function is unknown yet, but possibly is obtainable through field calculations by extending the CSRZ code to include the IR beamline.

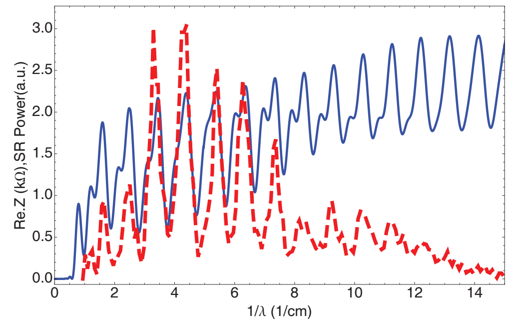

Figure 2: Comparison of the SR impedance and the measured incoherent SR power spectrum using interferometer at NSLS VUV ring. Blue solid line: Real part of the SR impedance; red dashed lines: Far IR power spectrum.

1D MODEL FOR SIMULATION OF CSR EFFECTS

It is possible to utilize the SR impedance in simulations of CSR effects. The general formula for calculating wake force from impedance is

$$F(z) = \frac{c}{2\pi} \int_{-\infty}^{\infty} Z_{\parallel SR}(k)\tilde{\lambda}(k)dk. \tag{8}$$

In the ultra-relativistic limit, one popular 1D model for the CSR wake potential per unit length inside a bending magnet is [15]

$$\frac{\partial W_{\parallel b}(z,s)}{\partial s} = T_1(z,R,s) + T_2(z,R,s), \tag{9}$$

where R is the bending radius, s is the orbit distance from the entrance of the magnet and z is the position within the bunch. The form of the above equation has been modified according to the notations of this paper. The quantities T_1 and T_2 represent the main part of CSR fields and the transient part at the entrance, respectively. They are defined by

$$T_1(z,R,s) = K \int_{z-z_L}^{z} \frac{d\lambda(z')}{dz'} \left(\frac{1}{z-z'}\right)^{1/3} dz', \tag{10}$$

$$T_2(z,R,s) = K \frac{\lambda(z-z_L) - \lambda(z-4z_L)}{z_L^{1/3}}, \tag{11}$$

where $\lambda(z')$ is the linear charge density, $z_L(R,s) = s^3/(24R^2)$ is the slippage length and the parameter K is defined as follows

$$K(R) = -\frac{1}{4\pi\epsilon_0} \frac{2}{(3R^2)^{1/3}}. \tag{12}$$

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Suppose that the bunch distribution $\lambda(z)$ equals Dirac delta function as a result of point charge, i.e. $\lambda(z) = \delta(z)$. Applying the delta function to Eq. (9) yields the s-dependent wake function of

$$\frac{\partial w_{\parallel b}(z,s)}{\partial s} = -K \times \left[\frac{1}{3}\left(\frac{1}{z}\right)^{4/3} H(z)H(z_L - z) + \frac{\delta(z-4z_L)}{z_L^{1/3}}\right], \quad (13)$$

where $H(z)$ is Heaviside step function defined by

$$H(z) = \int_{-\infty}^{z} \delta(t)\, dt. \quad (14)$$

Applying Laplace transform to the above wake function gives the s-dependent CSR impedance as follows

$$\frac{\partial Z_{\parallel b}(k,s)}{\partial s} = K\left\{\frac{e^{-4ikz_L}}{z_L^{1/3}} - (ik)^{1/3} \times \left[\Gamma\left(\frac{2}{3}\right) + \frac{1}{3}\Gamma\left(-\frac{1}{3}, ikz_L\right)\right]\right\}, \quad (15)$$

where $\Gamma(z)$ is Euler gamma function and $\Gamma(a,z)$ is incomplete gamma function, respectively defined by $\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}\, dt$, and $\Gamma(a,z) = \int_z^\infty t^{a-1}e^{-t}\, dt$. It is interesting to compare Eq. (15) with Eq. (6): Taking the derivative over s at both sides of Eq. (6) gives

$$\frac{\partial Z_{\parallel SR}(k,s)}{\partial s} = -\frac{1}{q}E_s(x_c, y_c, s). \quad (16)$$

Apparently the s-dependent longitudinal impedance is exactly the longitudinal fields being calculated by the CSRZ code.

Utilizing Eq. (9) to calculate CSR forces requires the derivative of the linear density, but using impedance only require the spectrum of the charge density. In the later case, the singularity near at $z - z' = 0$ due to the term of $(z - z')^{-1/3}$ in Eq. (10) is relaxed. Extremely high frequency CSR impedance related to incoherent synchrotron radiation (ISR) can be naturally cut off. Finally, the chamber shielding is naturally included if the SR impedance from numerical calculations with chamber shielding considered.

The capability of CSRZ is mainly limited by the mesh sizes in the $x - y$ plane and step size in the s direction. With explicit discretization scheme, the mesh and step sizes should be proportional to $k^{-2/3}$ and $k^{-1/3}$ respectively, due to necessary numerical stability conditions [3]. To obtain CSR impedance at extremely high frequency, the computations become unacceptably expensive. But the high-frequency impedance can be approximated by analytical formulas. Along the beam orbit, which may be formed by multi bends interleaved with drifts, the vacuum chamber is sliced into a series of segments. The low-frequency CSR impedance for each segment, in this case chamber shielding is significant, is obtained by numerical calculations. The high-frequency CSR impedance, in this case chamber shielding is negligible, is estimated by the analytical model of Eq. (15). The wake kick at each segment is computed via inverse Fourier transform of the impedance convolved the the beam spectrum. The most attractive merit of the method for CSR modeling lies in taking into account the realistic chamber shielding.

DISCUSSIONS

The 1D model for synchrotron radiation is extended in this paper. The SR impedance and power spectrum can be taken as linear response of the curved chamber. With information of the beam spectrum provided, the SR impedance can be used to explain the measured power spectrum. Or vise versa, with measured power spectrum, the information of the bunch profile can be extracted.

The SR impedance is distributed along the orbit. For magnet with finite length, it is strongly s-dependent. It is possible to utilize the well-defined SR impedance in simulations of CSR effects. The implementation of the scheme proposed in the paper to numerical simulations is under investigation.

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